Thermal Properties of Dense Matter

The Homogeneous Phase

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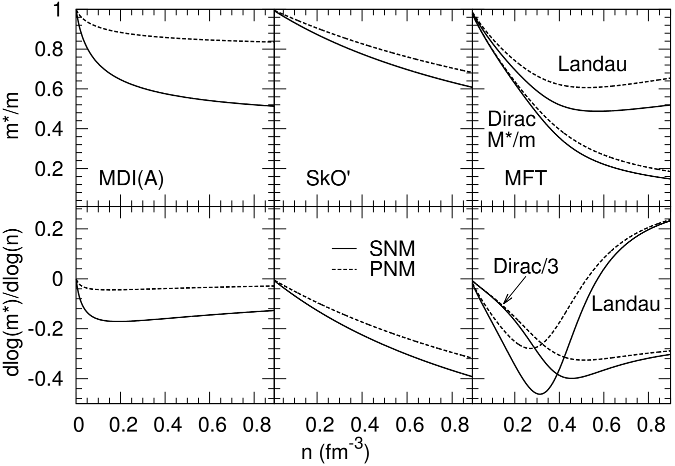
The Models



- For n large, $\frac{dm^*}{dn} \simeq 0$
- $m^* = \frac{m}{1+\beta(x)n}$
- $m^* = E_F^* = (p_F^2 + M^{*2})^{1/2}$
- Minimum at n s.t. $\frac{p_F}{M^*} + \frac{dM^*}{dp_F} = 0$

Degenerate Limit Thermodynamics

- Interaction switched-on adiabatically
- Entropy density and number density maintain their free Fermi-gas forms:

$$s = \frac{1}{V}\sum_p [f_p \ln f_p + (1-f_p)\ln(1-f_p)]$$

$$n = \frac{1}{V}\sum_p f_p(T)$$

- $\int d\varepsilon \frac{\delta s}{\delta T} \quad \Rightarrow \quad s = 2anT$

$$a = \frac{\pi^2}{2}\frac{m^*}{p_F^2}$$ level density parameter

Degenerate Limit Thermodynamics

Rest of thermodynamics via Maxwell's relations or other identities:

- Energy density
$$\frac{d\varepsilon}{ds} = T$$
$$\varepsilon_{th} = anT^2$$
- Pressure
$$\frac{dP}{dT} = -n^2 \frac{d(s/n)}{dn}$$
$$P_{th} = \frac{2}{3} nQT^2 \quad ; \quad Q = 1 - \frac{3}{2} \frac{n}{m^*} \frac{dm^*}{dn}$$
- Chemical potential
$$\frac{d\mu}{dT} = -\frac{ds}{dn}$$
$$\mu(n, T) = -a \left(1 - \frac{2Q}{3}\right) T^2$$
- Specific Heats
$$C_V = T \frac{d(s/n)}{dT}\bigg|_n = 2aT$$
$$C_P = T \frac{d(s/n)}{dT}\bigg|_P = 2aT$$

Degenerate Limit Thermodynamics Beyond Leading Order

Degenerate limit implications:

- $\eta = \frac{\mu - \epsilon(p=0)}{T} \gg 1 \Rightarrow$ Sommerfeld
- $\epsilon = \frac{p^2}{2m} + U(n,p;T) \rightarrow \frac{p^2}{2m} + U(n,p;0)$

For a general $U(n,p)$, define an effective mass function

$$\mathcal{M}(n,p) = m\left[1 + \frac{m}{p}\left.\frac{\partial U(n,p)}{\partial p}\right|_n\right]^{-1} .$$

Relation to Landau m^* : $\mathcal{M}(n, p = p_F) = m^*$
Applying the Sommerfeld expansion to the integral of the entropy density gives

$$s = 2anT - \frac{16}{5\pi^2}a^3 nT^3(1 - L_F)$$

$$L_F = \frac{7}{12}p_F^2\frac{\mathcal{M}_F'^2}{m^{*2}} + \frac{7}{12}p_F^2\frac{\mathcal{M}_F''}{m^*} + \frac{3}{4}p_F\frac{\mathcal{M}_F'}{m^*} \ ; \ \mathcal{M}_F' \equiv \left.\frac{\partial\mathcal{M}(n,p)}{\partial p}\right|_{p=p_F}$$

Degenerate Limit Thermodynamics Beyond Leading Order

- Thermal Energy:

$$E_{th} = aT^2 + \frac{12}{5\pi^2} a^3 T^4 (1 - L_F)$$

- Thermal Pressure:

$$P_{th} = \frac{2}{3} anQT^2 - \frac{8}{5\pi^2} a^3 nQT^4 \left(1 - L_F + \frac{n}{2Q}\frac{dL_F}{dn}\right)$$

- Thermal Chemical Potential:

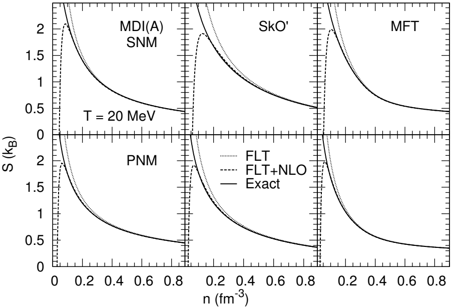
$$\mu_{th} = -a\left(1 - \frac{2Q}{3}\right) T^2 + \frac{4}{5\pi^2} a^3 T^4 \left[(1 - L_F)(1 - 2Q) - n\frac{dL_F}{dn}\right]$$

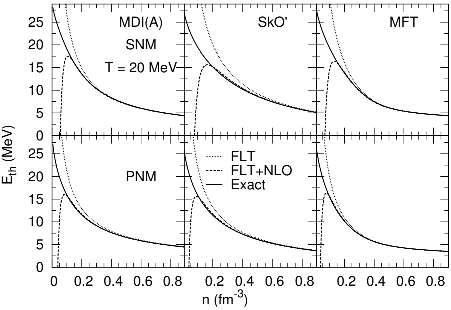
- Specific Heat at constant volume:

$$C_V = 2aT + \frac{48}{5\pi^2} a^3 T^3 (1 - L_F)$$

- Specific Heat at constant pressure: $C_P = C_V + \frac{T}{n^2} \frac{\left(\left.\frac{\partial P_{th}}{\partial T}\right|_n\right)^2}{\left.\frac{\partial P}{\partial n}\right|_T}$

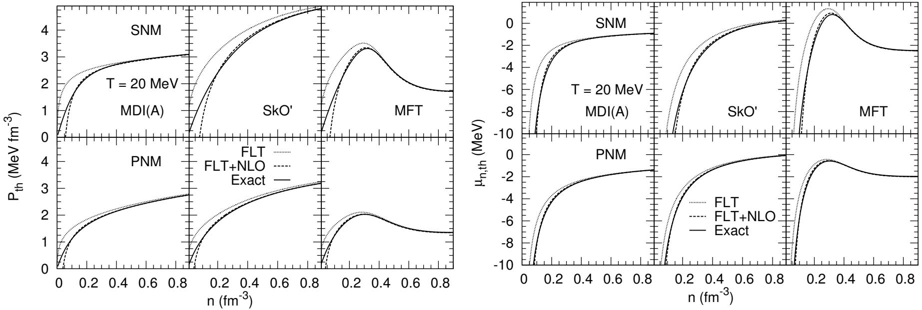
Results: S and E_{th}





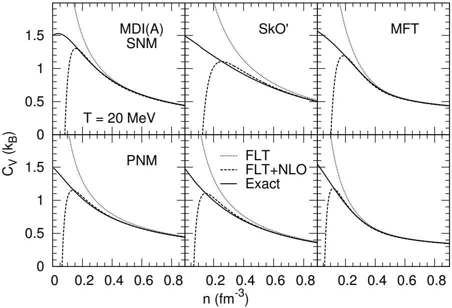
- The three models produce quantitatively similar results.
- Agreement with exact results extended to $n \simeq 0.1$ fm^{-3}.
- Better agreement for PNM than for SNM.

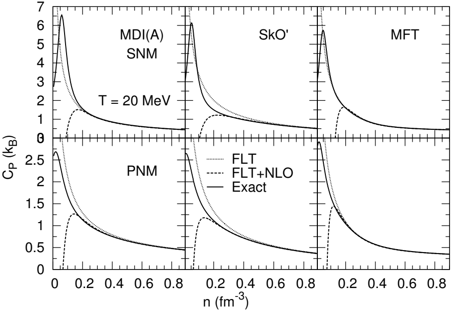
Results: P_{th} and μ_{th}



- Model dependence is evident- due to $\frac{dm^*}{dn}$.
- Agreement with exact results extended to $n \simeq 0.1$ fm^{-3}.
- Better agreement for PNM than for SNM.

Results: Specific Heats





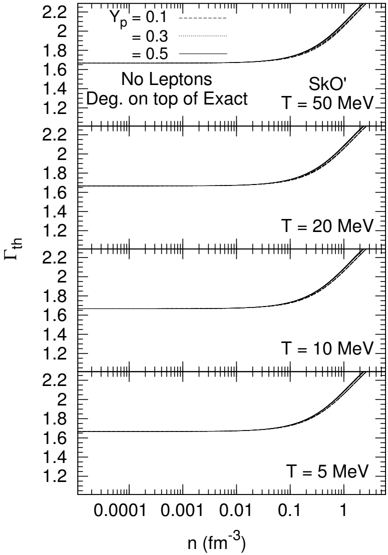
- The MDI and MFT C_V exceed the classical value of 1.5 in the nondegenerate limit. In this regime the T-dependence of the spectrum becomes important.
- The peaks in C_P are due to the proximity to the nuclear liquid-gas phase transition.

- The EOS is necessary for a complete description of the dynamics of a merger.
- The EOS is relevant to:
  - GW frequency
  - Size, type, and lifetime of remnant
- EOSs used in simulations:
  - Realistic
  - Polytropic, $P = \kappa n^{\Gamma_s}$
  - Ideal Fluid, $P_{th} = (\Gamma_{th} - 1)\varepsilon_{th}$

Γ_{th}-General Considerations

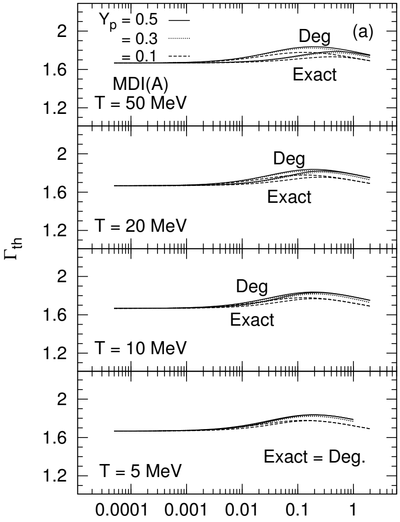
- $\Gamma_{th} = 1 + \frac{P_{th}}{\varepsilon_{th}}$
- Degenerate Limit
  - Nonrelativistic
    $\Gamma_{th} = 1 + \frac{2}{3}Q - \frac{4}{5\pi^2}a^2 n T^2 \frac{dL_F}{dn} \stackrel{n\to 0}{\longrightarrow} \frac{5}{3}$
    $Q = 1 + \frac{3}{2}\frac{n}{m^*}\frac{dm^*}{dn}$
  - Relativistic
    $\Gamma_{th} = 1 + \frac{Q}{3} + \frac{8}{15\pi^2}a^2 T^2 (1-Q)\left(L_F - \frac{5}{3}\frac{p_F^4}{E_F^{*4}}\right) \stackrel{n\to\infty}{\longrightarrow} \frac{4}{3}$
    $Q = 1 + \left(\frac{M^*}{E_F^*}\right)^2 \left(1 - \frac{3n}{M^*}\frac{dM^*}{dn}\right)$
- CC, B. Muccioli, M. Prakash & J.M. Lattimer, arXiv:1504.03982

Γ_{th}-SkO'



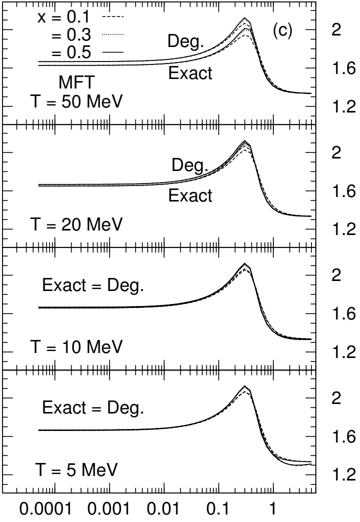
- No T dependence:
  For Skyrme models,
  $P_{th}(n, T) = P_{th}^{id}(n, T; m^*)Q$
  $\varepsilon_{th}(n, T) = \varepsilon_{th}^{id}(n, T; m^*)$
  $\frac{P_{th}^{id}}{\varepsilon_{th}^{id}} = \frac{2}{3}$
  $\Gamma_{th} = \frac{8}{3} - \frac{m^*}{m}$
- Weak x dependence
- Sharp rise in homogeneous phase

Γ_{th}-MDI



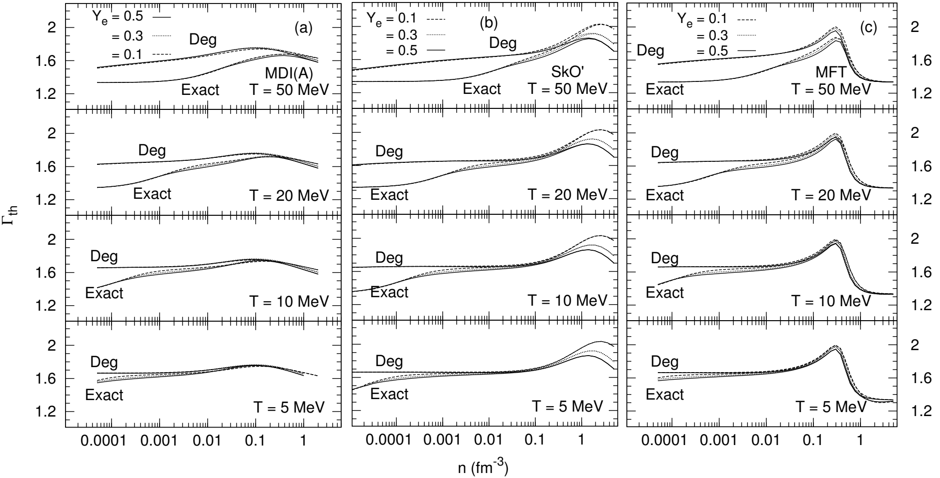
- Weak T dependence
- Weak x dependence
- Maximum around $n \sim n_0$:
  $\frac{d\Gamma_{th}}{dn} = 0 \Rightarrow$
  $\frac{dm^*}{dn}\left(1 - \frac{n}{m^*}\frac{dm^*}{dn}\right) + n\frac{d^2m^*}{dn^2} = 0$

Γ_{th}-MFT



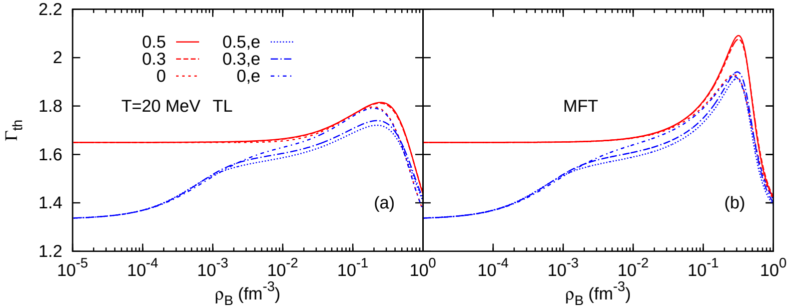
- Weak T dependence
- Weak x dependence
- Maximum around $2n_0$

Γ_{th}-Leptons and photons included



- $\Gamma_{th} \xrightarrow{n \to 0} \frac{4}{3}$
- Maximum even for Skyrme

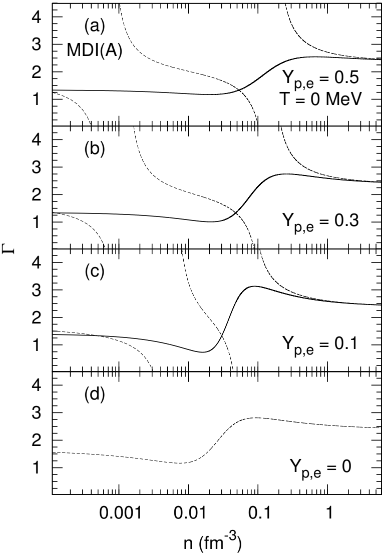
Γ_{th}-Beyond MFT



- X. Zhang & M. Prakash, work in progress
- Finite range effects via 2-loop calculation
- Lower peak relative to a similarly-calibrated MFT

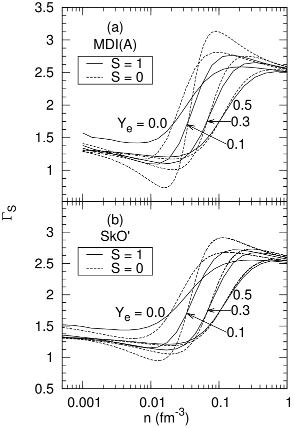
Γ_S-General Comments

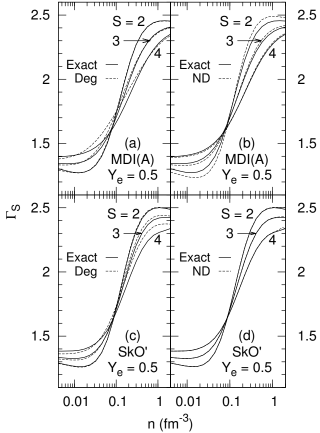
- $\Gamma_S(n,S) = \frac{\partial \ln P}{\partial \ln n}\Big|_S = \frac{n}{P}\ \frac{\partial P}{\partial n}\Big|_S$
- $\Gamma_S(n,T) = \frac{C_P}{C_V}\frac{n}{P}\ \frac{\partial P}{\partial n}\Big|_T$
- Relation to sound speed, c_s:
  $\left(\frac{c_s}{c}\right)^2 = \Gamma_S\ \frac{P}{h+mn}$
- Degenerate Limit:
  $\Gamma_S(n,S) = \frac{n}{P_0+\frac{nQS^2}{6a}}\left[\frac{dP_0}{dn} + \frac{QS^2}{6a}\left(1+\frac{2}{3}Q + \frac{Q}{n}\frac{dQ}{dn}\right)\right]$
  $\Gamma_S(n,T) = \frac{n}{P_0+P_{th}}\left[\frac{K}{9} + \frac{\partial P_{th}}{\partial n}\Big|_T + \frac{T}{n^2 C_V}\left(\frac{\partial P_{th}}{\partial T}\Big|_n\right)^2\right]$
  (using $C_P = C_V + \frac{T}{n^2}\frac{\left(\frac{\partial P}{\partial T}\big|_n\right)^2}{\frac{\partial P}{\partial n}\big|_T}$)



- Only nucleons $\Rightarrow$ mechanical instability
- With leptons, nuclear matter is stable
- At $S=0$, $P_l \sim n^{4/3}$
  $\Rightarrow\ \Gamma_{S=0} = \left(\frac{4}{3} + \frac{n}{P_l}\frac{dP_b}{dn}\right)\left(1+\frac{P_b}{P_l}\right)^{-1}$
  For $n \simeq n_0$, $\quad P_b(n,\alpha) \simeq$
  $\frac{n^2(n-n_0)}{9n_0^2}\left\{K_0 + \alpha^2\left[\frac{3n_0L_v}{(n-n_0)} + K_v\right]\right\}$
- SNM ($\alpha=0$, $P_b=0$)
  $\Gamma_{S=0}(n=n_0) = \frac{4}{3} + \frac{K_0}{9n_0P_l} \sim 2.1,$
  $n^{(sp)} = \frac{2n_0}{3}$
- PNM ($\alpha=1$, $P_l=0$)
  $\Gamma_{S=0}(n=n_0) = 2 + \frac{K_0+K_v}{3L_v} \sim 2.8$
  $n^{(sp)} = n_0\ \frac{3(K_0+K_v)}{2(K_0+K_v)-6L_v} < 0$

$\Gamma_{S\neq0}$





- Low n, $\Gamma_S > \Gamma_0$
  High n, $\Gamma_S < \Gamma_0$

- n_X such that
  $\frac{1}{P_0}\frac{dP_0}{dn} = \frac{1}{P_{th}} \left.\frac{\partial P_{th}}{\partial n}\right|_S$
  (indep. of S in Deg. Limit)

Conclusions

- m^* is crucial in the determination of thermal effects.
- Both Γ_{th} and Γ_S depend weakly on T but their density dependence cannot be ignored.
- Finite-range effects suppress the density dependence of Γ_{th}.
- Finite entropy restricts the range of values Γ_S can attain.
- Leptons are essential to the stability of nuclear matter.