Parameter study of r-process lanthanide production and heating rates in kilonovae with SkyNet

arXiv:1508.03133

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Caltech

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Solar System Abundances

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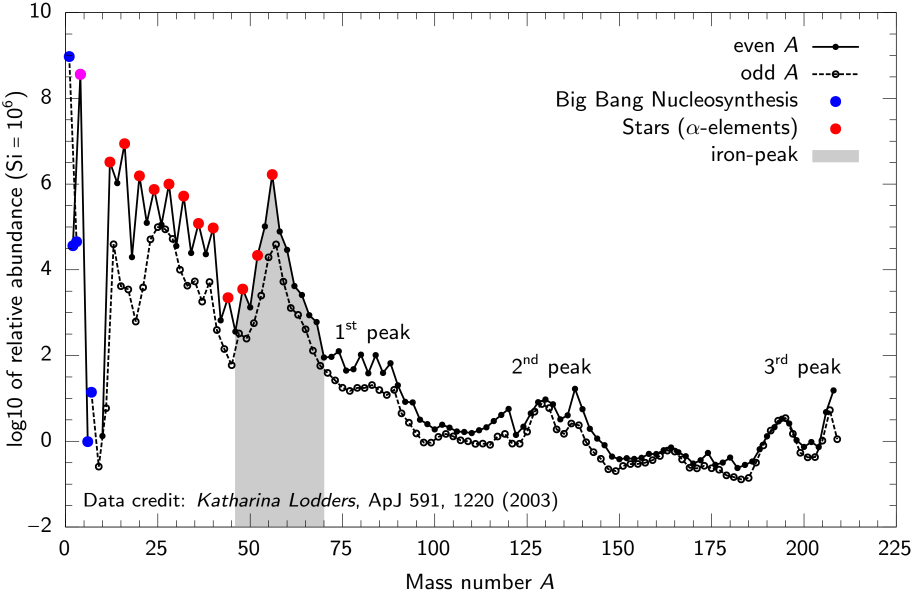

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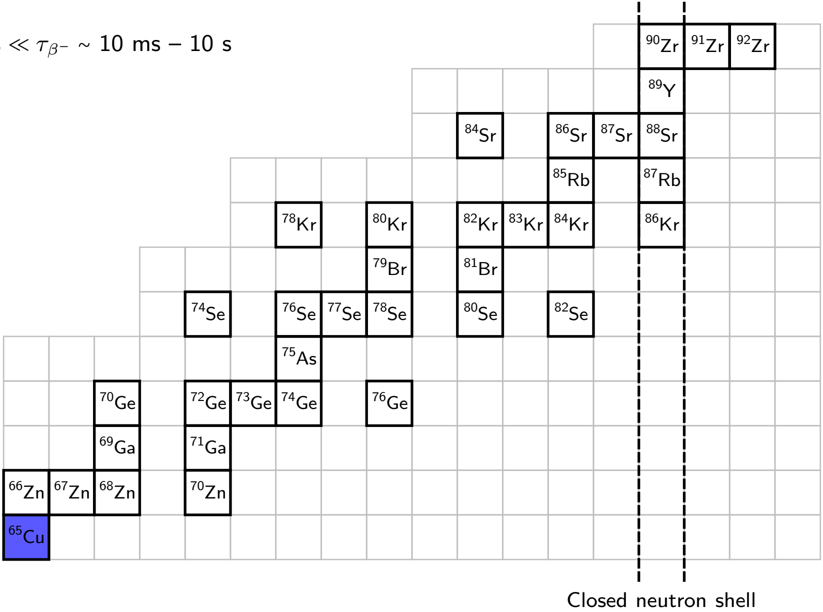
$\tau_n \ll \tau_{\beta^-} \sim 10$ ms – 10 s



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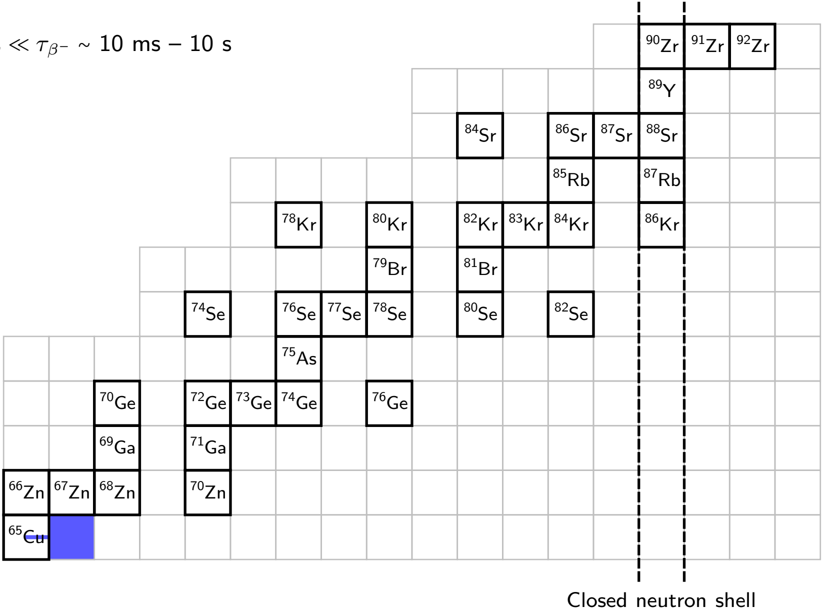

Caltech

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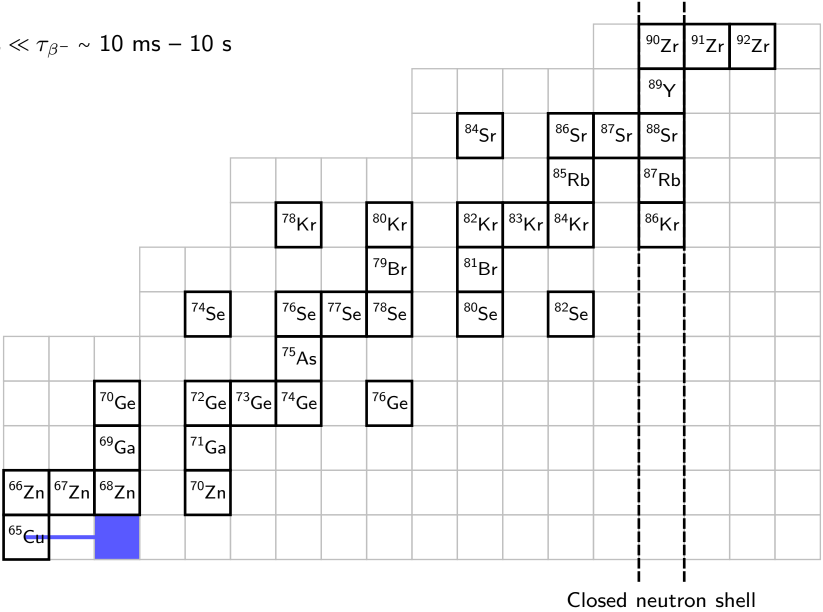

The r-process

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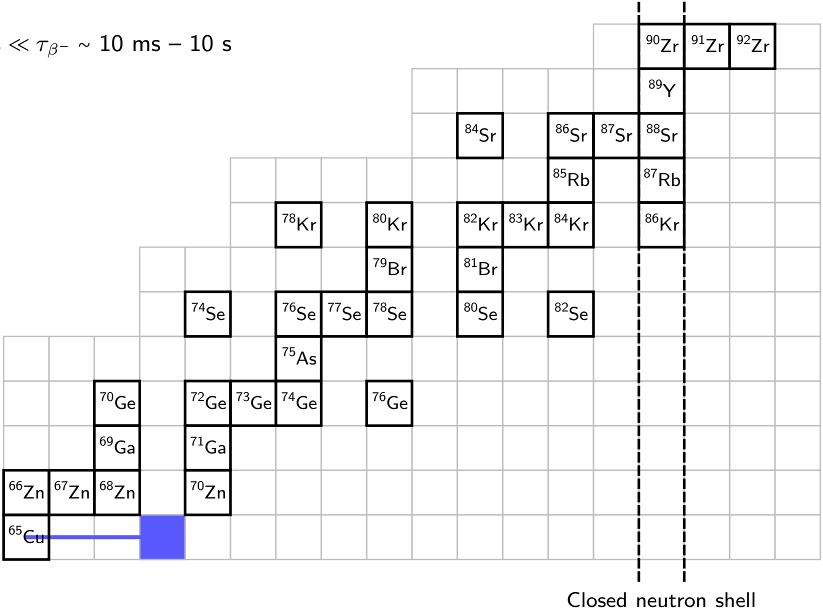

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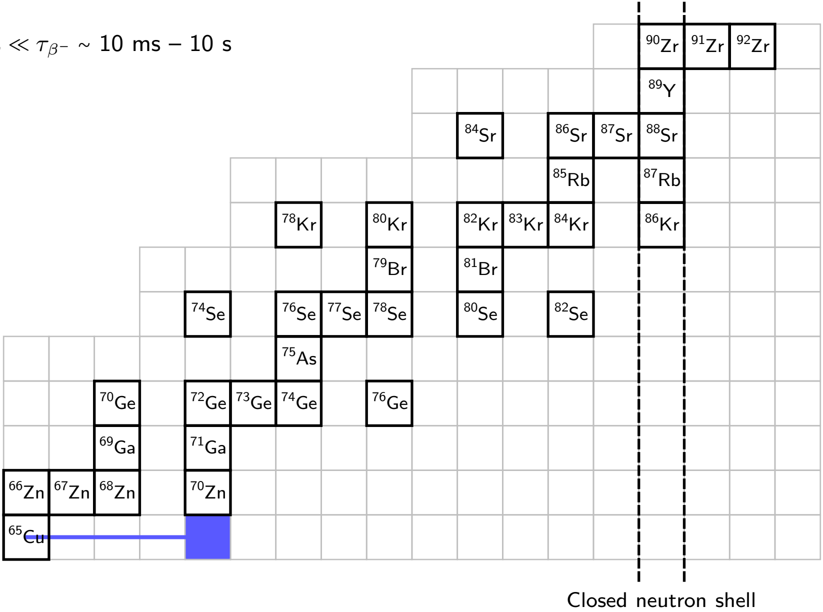

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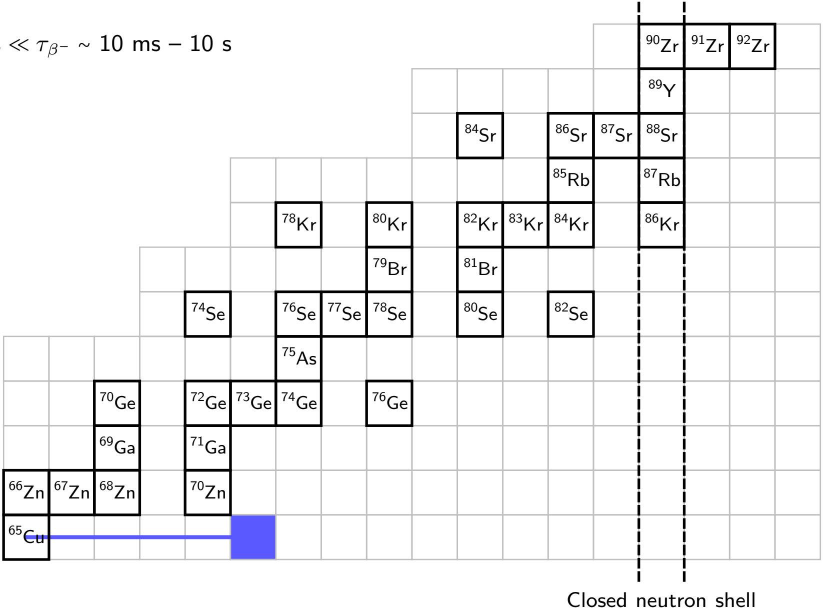

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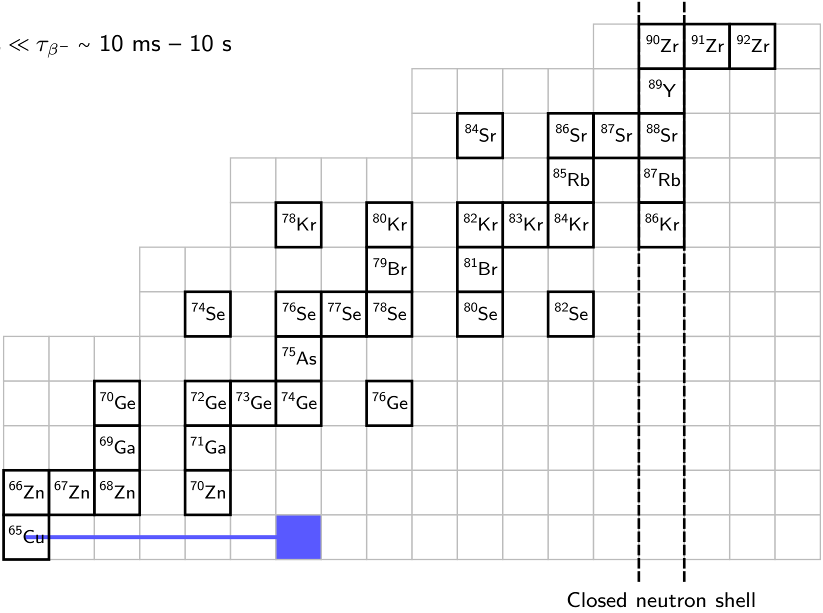

Caltech

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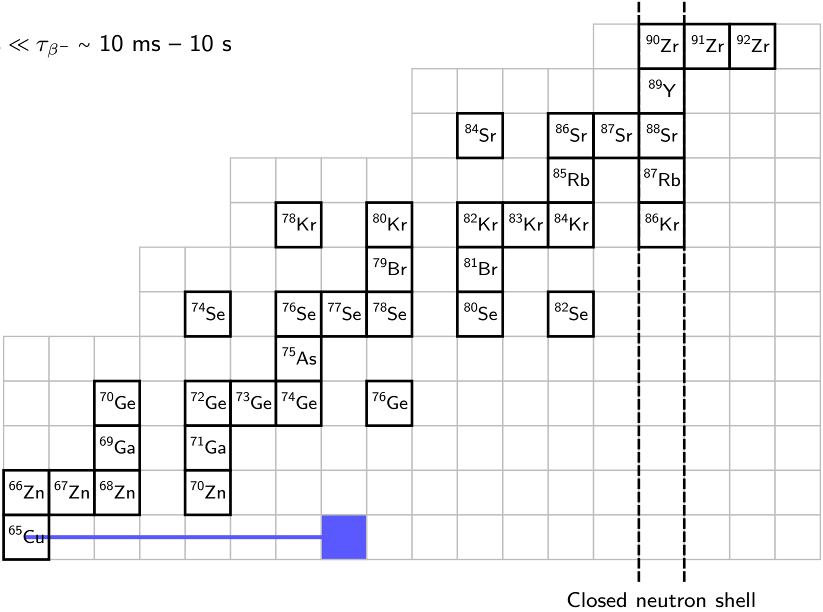

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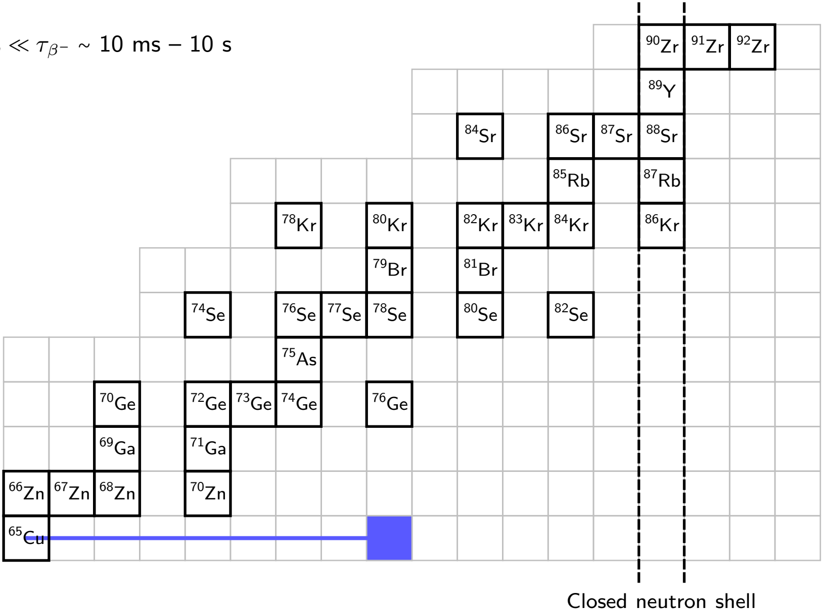

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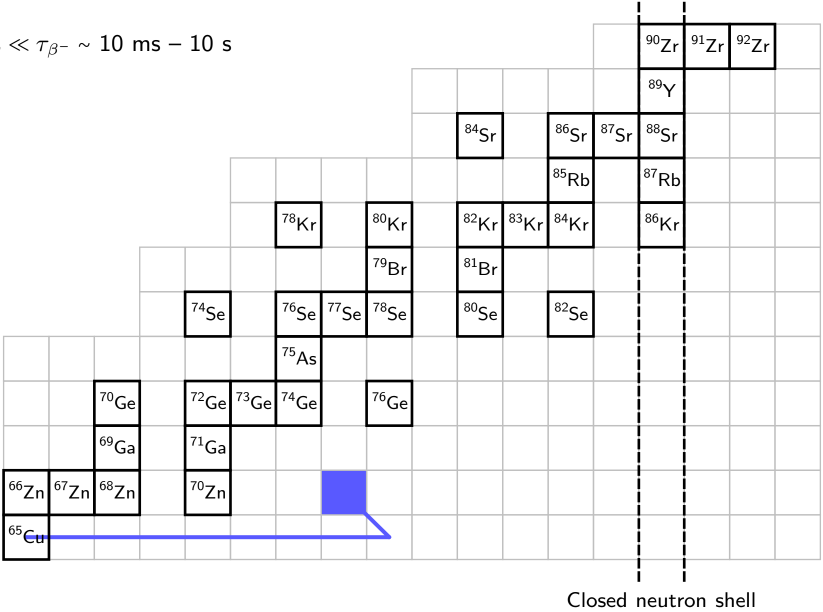

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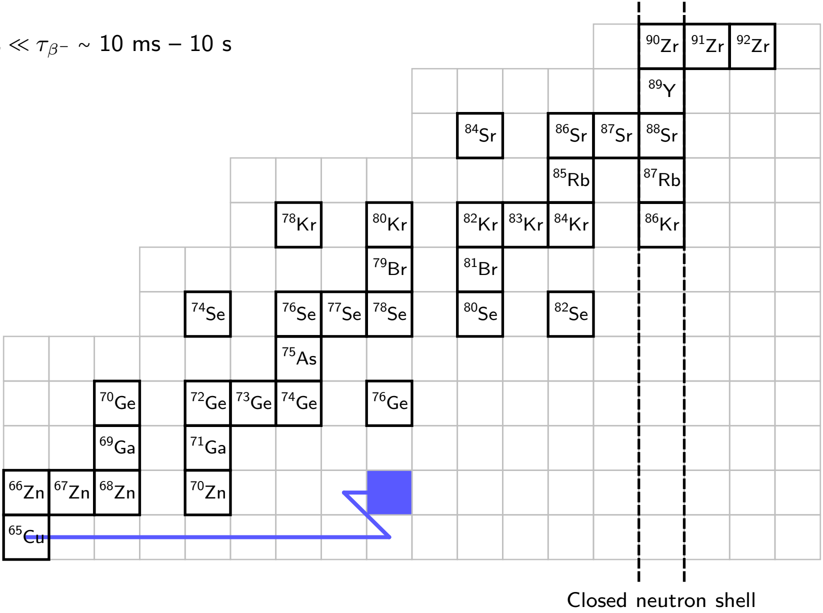

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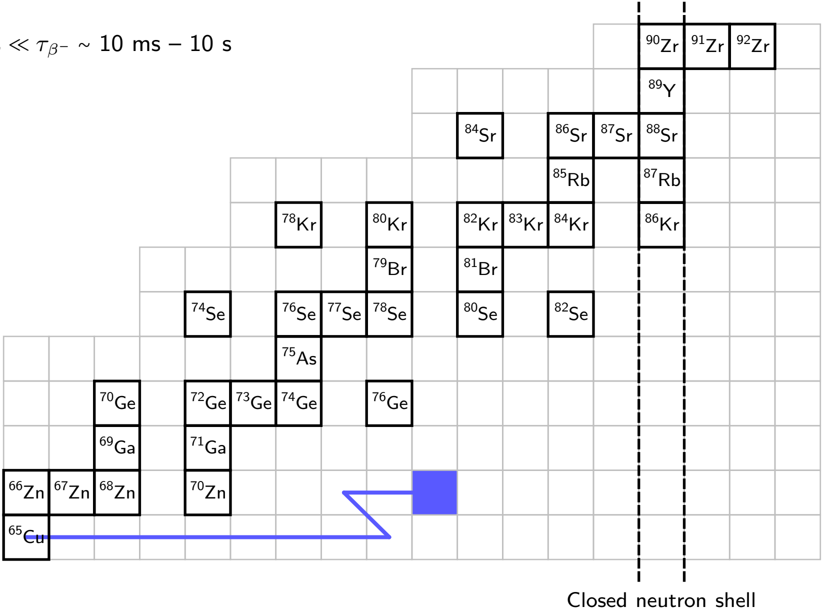

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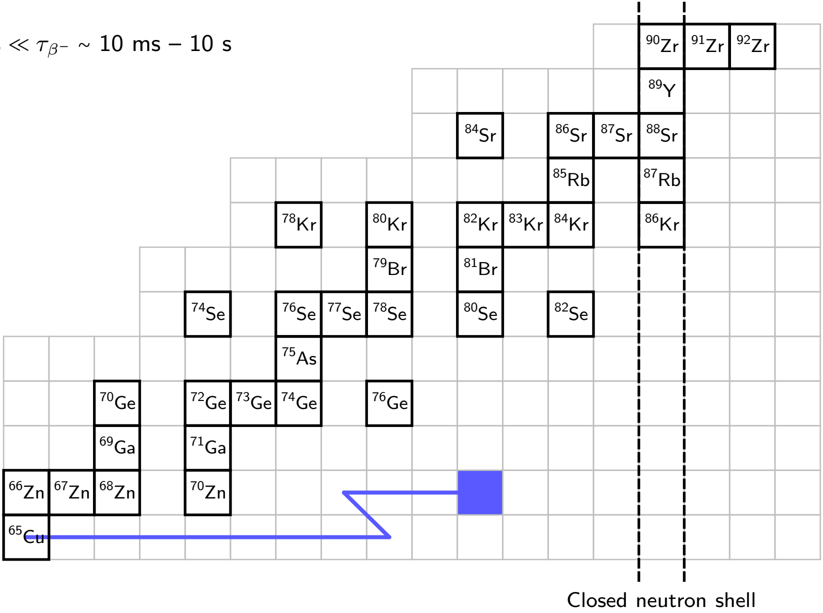

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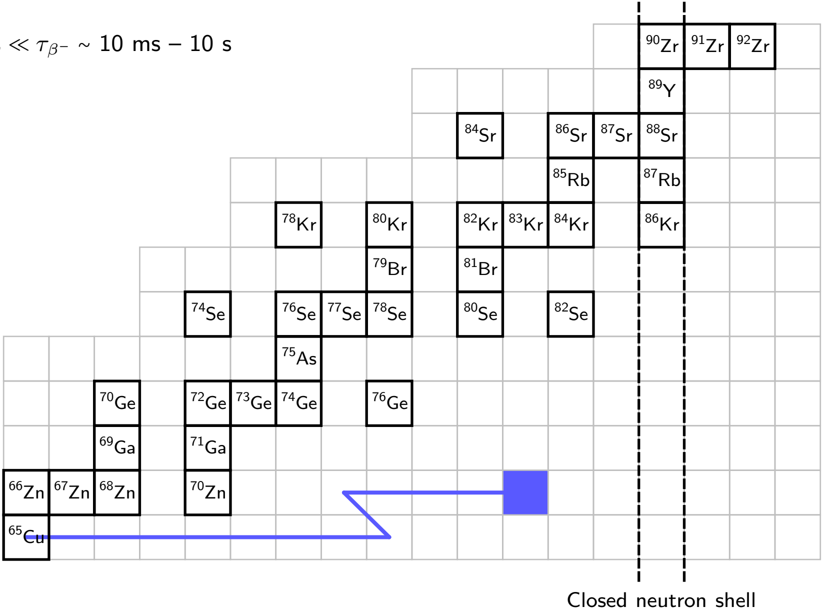

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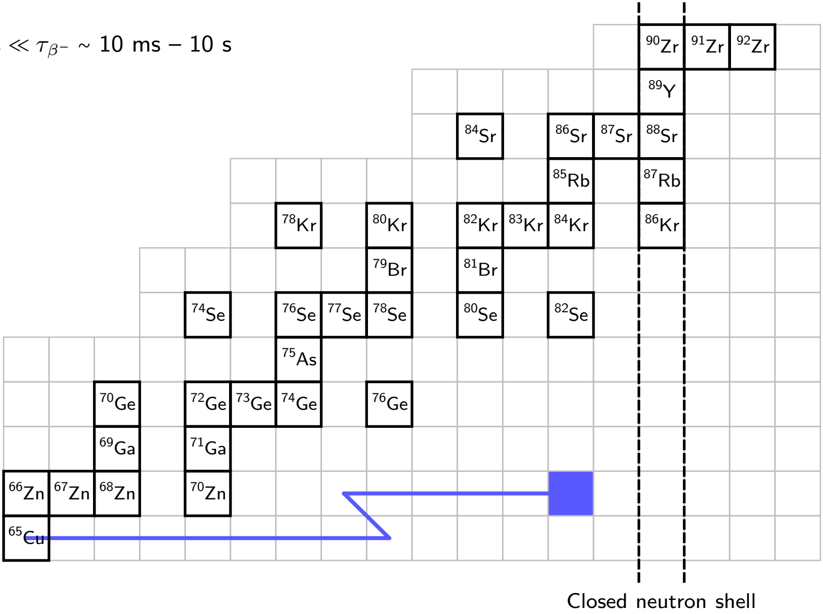

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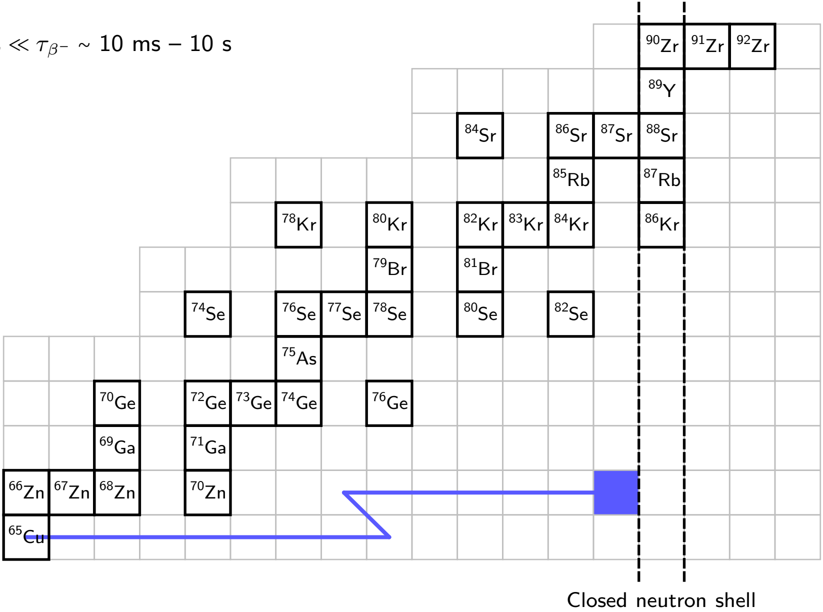

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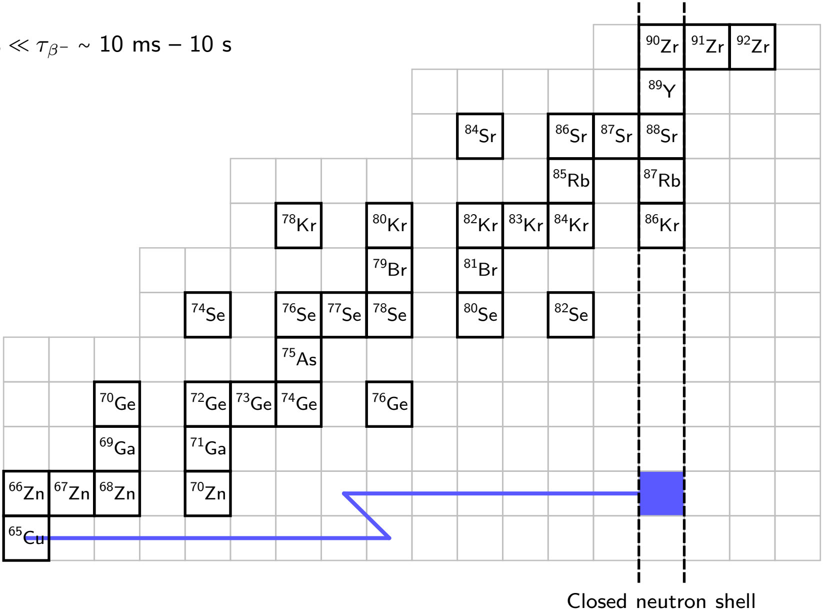

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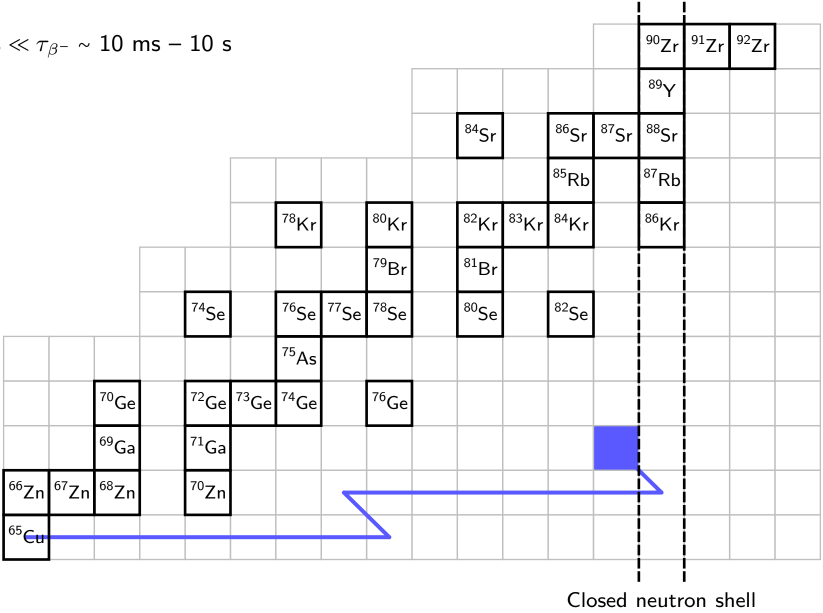

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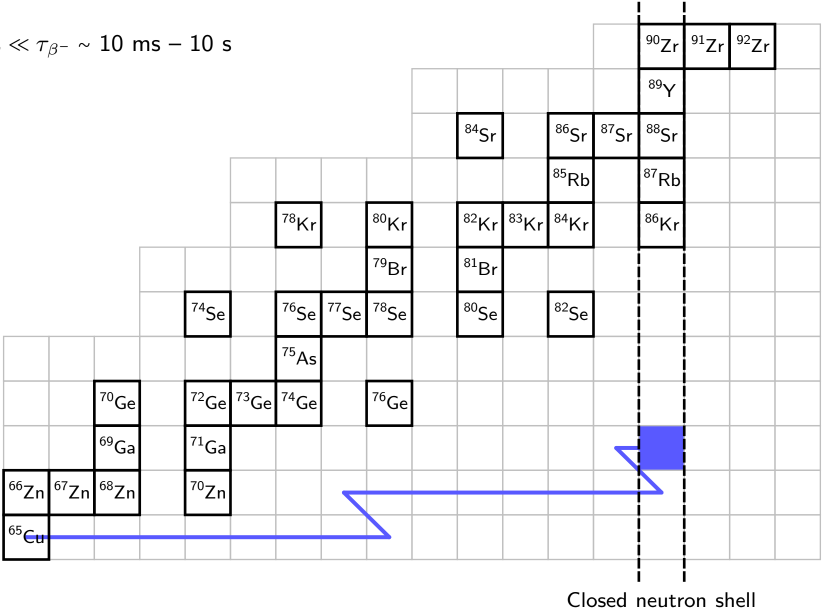

The r-process

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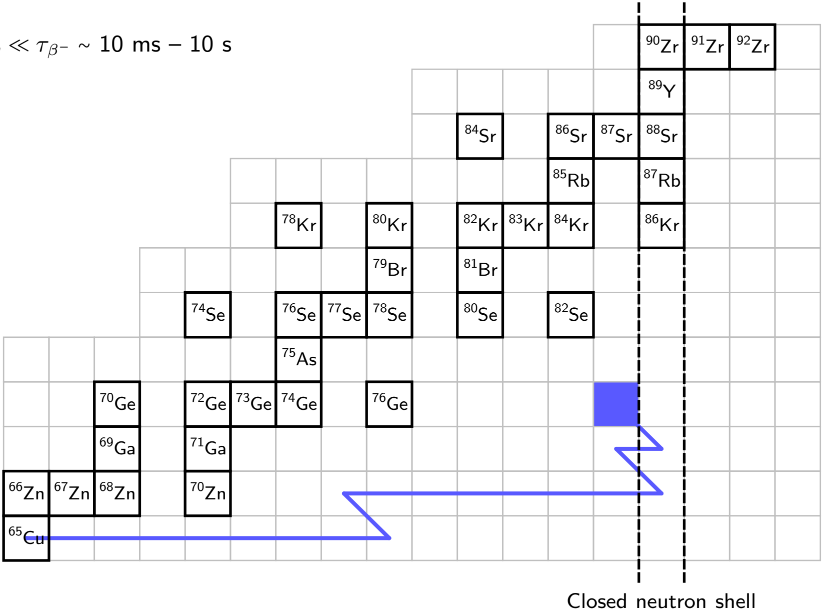

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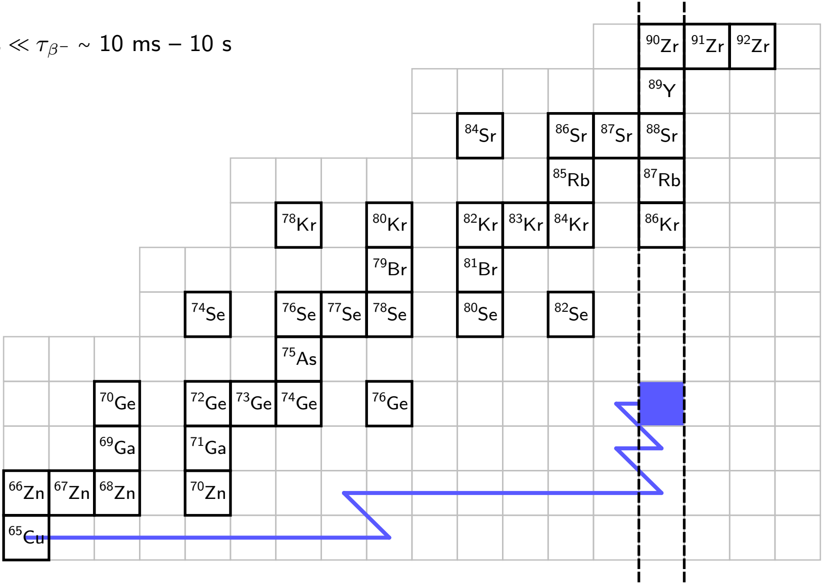

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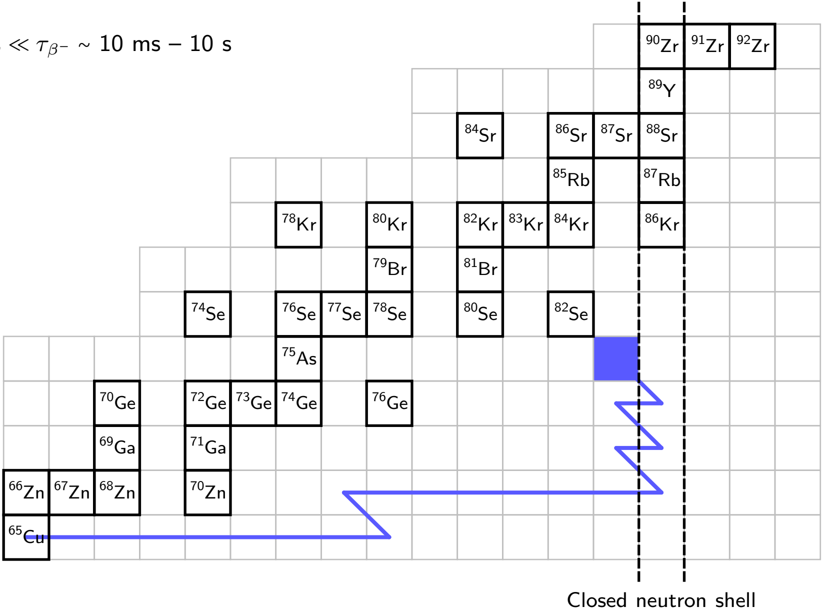

Caltech

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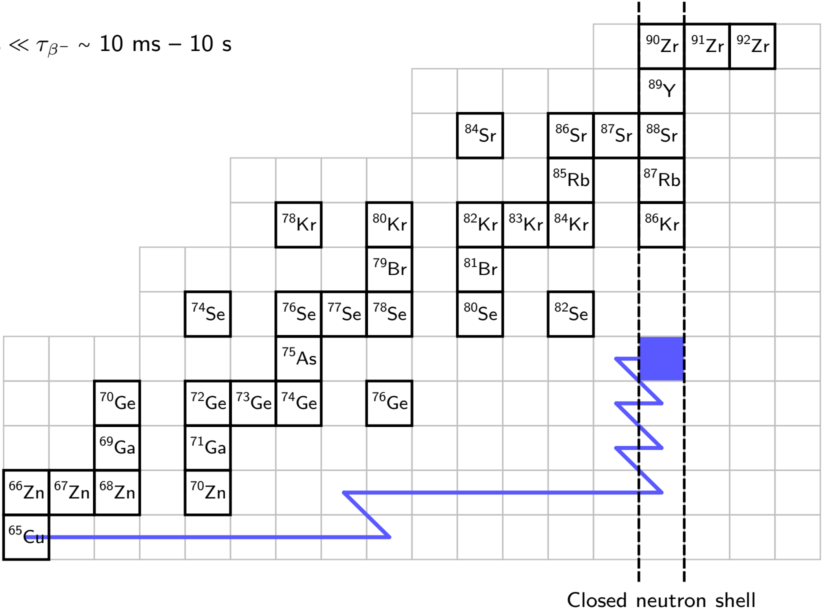

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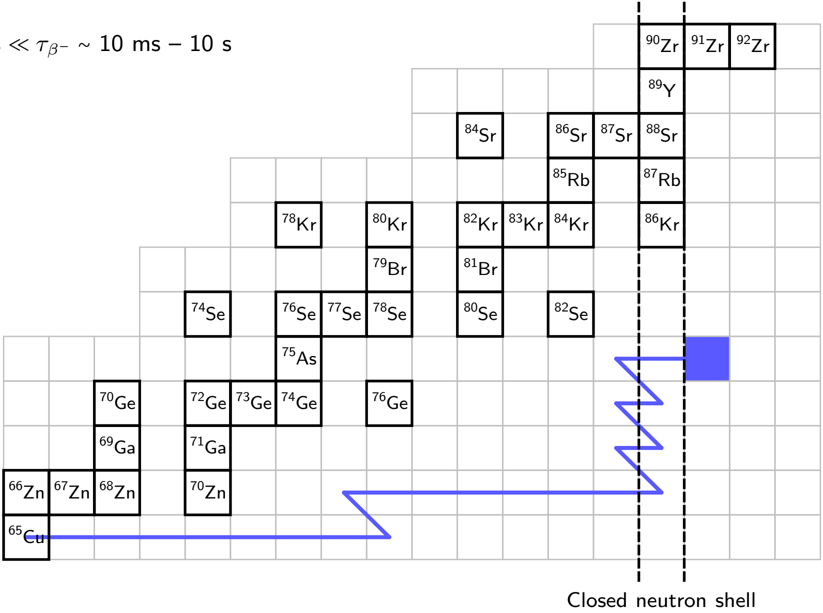

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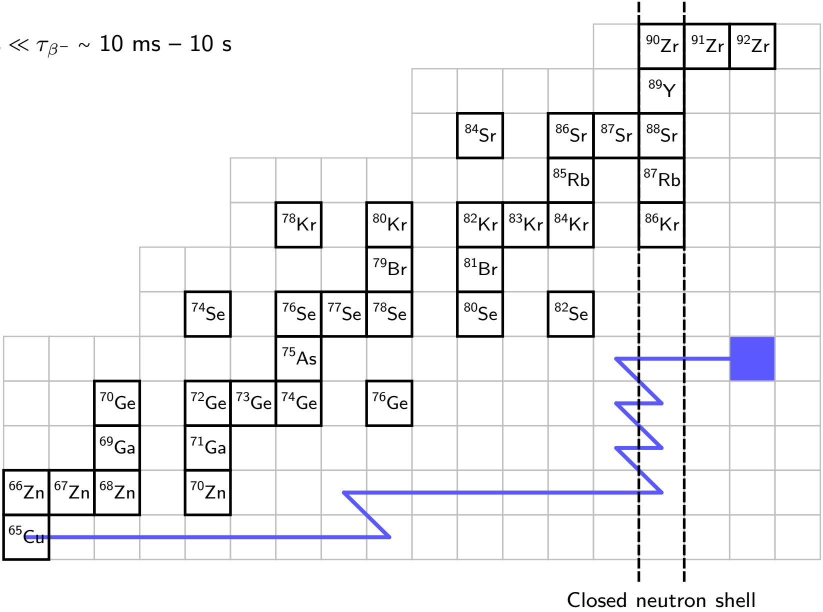

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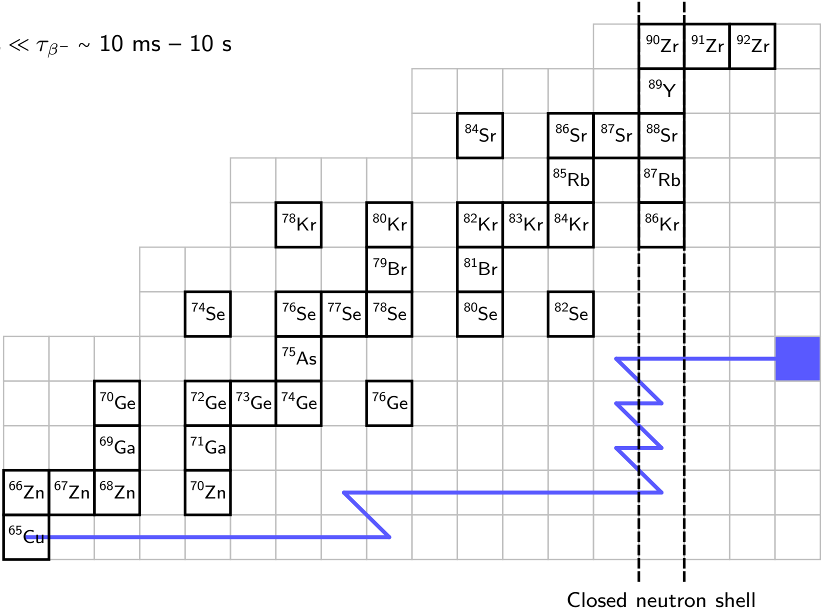

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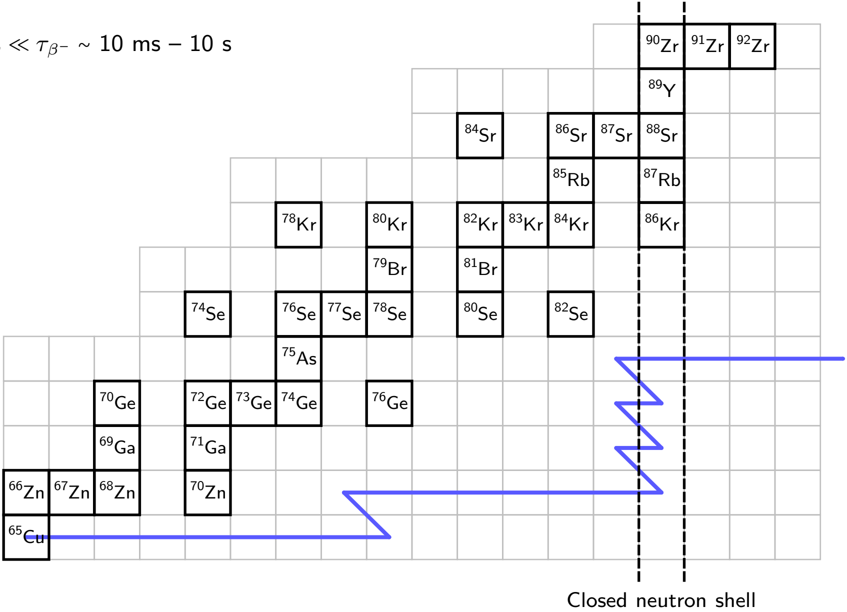

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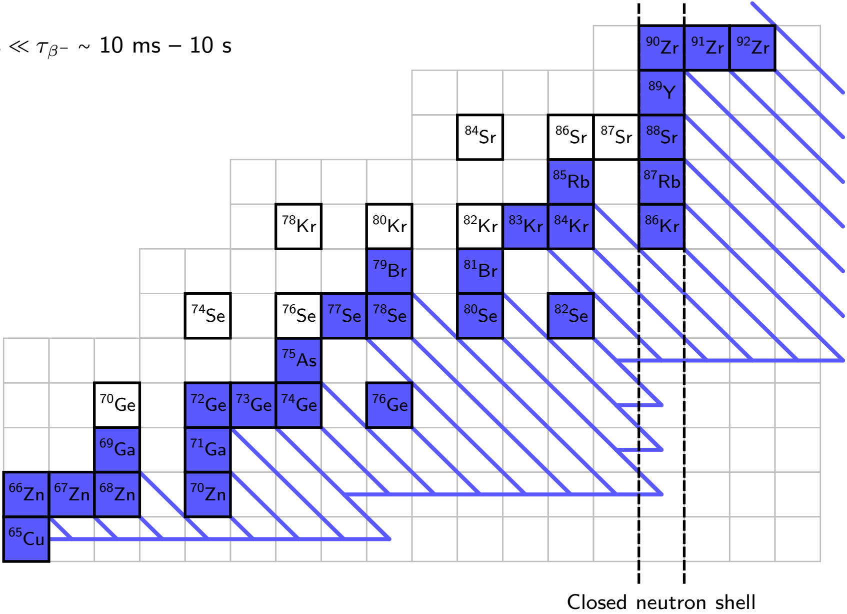

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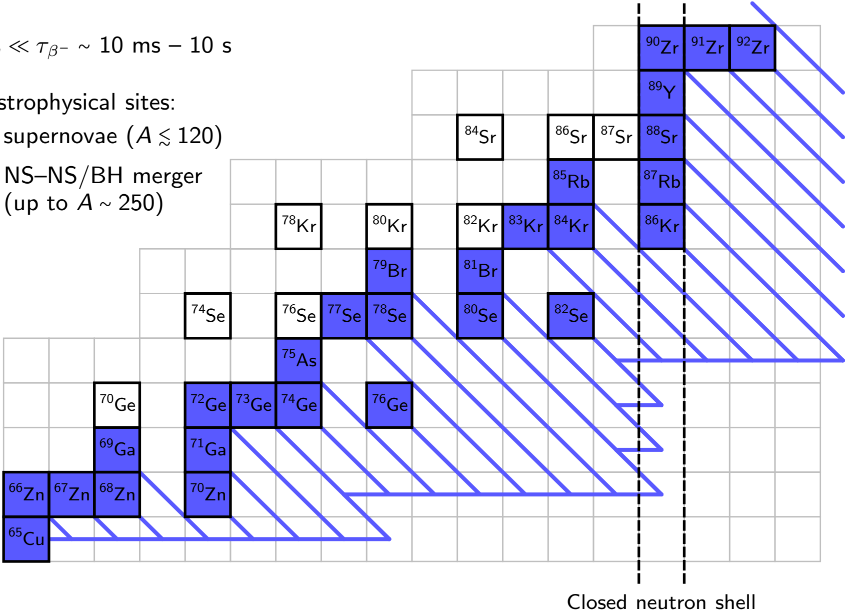
Astrophysical sites:

- supernovae ($A \lesssim 120$)
- NS–NS/BH merger (up to $A \sim 250$)



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- Radioactively powered transient after r-process nucleosynthesis
- Triple coincidence: Gravitational wave signal, short GRB, kilonova
- Possibly observed after GRB060614 and GRB130603B
- Observational signature:
    - Heating rate → luminosity
    - Amount of lanthanides and actinides → opacity

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Lanthanides (between 2nd and 3rd peak) and actinides (beyond 3rd peak) have open f-shells → very high line opacities

	lanthanide-free	lanthanide-rich
opacity	$\sim 0.1\,\mathrm{cm}^2\,\mathrm{g}^{-1}$	$\sim 10\,\mathrm{cm}^2\,\mathrm{g}^{-1}$
luminosity	bright	dim
timescale	∼ day	∼ week
band	blue / optical	red / infrared

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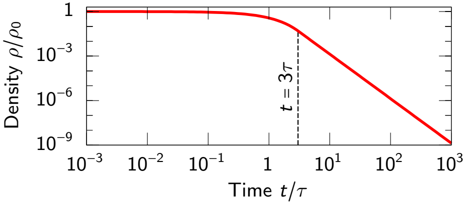
Parameters

$0.01 \le Y_e \le 0.50$	initial electron fraction
$1\,k_B\,\text{baryon}^{-1} \le s \le 100\,k_B\,\text{baryon}^{-1}$	initial specific entropy
$0.1\,\text{ms} \le \tau \le 500\,\text{ms}$	expansion time scale

Density profile

$$\rho(t,\tau) = \begin{cases} \rho_0 e^{-t/\tau} & t \le 3\tau \\ \rho_0 \left(\frac{3\tau}{te}\right)^3 & t \ge 3\tau \end{cases}$$



Initial conditions

- Choose initial temperature $T_0 = 6\,\text{GK}$
- Find ρ_0 by solving for NSE at T_0 and Y_e that produces specified s

SkyNet

github.com/jlippuner/SkyNet

- General-purpose nuclear reaction network
- ~8000 isotopes, ~110,000 nuclear reactions
- Evolves temperature and entropy based on nuclear reactions
- Input: $\rho(t)$, initial composition, initial entropy or temperature
- Open source (soon)

Science

- Helmholtz equation of state (EOS)
- Calculate nuclear statistical equilibrium (NSE)
- Calculate inverse rates from *detailed balance* to be consistent with NSE
- NSE evolution mode

Code

- Object-oriented C++11
- Python bindings
- Uses REACLIB rates, but can easily be extended
- Convenient HDF5 output
- Make movie with chart of nuclides

Final abundances vs. electron fraction

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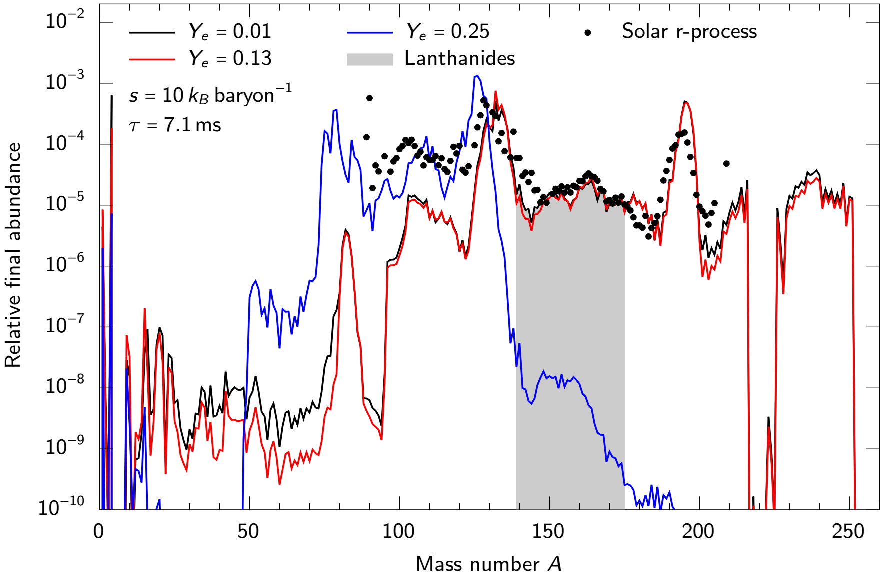
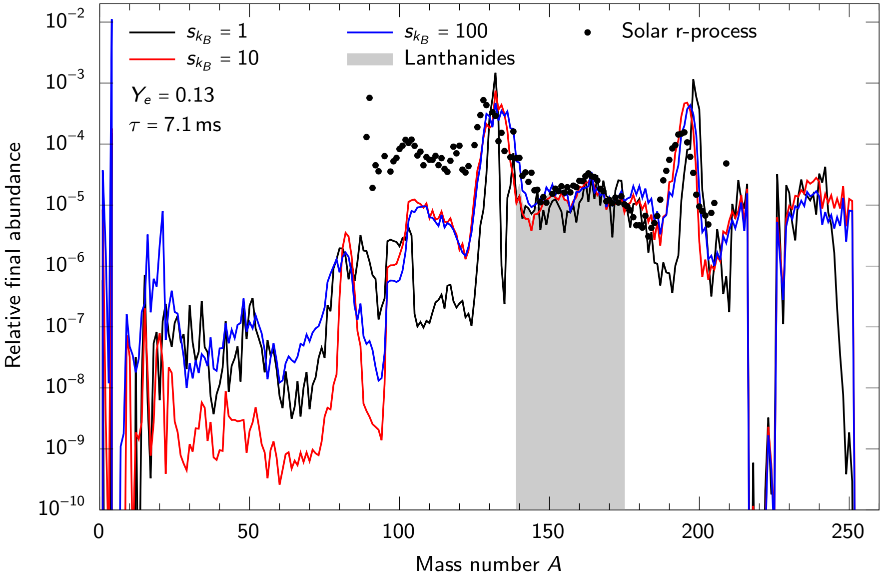


Final abundances vs. entropy

Caltech



Example light curves

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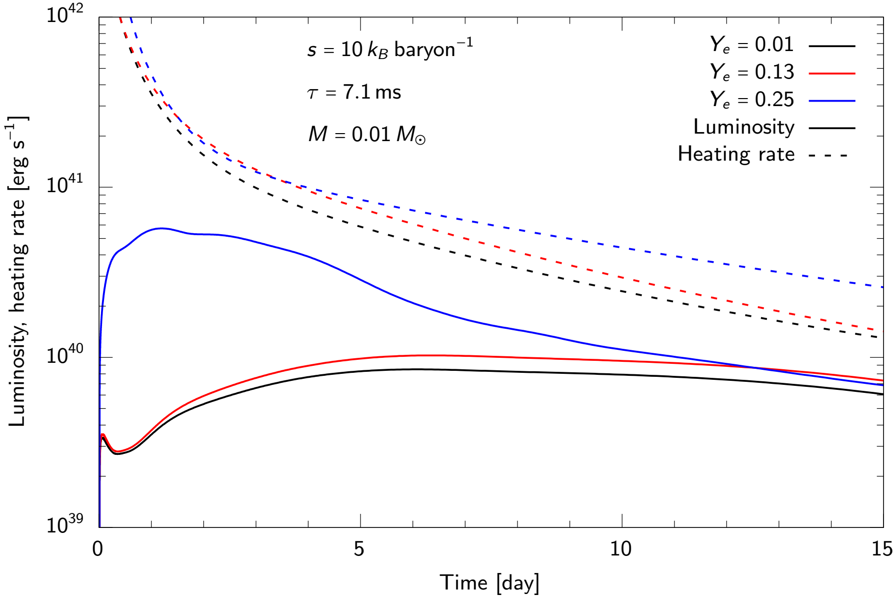
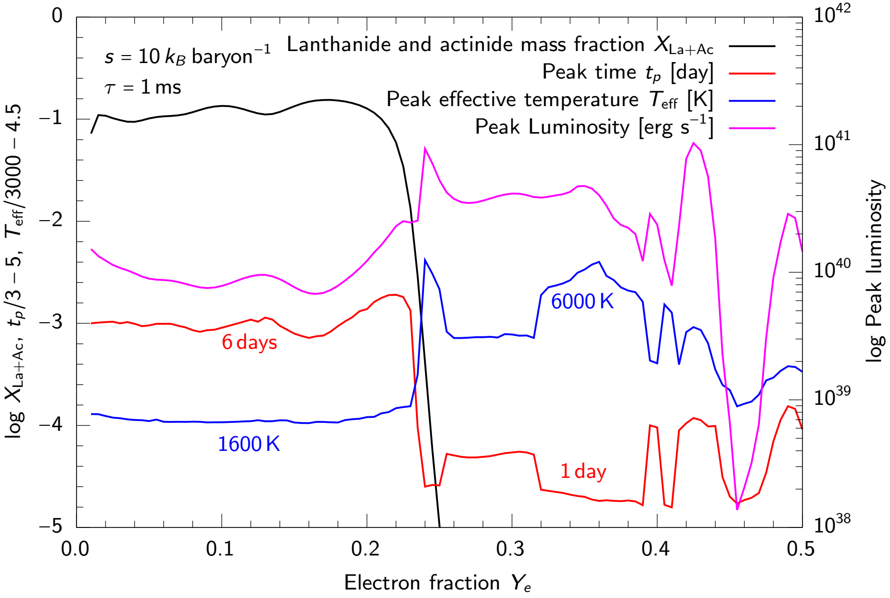
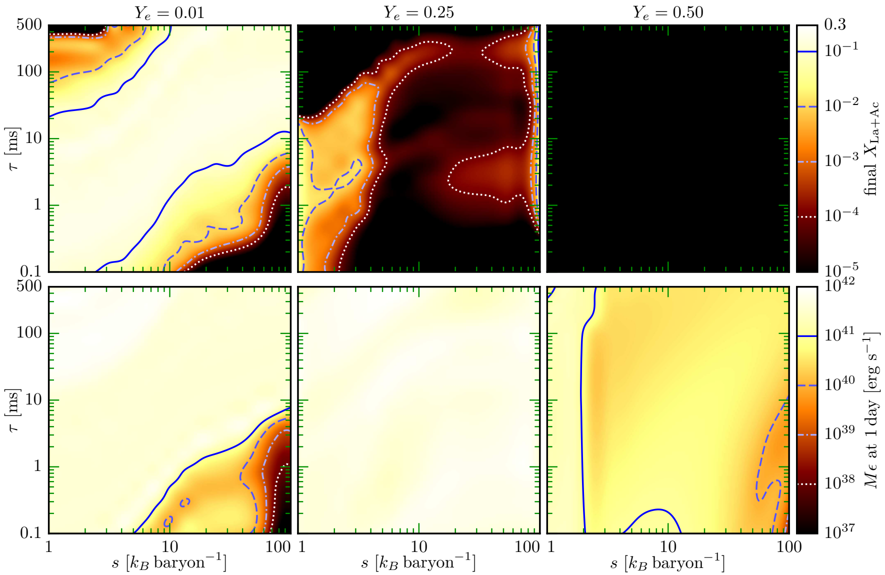


Light curves vs. electron fraction

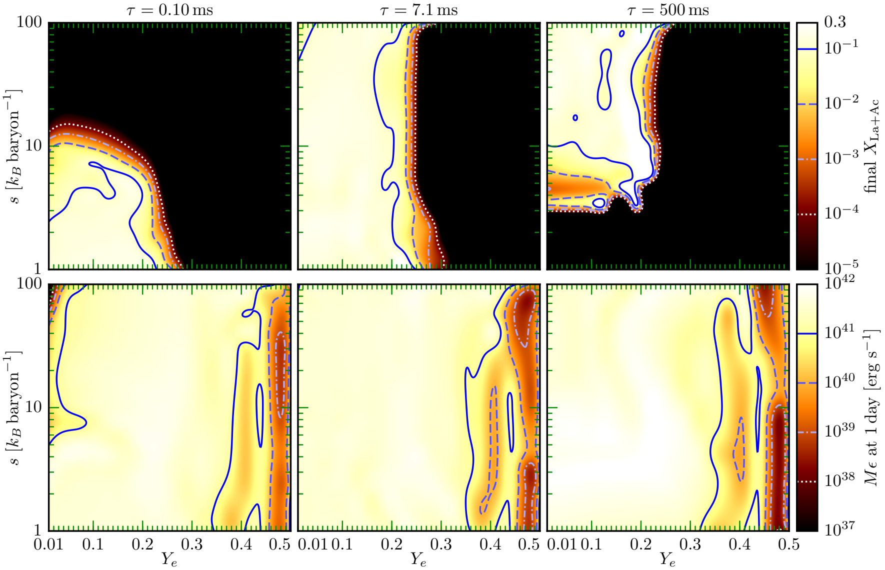
Caltech



Y_e slices



$Y_e = 0.01$
$Y_e = 0.25$
$Y_e = 0.50$
500
100
10
1
0.1
τ [ms]
1
10
100
s [k_B baryon^{-1}]
0.3
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
final $X_{\mathrm{La+Ac}}$
10^{42}
10^{41}
10^{40}
10^{39}
10^{38}
10^{37}
$M\epsilon$ at 1 day [erg s^{-1}]


$\tau = 0.10\,\mathrm{ms}$
$\tau = 7.1\,\mathrm{ms}$
$\tau = 500\,\mathrm{ms}$
$s\ [k_B\ \mathrm{baryon}^{-1}]$
100
10
1
Y_e
0.01 0.1 0.2 0.3 0.4 0.5
final $X_{\mathrm{La+Ac}}$
0.3
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
$M\epsilon$ at 1 day $[\mathrm{erg\ s}^{-1}]$
10^{42}
10^{41}
10^{40}
10^{39}
10^{38}
10^{37}

Summary and conclusions

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- Lanthanides and actinides have big impact on kilonova light curve
- Lanthanide-rich for $Y_e \lesssim 0.22 - 0.30$, except:
  - High s and small τ: neutron-rich freeze-out
  - Low s and large τ: restarted r-process at high Y_e
- Uniform heating rate for $Y_e \lesssim 0.4$

Additional slides

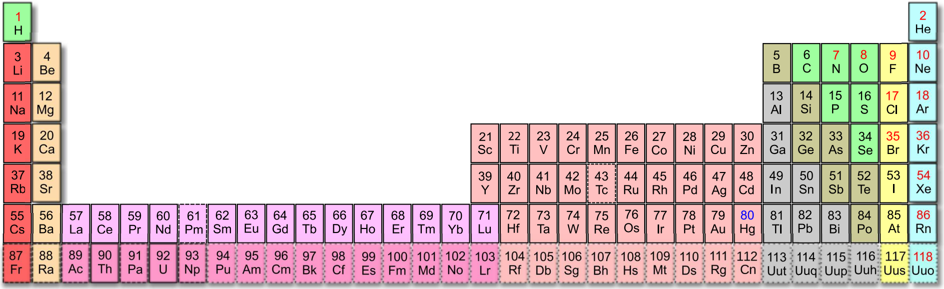
Lanthanides and actinides

1 H																	2 He
3 Li	4 Be											5 B	6 C	7 N	8 O	9 F	10 Ne
11 Na	12 Mg											13 Al	14 Si	15 P	16 S	17 Cl	18 Ar
19 K	20 Ca	21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn	31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr
37 Rb	38 Sr	39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd	49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe
55 Cs	56 Ba	*	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg	81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn
87 Fr	88 Ra	**	104 Rf	105 Db	106 Sg	107 Bh	108 Hs	109 Mt	110 Ds	111 Rg	112 Cn	113 Uut	114 Fl	115 Uup	116 Lv	117 Uus	118 Uuo
119 Uun																	

* Lanthanides	57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	71 Lu
** Actinides	89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	103 Lr

Credit: http://www.sciencegeek.net

Lanthanides and actinides



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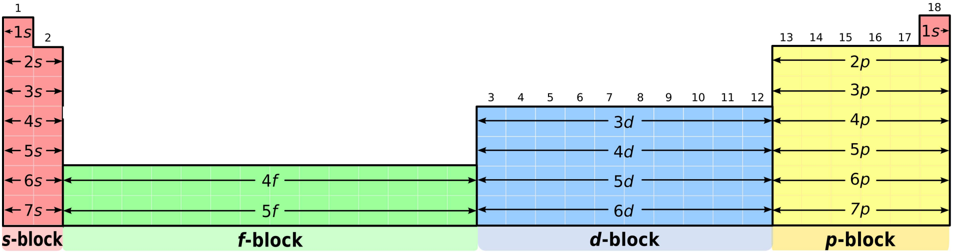
Lanthanides and actinides



Credit: User:DePiep (Wikipedia)

Nucleosynthesis: Overview

- Creating nuclides out of nucleons (protons and neutrons)
- Cost: overcoming Coulomb barrier (but there is a loophole)
- Reward: ~ 8 MeV per nucleon binding energy
- Sources
  - Big Bang
  - Stellar fusion
  - Supernovae
  - Neutron star mergers (NS–NS and NS–BH)

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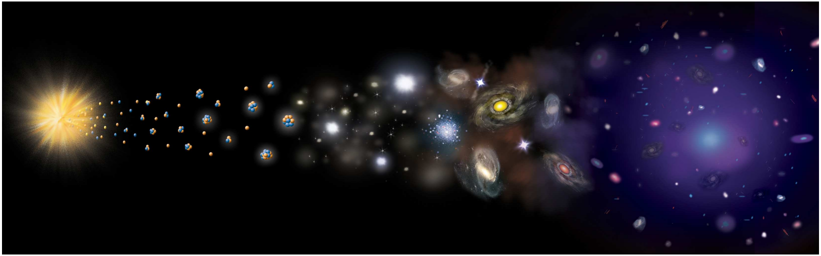
- ~ 100 s after Big Bang, $T \sim 1$ GK

H	^{4}He	D/H	^{3}He/H	^{7}Li/H
75.2% by mass	24.8% by mass	3×10^{-5}	1×10^{-5}	1.5×10^{-10} predicted 5×10^{-10}, 4σ discrepancy



Credit: NASA / CXC / M. Weiss, note that such big nuclides where not produced in BBN

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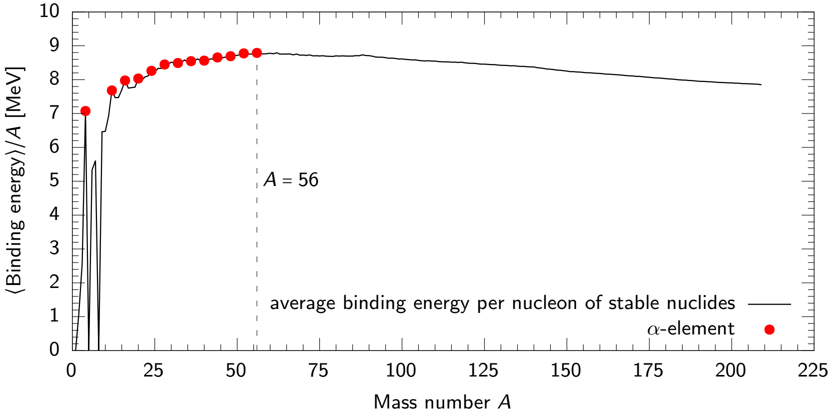
- p–p Coulomb barrier: ~ 1 MeV $\sim 10^{10}$ K
- Sun's core temperature: $\sim 1.6 \times 10^{7}$ K
- Produces mostly α-elements: ^{4}He, ^{12}C, ^{16}O, ^{20}Ne, ^{24}Mg, ^{28}Si, ^{32}S, ^{36}Ar, ^{40}Ca, ^{44}Ti*, ^{48}Cr*, ^{52}Fe*, ^{56}Ni* (* = unstable)



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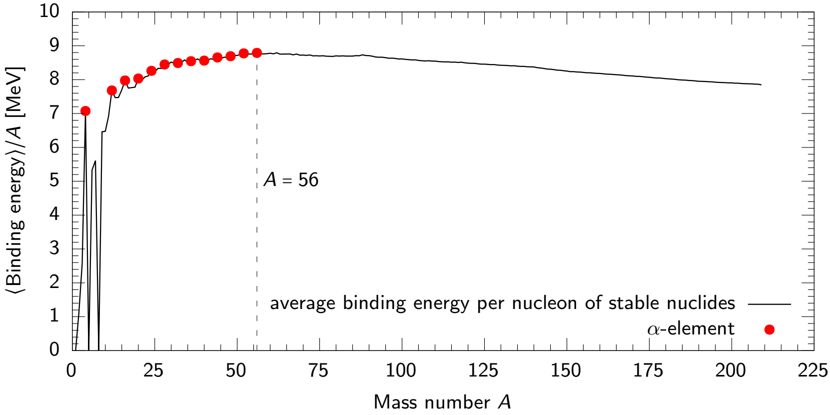
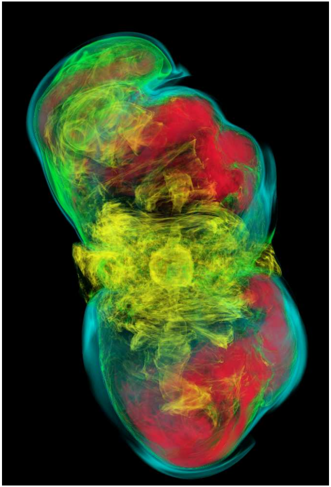


- Thermonuclear (Type Ia)
  - Thermonuclear explosion of one or two (?) white dwarf(ves)
  - Produces mostly iron-peak elements through nuclear statistical equilibrium (NSE): Sc, Ti, V, Cr, Mn, Fe, Co, Ni, Cu, Zn
- Core-collapse (Type II, Ib/c)
  - Fluorine (ν knocks out a p from ^{20}Ne)
  - Produces some iron-peak elements
  - Ejects s-process elements
  - Maybe r-process



Credit: S. Richers, P. Mösta

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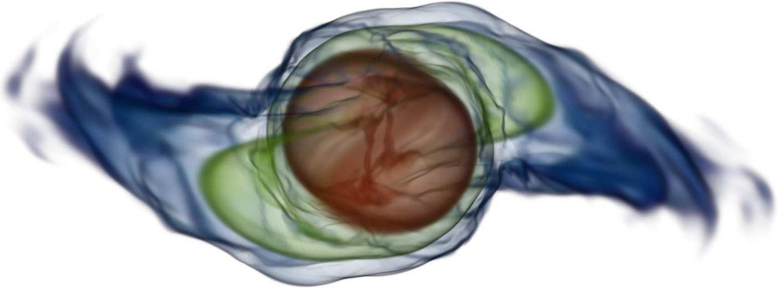
- r-process in neutron-rich ejecta
- Ejecta: $\sim 10^{-3} - 10^{-2}\ M_\odot$



Credit: R. Haas

s-process Path

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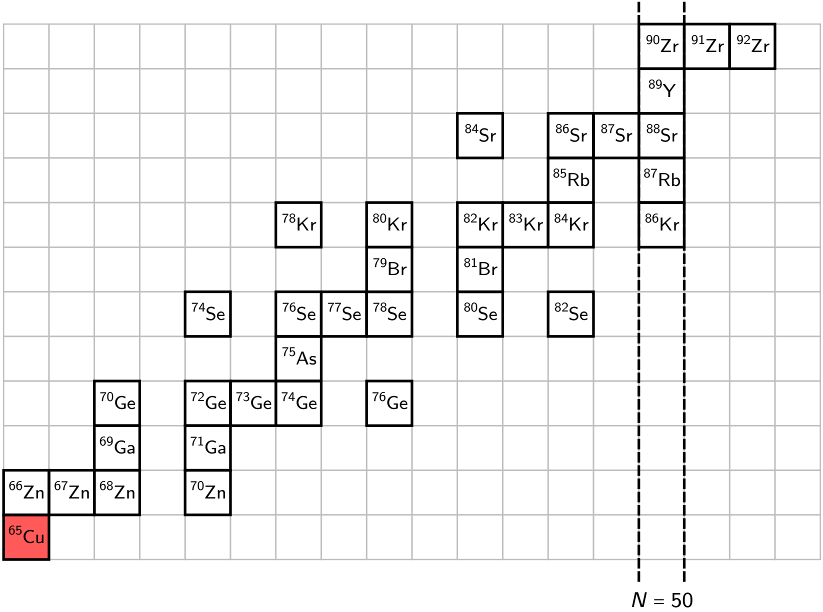
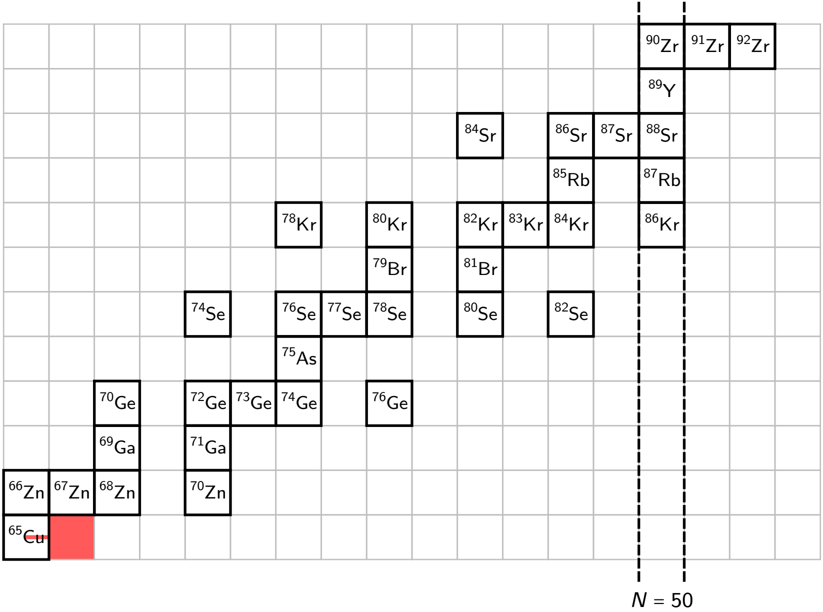


s-process Path

Caltech



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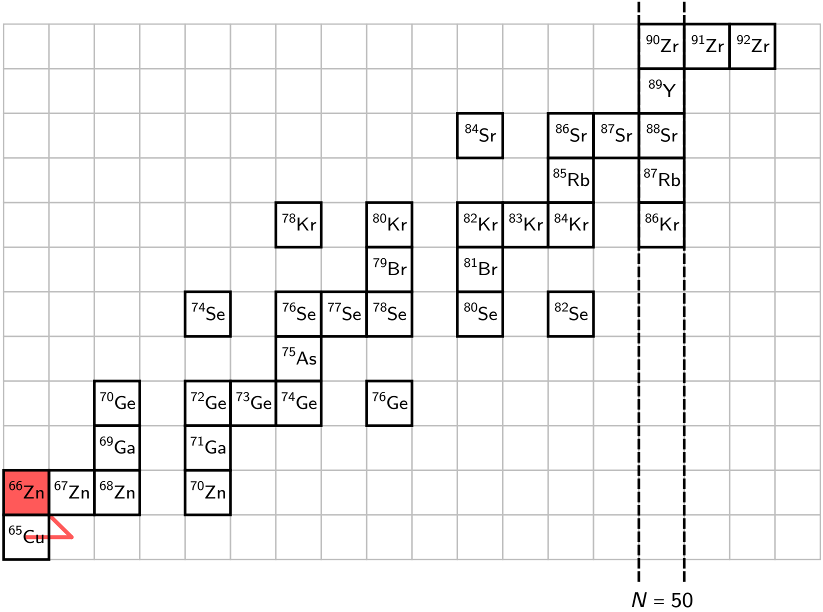

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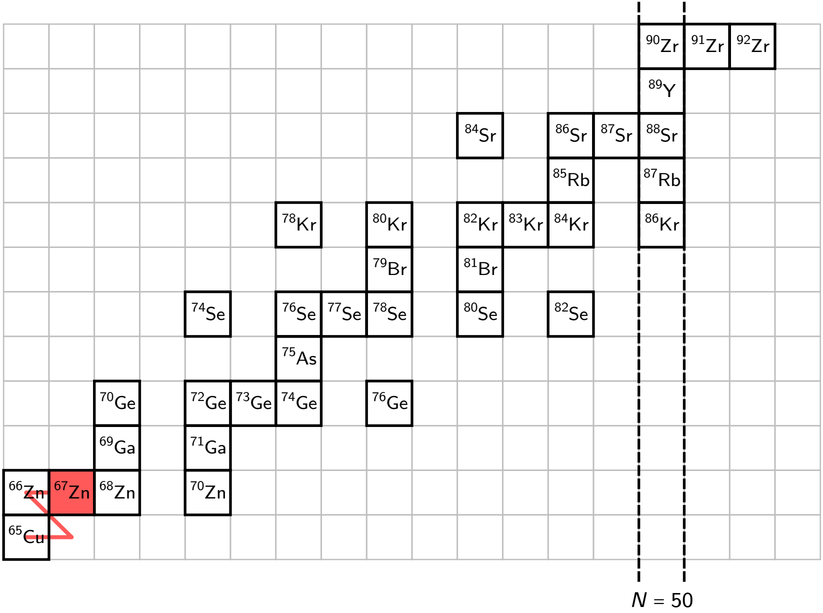
90Zr
91Zr
92Zr
89Y
84Sr
86Sr
87Sr
88Sr
85Rb
87Rb
78Kr
80Kr
82Kr
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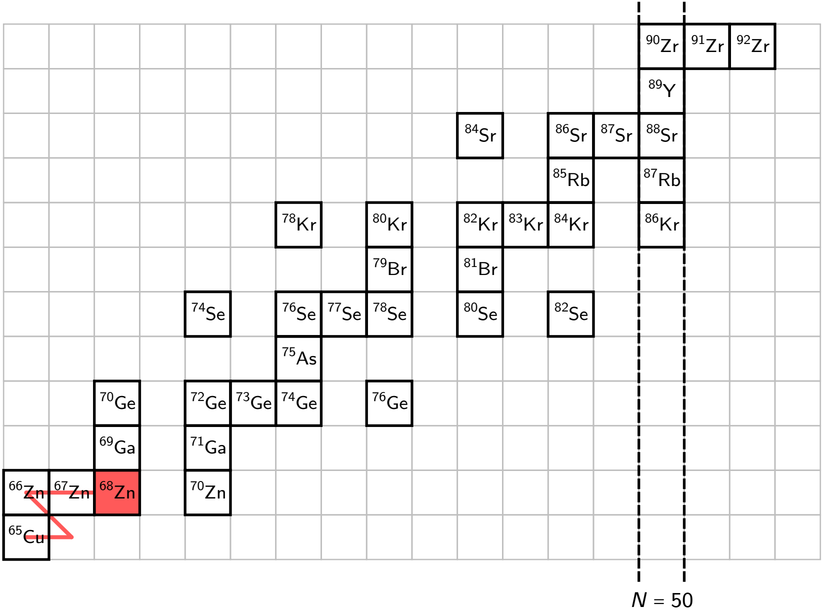
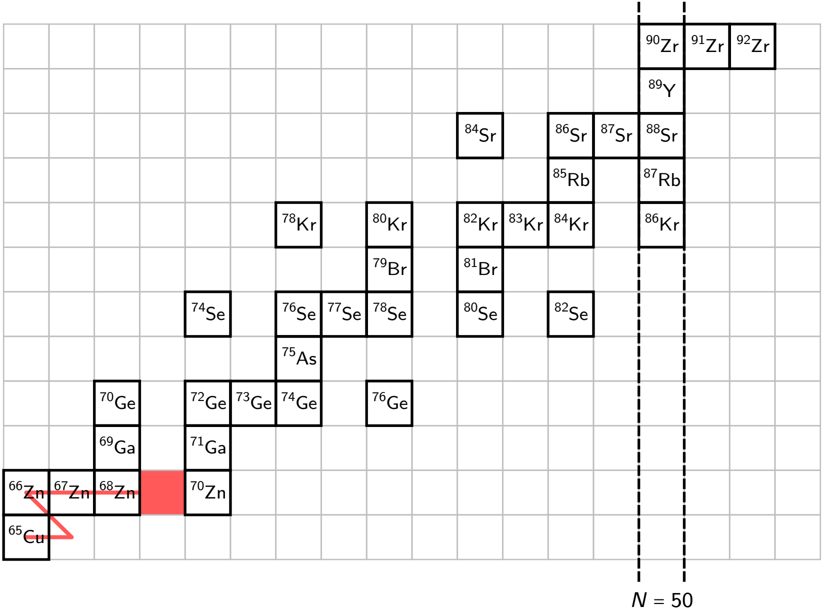
84Kr
86Kr
79Br
81Br
74Se
76Se
77Se
78Se
80Se
82Se
75As
70Ge
72Ge
73Ge
74Ge
76Ge
69Ga
71Ga
66Zn
67Zn
68Zn
70Zn
65Cu
N = 50

s-process Path




^{90}Zr
^{91}Zr
^{92}Zr
^{89}Y
^{84}Sr
^{86}Sr
^{87}Sr
^{88}Sr
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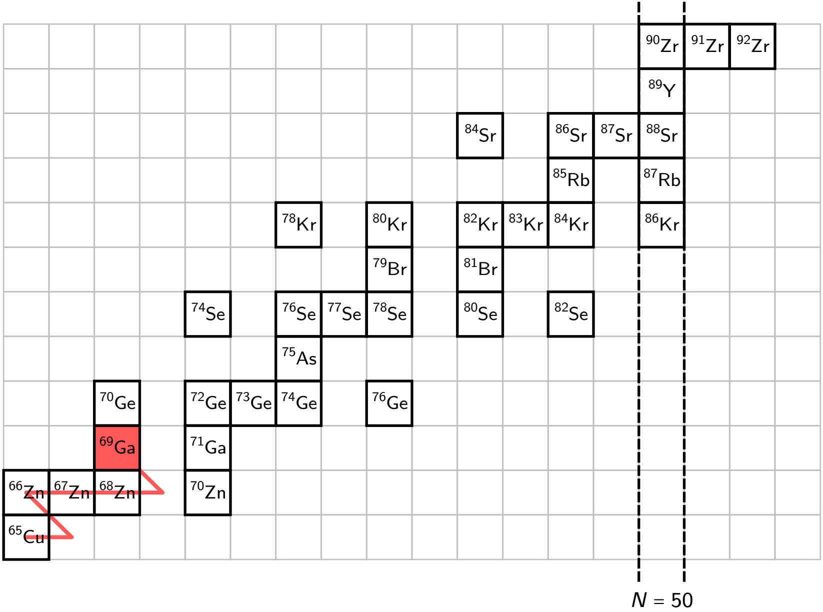
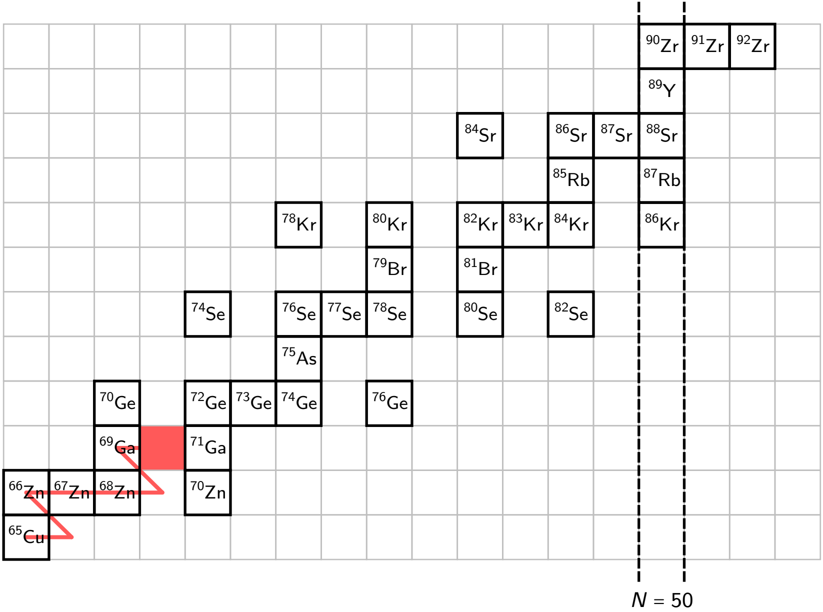
^{78}Kr
^{80}Kr
^{82}Kr
^{83}Kr
^{84}Kr
^{86}Kr
^{79}Br
^{81}Br
^{74}Se
^{76}Se
^{77}Se
^{78}Se
^{80}Se
^{82}Se
^{75}As
^{70}Ge
^{72}Ge
^{73}Ge
^{74}Ge
^{76}Ge
^{69}Ga
^{71}Ga
^{66}Zn
^{67}Zn
^{68}Zn
^{70}Zn
^{65}Cu
$N = 50$

s-process Path



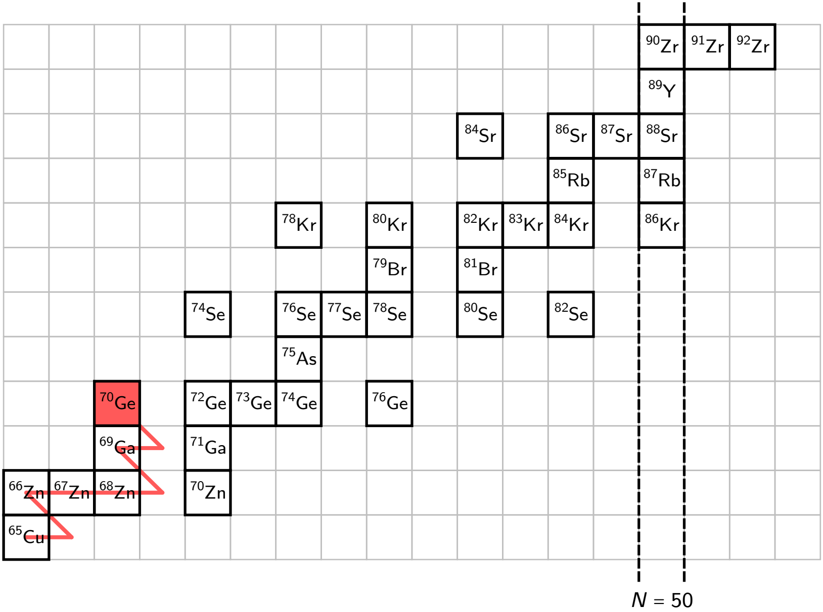
s-process Path

Caltech



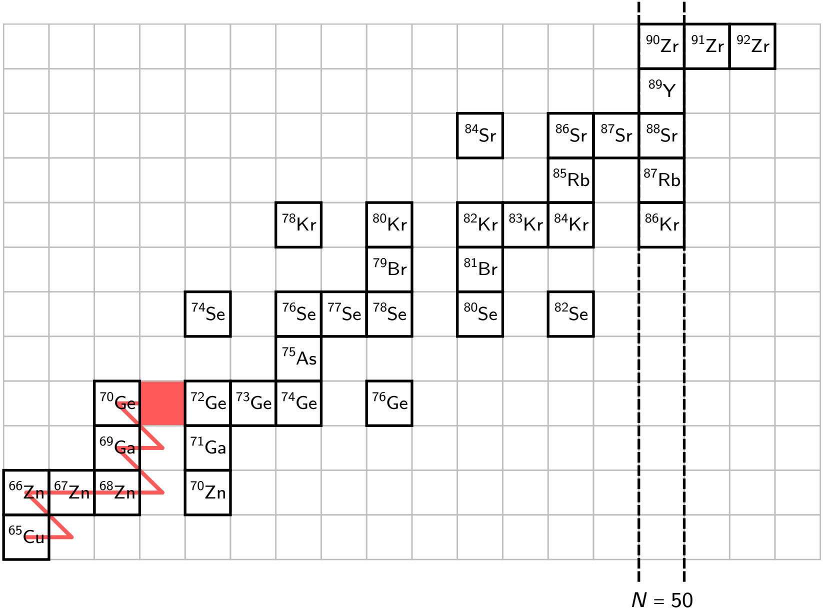
s-process Path

Caltech



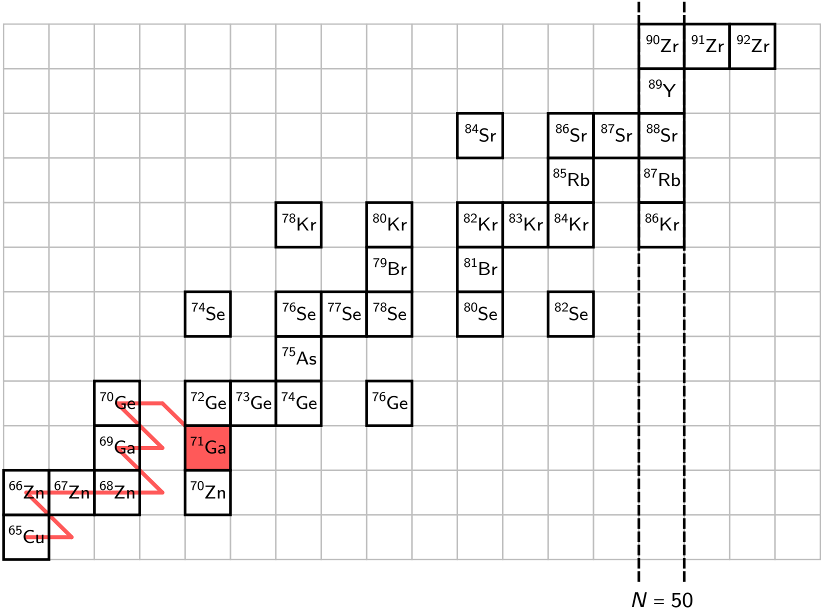
s-process Path

Caltech

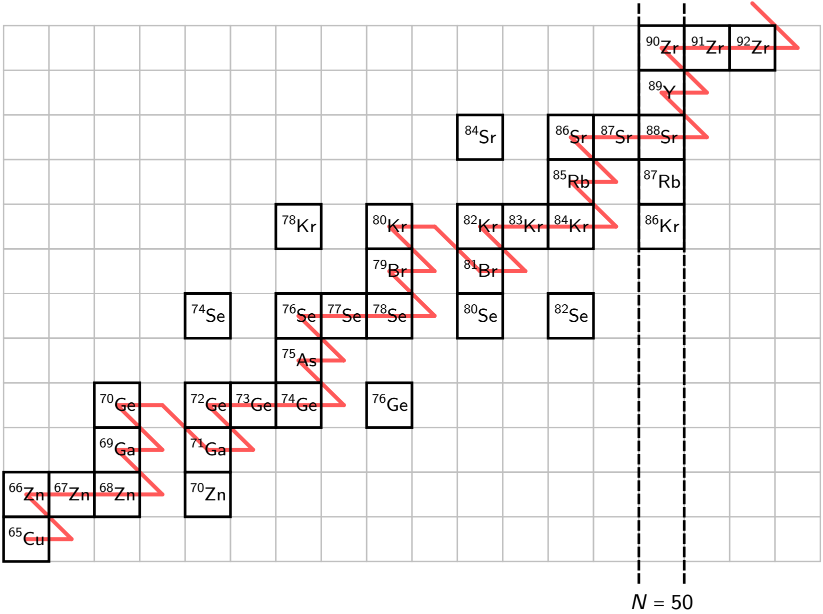


s-process Path

Caltech

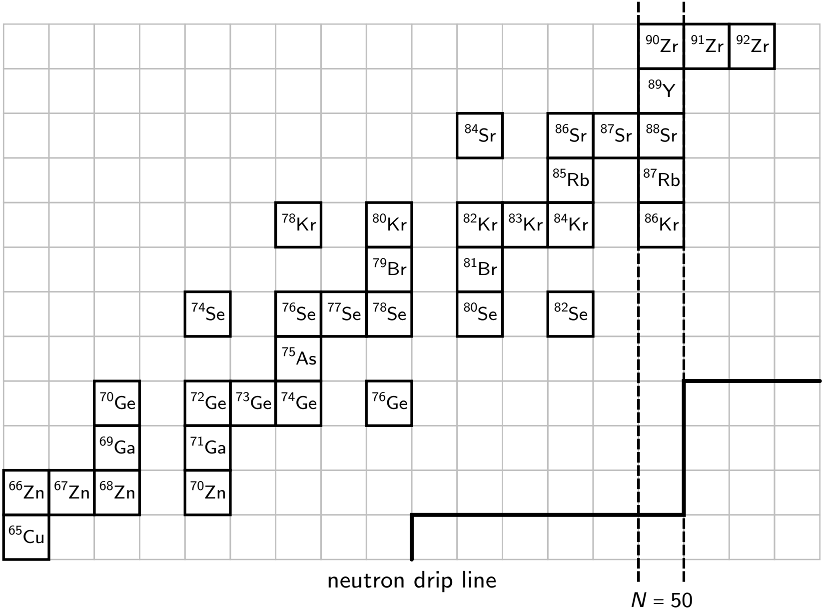


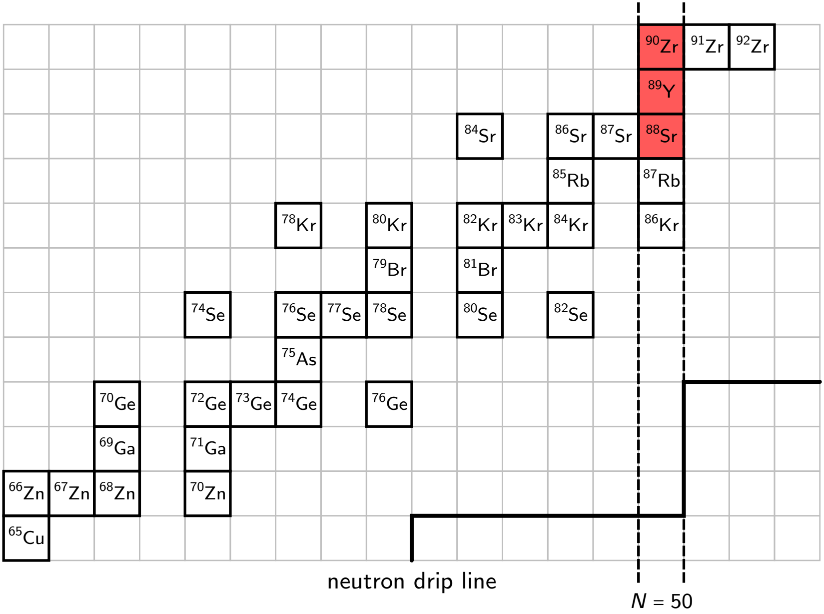
s-process Path



Double Peaks due to Closed Neutron Shells

Caltech





90Zr
91Zr
92Zr
89Y
84Sr
86Sr
87Sr
88Sr
85Rb
87Rb
78Kr
80Kr
82Kr
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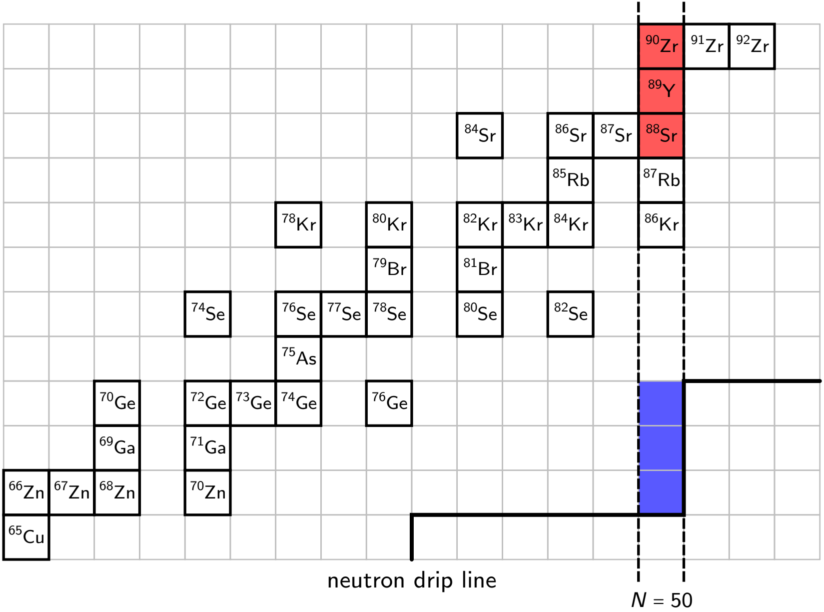
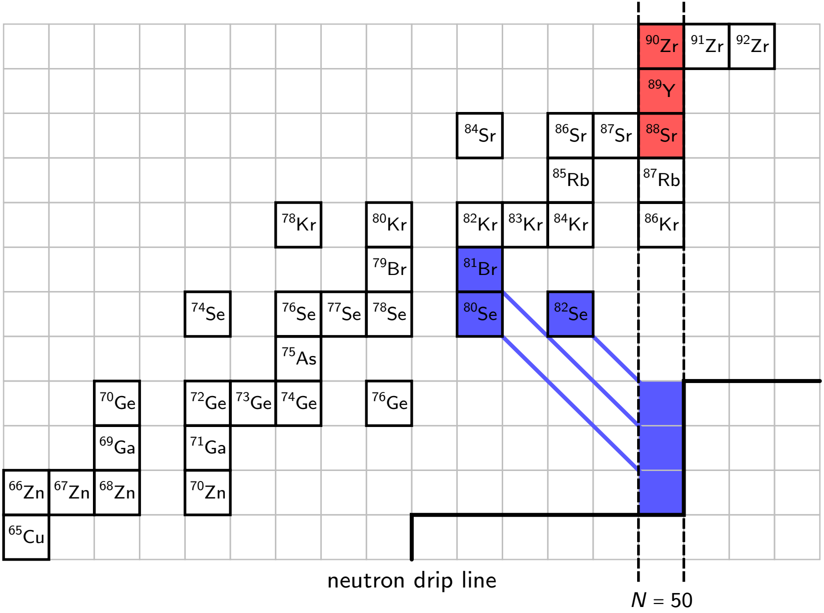
84Kr
86Kr
79Br
81Br
74Se
76Se
77Se
78Se
80Se
82Se
75As
70Ge
72Ge
73Ge
74Ge
76Ge
69Ga
71Ga
66Zn
67Zn
68Zn
70Zn
65Cu
neutron drip line
N = 50

Double Peaks due to Closed Neutron Shells

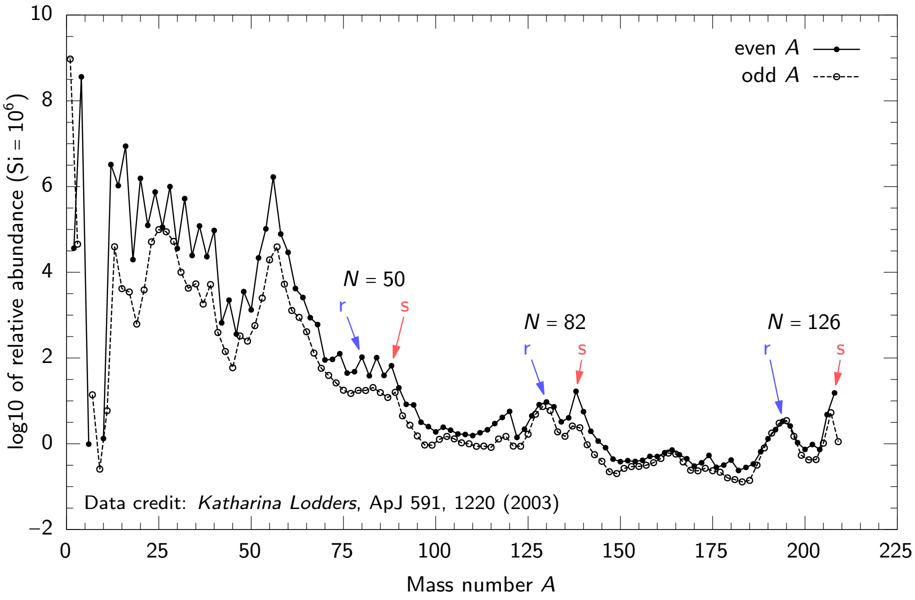


Double Peaks due to Closed Neutron Shells

Caltech



Caltech



10
8
6
4
2
0
−2
log10 of relative abundance (Si = 10^6)
0
25
50
75
100
125
150
175
200
225
Mass number *A*
even *A*
odd *A*
N = 50
r
s
N = 82
r
s
N = 126
r
s
Data credit: *Katharina Lodders*, ApJ 591, 1220 (2003)

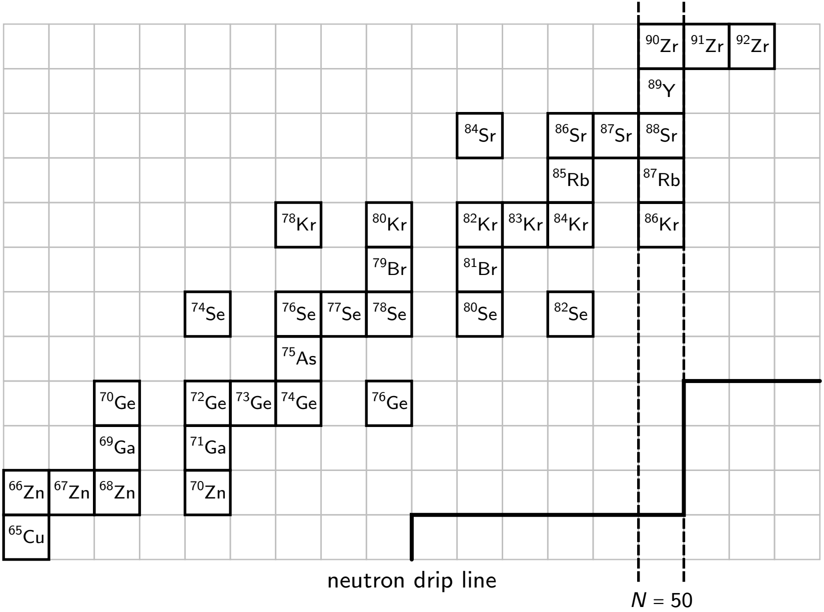
Caltech

	s-process	r-process
mechanism	neutron capture, β^- decay	No Coulomb barrier!
τ_n	$10^2 - 10^5$ yr	$\ll \tau_{\beta^-}$
τ_{β^-}	$\ll \tau_n$	$0.01 - 10$ s
site	inside massive stars	supernovae? NS-NS/BH mergers?
neutron source	$^{13}C + {}^4He \to {}^{16}O + n$ $^{22}Ne + {}^4He \to {}^{25}Mg + n$	neutrino driven wind tidal ejecta of NS material
path	valley of stability	neutron drip line
peaks*	$A = 88, 138, 208$ strontium, barium, lead	$A = 80, 130, 194$ selenium, xenon, platinum

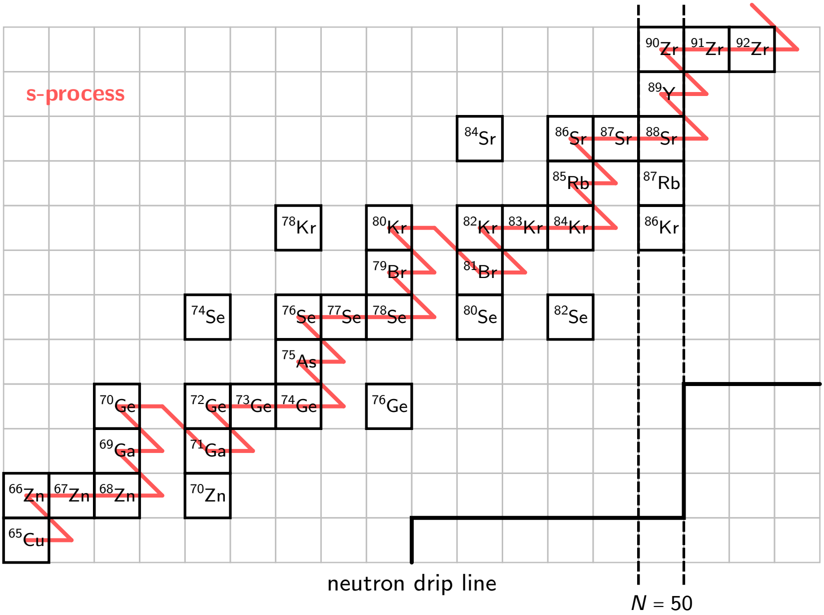
* due to closed neutron shells at $N = 50, 82, 126$

Shielding

Caltech

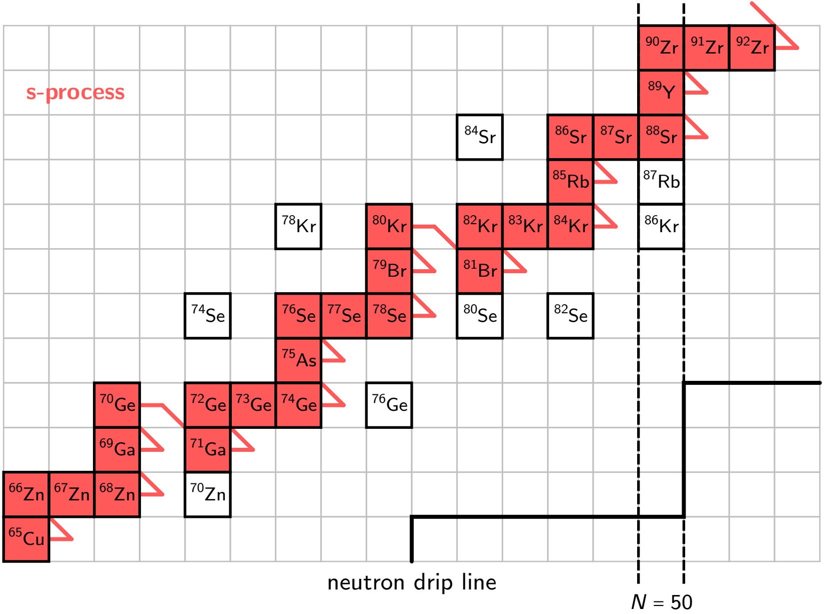


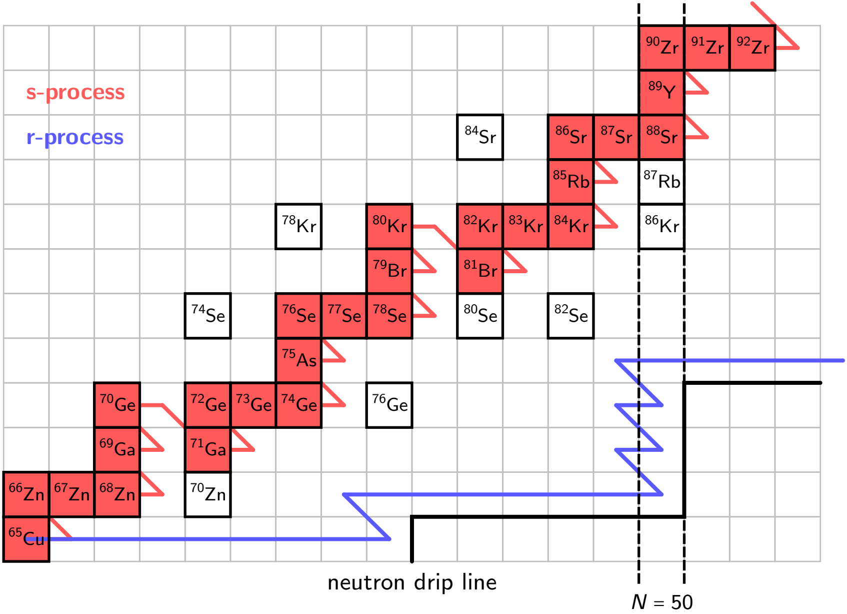
Shielding



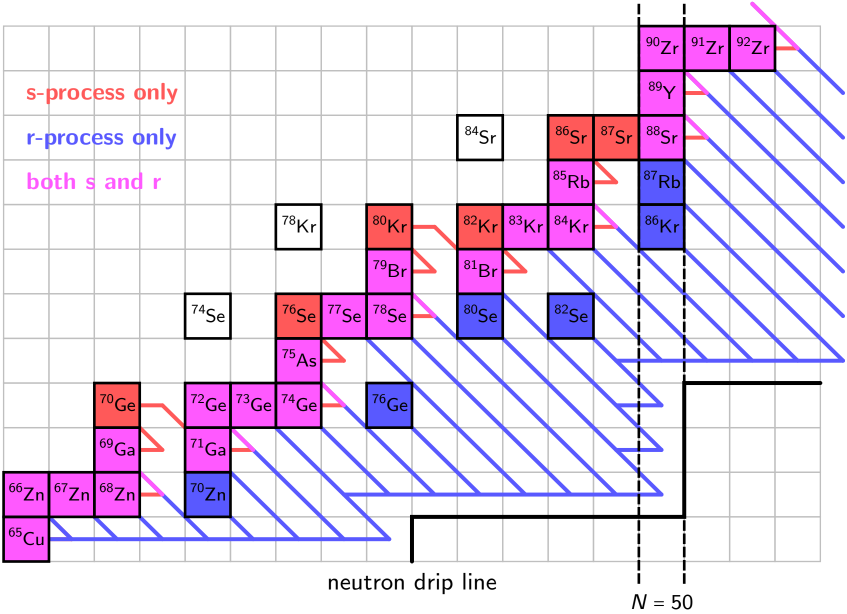
s-process
90Zr
91Zr
92Zr
89Y
84Sr
86Sr
87Sr
88Sr
85Rb
87Rb
78Kr
80Kr
82Kr
83Kr
84Kr
86Kr
79Br
81Br
74Se
76Se
77Se
78Se
80Se
82Se
75As
70Ge
72Ge
73Ge
74Ge
76Ge
69Ga
71Ga
66Zn
67Zn
68Zn
70Zn
65Cu
neutron drip line
N = 50

Shielding


s-process
90Zr
91Zr
92Zr
89Y
84Sr
86Sr
87Sr
88Sr
85Rb
87Rb
78Kr
80Kr
82Kr
83Kr
84Kr
86Kr
79Br
81Br
74Se
76Se
77Se
78Se
80Se
82Se
75As
70Ge
72Ge
73Ge
74Ge
76Ge
69Ga
71Ga
66Zn
67Zn
68Zn
70Zn
65Cu
neutron drip line
N = 50



s-process
r-process
90Zr
91Zr
92Zr
89Y
84Sr
86Sr
87Sr
88Sr
85Rb
87Rb
78Kr
80Kr
82Kr
83Kr
84Kr
86Kr
79Br
81Br
74Se
76Se
77Se
78Se
80Se
82Se
75As
70Ge
72Ge
73Ge
74Ge
76Ge
69Ga
71Ga
66Zn
67Zn
68Zn
70Zn
65Cu
neutron drip line
N = 50


s-process only
r-process only
both s and r
90Zr
91Zr
92Zr
89Y
84Sr
86Sr
87Sr
88Sr
85Rb
87Rb
78Kr
80Kr
82Kr
83Kr
84Kr
86Kr
79Br
81Br
74Se
76Se
77Se
78Se
80Se
82Se
75As
70Ge
72Ge
73Ge
74Ge
76Ge
69Ga
71Ga
66Zn
67Zn
68Zn
70Zn
65Cu
neutron drip line
N = 50

- General purpose nuclear reaction network
- Inputs:
  - List of nuclides ($\sim$ 8000)
  - List of nuclear reactions and rates ($\sim$ 100,000)
  - Initial composition, initial entropy / temperature
  - Density vs. time
- Outputs:
  - Composition vs. time
  - Temperature, entropy, heating rate vs. time

Science

- Helmholtz equation of state (EOS)
- Calculate nuclear statistical equilibrium (NSE)
- Calculate inverse rates from *detailed balance* to be consistent with NSE
- NSE evolution mode

Code

- Object-oriented C++11 (with CMake build system)
- Python bindings (with SWIG)
- Support for different matrix solver packages
  - Sparse: Intel MKL, Trilinos (KLU, UMFPACK, SuperLU), Pardiso
  - Dense: LAPACK, Trilinos (LAPACK), Armadillo
- Convenient HDF5 output
- Make movie with chart of nuclides
- Open source (soon)

Consider reaction

$$[\mathrm{j}] + [\mathrm{k}] \to [\mathrm{m}] \tag{1}$$

$$\begin{aligned}\text{cross section} = \sigma &= \frac{\#\text{ of reactions per target [j] per second}}{\text{flux of projectiles [k]}} \\ &= \frac{R/(Vn_j)}{n_k v} = \frac{r}{n_j n_k v},\end{aligned} \tag{2}$$

and so

$$r = \frac{R}{V} = \sigma v n_j n_k = \#\text{ of reactions per second per volume}, \tag{3}$$

where

$$\begin{aligned} R &= \#\text{ of reactions per second}, \\ V &= \text{volume}, \\ n_{j,k} &= \text{number density of species [j], [k]}, \\ v &= \text{relative speed between [j] and [k]}. \end{aligned}$$

In general

$$r_{j,k} = \int \sigma(\|\boldsymbol{v}_j - \boldsymbol{v}_k\|)\|\boldsymbol{v}_j - \boldsymbol{v}_k\| d^3 n_j d^3 n_k, \qquad (4)$$

using Boltzmann distribution

$$r_{j,k} = n_j n_k \langle \sigma v \rangle_{j,k} = n_j n_k \left(\frac{8}{\mu\pi}\right)^{1/2} (k_B T)^{-3/2} \int_0^\infty E\sigma(E) e^{-E/(k_B T)} dE, \qquad (5)$$

where

$$\mu = \text{reduced mass} = \frac{m_j m_k}{m_j + m_k},$$

$$T = \text{temperature},$$

$$k_B = \text{Boltzmann constant}.$$

Note that $\langle \sigma v \rangle_{j,k} = \langle \sigma v \rangle_{j,k}(T)$.

Define *abundance*

$$Y_i = \frac{n_i}{n_B} = \frac{\#\text{ of species [i]}}{\#\text{ of baryons}}, \tag{6}$$

where n_B is baryon number density, then for [j] + [k] → [m]

$$\dot{Y}_m = \frac{r_{j,k} V}{\#\text{ of baryons}} = \frac{r_{j,k}}{n_B} = \frac{Y_j n_B Y_k n_B \langle \sigma v \rangle_{j,k}}{n_B} = Y_j Y_k \lambda_{j,k}, \tag{7}$$

where

$$\lambda_{j,k} = n_B \langle \sigma v \rangle_{j,k} = N_A \rho \langle \sigma v \rangle_{j,k}(T) = \lambda_{j,k}(T, \rho), \tag{8}$$

where N_A is Avogadro's number, and ρ is the mass density.

And, of course

$$\dot{Y}_j = \dot{Y}_k = -\dot{Y}_m. \tag{9}$$

In general

$$\dot{Y}_i = \sum_\alpha N_i^\alpha \lambda_\alpha(T,\rho) \prod_{m\in\mathcal{R}_\alpha} Y_m^{|N_m^\alpha|}, \tag{10}$$

where

$$\begin{aligned} \alpha &= \text{index running over all reactions}, \\ N_i^\alpha &= \#\text{ of species [i] destroyed/created in } \alpha, \\ \mathcal{R}_\alpha &= \text{set of reactants of } \alpha. \end{aligned}$$

Example:

$$\dot{Y}_{^4\mathrm{He}} = \underbrace{}_{\substack{\text{decay} \\ ^4\mathrm{He}\to 2\,\mathrm{d}}} \underbrace{}_{\substack{\text{producing reaction} \\ \mathrm{p}+{}^7\mathrm{Li}\to 2\,{}^4\mathrm{He}}} \underbrace{}_{\substack{\text{destroying reaction} \\ \mathrm{n}+\mathrm{p}+2\,{}^4\mathrm{He}\to {}^7\mathrm{Li}+{}^3\mathrm{He}}} + \cdots \tag{11}$$

In general

$$\dot{Y}_i = \sum_\alpha N_i^\alpha \lambda_\alpha(T,\rho) \prod_{m\in\mathcal{R}_\alpha} Y_m^{|N_m^\alpha|}, \tag{10}$$

where

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Example:

$$\dot{Y}_{^4\mathrm{He}} = \underbrace{-\lambda_{^4\mathrm{He}}}_{\substack{\text{decay} \\ ^4\mathrm{He}\to 2\,\mathrm{d}}} \quad \underbrace{}_{\substack{\text{producing reaction} \\ \mathrm{p}+{}^7\mathrm{Li}\to 2\,{}^4\mathrm{He}}} \quad \underbrace{}_{\substack{\text{destroying reaction} \\ \mathrm{n}+\mathrm{p}+2\,{}^4\mathrm{He}\to {}^7\mathrm{Li}+{}^3\mathrm{He}}} + \cdots \tag{11}$$

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In general

$$\dot{Y}_i = \sum_\alpha N_i^\alpha \lambda_\alpha(T,\rho) \prod_{m\in\mathcal{R}_\alpha} Y_m^{|N_m^\alpha|}, \tag{10}$$

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Example:

$$\dot{Y}_{^4\mathrm{He}} = \underbrace{-\lambda_{^4\mathrm{He}} Y_{^4\mathrm{He}}}_{\substack{\text{decay}\\ ^4\mathrm{He}\to 2\,\mathrm{d}}} \quad \underbrace{\qquad\qquad}_{\substack{\text{producing reaction}\\ \mathrm{p}+{}^7\mathrm{Li}\to 2\,{}^4\mathrm{He}}} \quad \underbrace{\qquad\qquad}_{\substack{\text{destroying reaction}\\ \mathrm{n}+\mathrm{p}+2\,{}^4\mathrm{He}\to {}^7\mathrm{Li}+{}^3\mathrm{He}}} + \cdots \tag{11}$$

In general

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Example:

$$\dot{Y}_{^4\mathrm{He}} = \underbrace{-\lambda_{^4\mathrm{He}} Y_{^4\mathrm{He}}}_{\substack{\text{decay} \\ ^4\mathrm{He}\to 2\,\mathrm{d}}} \underbrace{+\,2\lambda_{\mathrm{p},^7\mathrm{Li}}}_{\substack{\text{producing reaction} \\ \mathrm{p}+{}^7\mathrm{Li}\to 2\,{}^4\mathrm{He}}} \underbrace{\qquad\qquad}_{\substack{\text{destroying reaction} \\ \mathrm{n}+\mathrm{p}+2\,{}^4\mathrm{He}\to {}^7\mathrm{Li}+{}^3\mathrm{He}}} + \cdots \qquad (11)$$

In general

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Example:

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In general

$$\dot{Y}_i = \sum_\alpha N_i^\alpha \lambda_\alpha(T,\rho) \prod_{m\in\mathcal{R}_\alpha} Y_m^{|N_m^\alpha|}, \tag{10}$$

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Example:

$$\dot{Y}_{^4\mathrm{He}} = \underbrace{-\lambda_{^4\mathrm{He}} Y_{^4\mathrm{He}}}_{\substack{\text{decay} \\ ^4\mathrm{He}\to 2\,\mathrm{d}}} \underbrace{+\,2\lambda_{\mathrm{p},^7\mathrm{Li}} Y_\mathrm{p} Y_{^7\mathrm{Li}}}_{\substack{\text{producing reaction} \\ \mathrm{p}+{}^7\mathrm{Li}\to 2\,{}^4\mathrm{He}}} \underbrace{-\,2\lambda_{\mathrm{n,p,2}\,^4\mathrm{He}}}_{\substack{\text{destroying reaction} \\ \mathrm{n}+\mathrm{p}+2\,{}^4\mathrm{He}\to {}^7\mathrm{Li}+{}^3\mathrm{He}}} + \cdots \tag{11}$$

In general

$$\dot{Y}_i = \sum_\alpha N_i^\alpha \lambda_\alpha(T,\rho) \prod_{m\in\mathcal{R}_\alpha} Y_m^{|N_m^\alpha|}, \tag{10}$$

where

$$\begin{aligned} \alpha &= \text{index running over all reactions}, \\ N_i^\alpha &= \#\text{ of species [i] destroyed/created in } \alpha, \\ \mathcal{R}_\alpha &= \text{set of reactants of } \alpha. \end{aligned}$$

Example:

$$\dot{Y}_{^4\mathrm{He}} = \underbrace{-\lambda_{^4\mathrm{He}} Y_{^4\mathrm{He}}}_{\substack{\text{decay} \\ ^4\mathrm{He} \to 2\,\mathrm{d}}} \; \underbrace{+\,2\lambda_{\mathrm{p},{}^7\mathrm{Li}} Y_\mathrm{p} Y_{^7\mathrm{Li}}}_{\substack{\text{producing reaction} \\ \mathrm{p} + {}^7\mathrm{Li} \to 2\,{}^4\mathrm{He}}} \; \underbrace{-\,2\lambda_{\mathrm{n},\mathrm{p},2\,{}^4\mathrm{He}} Y_\mathrm{n} Y_\mathrm{p} Y_{^4\mathrm{He}}^2}_{\substack{\text{destroying reaction} \\ \mathrm{n} + \mathrm{p} + 2\,{}^4\mathrm{He} \to {}^7\mathrm{Li} + {}^3\mathrm{He}}} + \cdots \tag{11}$$

Given $\boldsymbol{Y}$, T, and ρ, we have a (big) system of coupled ODEs:

$$\dot{\boldsymbol{Y}} = \dot{\boldsymbol{Y}}(\boldsymbol{Y}, T, \rho). \tag{12}$$

Use implicit method due to stiffness

$$\dot{\boldsymbol{Y}}(t + \Delta t) = \frac{\boldsymbol{Y}(t + \Delta t) - \boldsymbol{Y}(t)}{\Delta t}. \tag{13}$$

Use Newton–Raphson to find unknown $\boldsymbol{x} = \boldsymbol{Y}(t + \Delta t)$ by finding root of

$$\boldsymbol{F}(\boldsymbol{x}) = \frac{\boldsymbol{x} - \boldsymbol{c}}{\Delta t} - \dot{\boldsymbol{Y}}(\boldsymbol{x}, T, \rho) = \boldsymbol{0}, \tag{14}$$

where $\boldsymbol{c} = \boldsymbol{Y}(t)$ is a known constant. Update guess with

$$\boldsymbol{x}_{n+1} = \boldsymbol{x}_n - \left[J_{\boldsymbol{F}}(\boldsymbol{x}_n)\right]^{-1} \boldsymbol{F}(\boldsymbol{x}_n), \tag{15}$$

where the (sparse) Jacobian is

$$(J_{\boldsymbol{F}})_{ij} = \frac{\partial F_i}{\partial Y_j} = \frac{\delta_{ij}}{\Delta t} - \frac{\partial \dot{Y}_i}{\partial Y_j}. \tag{16}$$

Time Stepping Method

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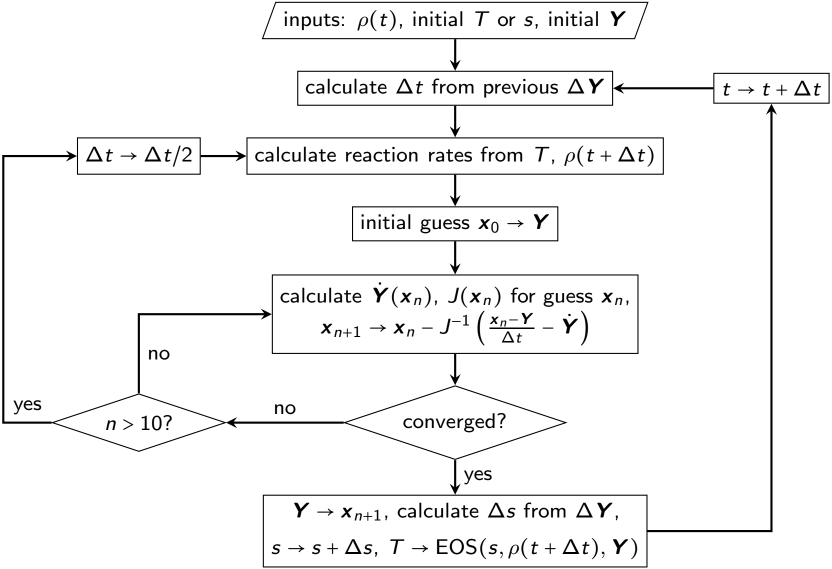


Let species [i] have N_i neutrons and Z_i protons. NSE means the reactions

$$[\mathrm{i}] \leftrightarrow N_i\,\mathrm{n} + Z_i\,\mathrm{p} \tag{17}$$

are in equilibrium so

$$\mu_i = N_i \mu_\mathrm{n} + Z_i \mu_\mathrm{p}, \tag{18}$$

where μ_x are chemical potentials. Abundance is

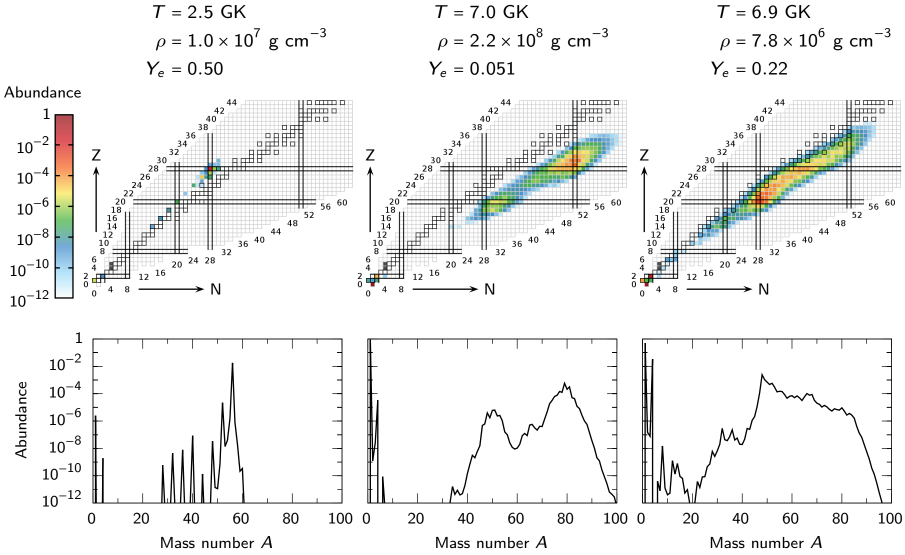
$$Y_i = e^{\mu_i/(k_B T)} \frac{G_i(T)}{n_B} \left(\frac{m_i k_B T}{2\pi\hbar^2}\right)^{3/2}, \tag{19}$$

where $G_i(T)$ is internal partition function. Now find μ_n and μ_p such that

$$1 = \sum_i A_i Y_i \qquad \text{and} \qquad Y_e = \sum_i Z_i Y_i, \tag{20}$$

where $A_i = N_i + Z_i$.

Nuclear Statistical Equilibrium (NSE)



Neutron Star–Black Hole Merger

Caltech

1. Full GR neutron star–black hole merger simulation (right), up to 10's of ms
   Francois Foucart (LBL), *Foucart et al.*, Phys. Rev. D 90, 024026 (2014)
2. Follow ejecta (few $\times 10^{-2} M_{\odot}$) in SPH simulation, up to 10 s, get many $\rho(t)$ histories,
   Matt Duez (WSU)
3. Nucleosynthesis with SkyNet, extend $\rho \propto t^{-3}$, 7841 isotopes, 95,467 reactions
   JL with Luke Roberts (Caltech)

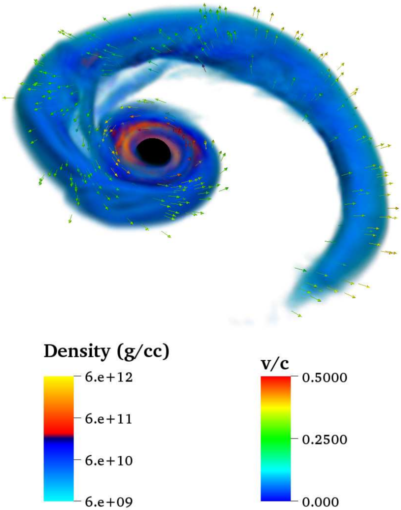


Figure credit: F. Foucart

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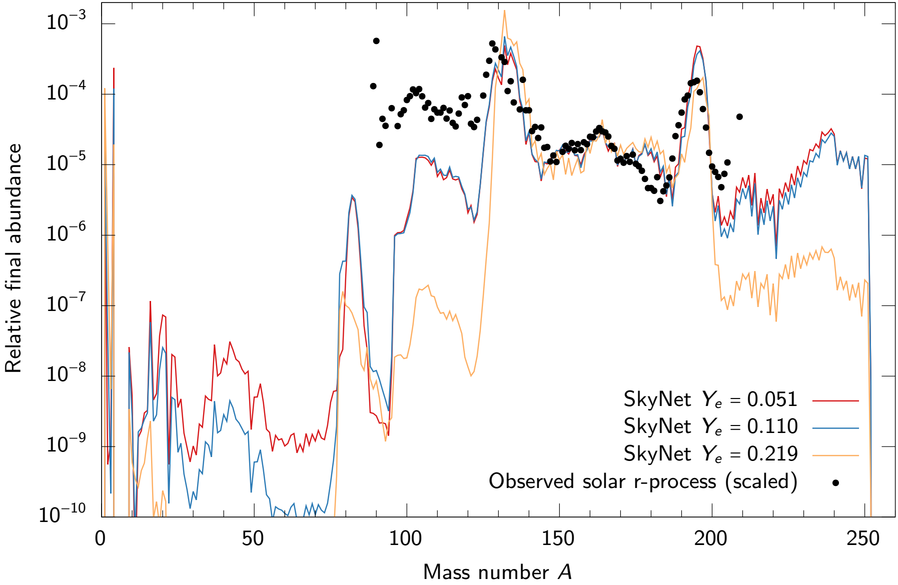
Relative final abundance
Mass number A
SkyNet Ye = 0.051
SkyNet Ye = 0.110
SkyNet Ye = 0.219
Observed solar r-process (scaled)
10−3
10−4
10−5
10−6
10−7
10−8
10−9
10−10
0
50
100
150
200
250

Parameter Study: Running SkyNet with Python

Caltech

```python
#!/usr/bin/env python
from SkyNet import *
import numpy as np

init_T9 = 6.0; Ye = 0.01; s = 10.0; tau_ms = 7.1
opts = NetworkOptions()
opts.ConvergenceCriterion = NetworkConvergenceCriterion.Mass
opts.MassDeviationThreshold = 1.0E-10
opts.IsSelfHeating = True

nuclib = NuclideLibrary.CreateFromWebnucleoXML(SkyNetRoot + "/data/webnucleo_nuc_v2.0.xml")
helm = HelmholtzEOS(SkyNetRoot + "/data/helm_table.dat")
weakReacs = REACLIBReactionLibrary(SkyNetRoot + "/data/reaclib",
    ReactionType.Weak, False, "Weak reactions", nuclib, opts)
strongReacs = REACLIBReactionLibrary(SkyNetRoot + "/data/reaclib",
    ReactionType.Strong, True, "Strong reactions", nuclib, opts)
neutFissReacs = REACLIBReactionLibrary(SkyNetRoot + "/data/netsu_panov_symmetric_0neut",
    ReactionType.Strong, False, "Symmetric neutron induced fission reactis", nuclib, opts)
spontFissReacs = REACLIBReactionLibrary(SkyNetRoot + "/data/netsu_sfis_Roberts2010rates",
    ReactionType.Strong, False, "Spontaneous fission reactions", nuclib, opts)

net = ReactionNetwork(nuclib, [weakReacs, strongReacs, neutFissReacs, spontFissReacs], helm, opts)
nse = NSE.CalcFromTemperatureAndEntropy(init_T9, s, Ye, net.GetNuclideLibrary(), helm)
density_profile = ExpTMinus3(nse.Rho(), tau_ms / 1000.0)
output = net.EvolveSelfHeatingWithInitialTemperature(nse.Y(), 0.0, 1.0E9, init_T9, density_profile,
    "Ye_%4.2f_s_%07.3f_tau_%07.3f" % (Ye, s, tau_ms)); final_y = output.FinalYVsA()
np.savetxt("Y_vs_A", np.column_stack((np.arange(len(final_y)), final_y)), "%6i  %30.20E")
```