# Equation of state and neutron star properties constrained by nuclear physics and observation

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Stockholm, August 17, 2015

TECHNISCHE UNIVERSITÄT DARMSTADT

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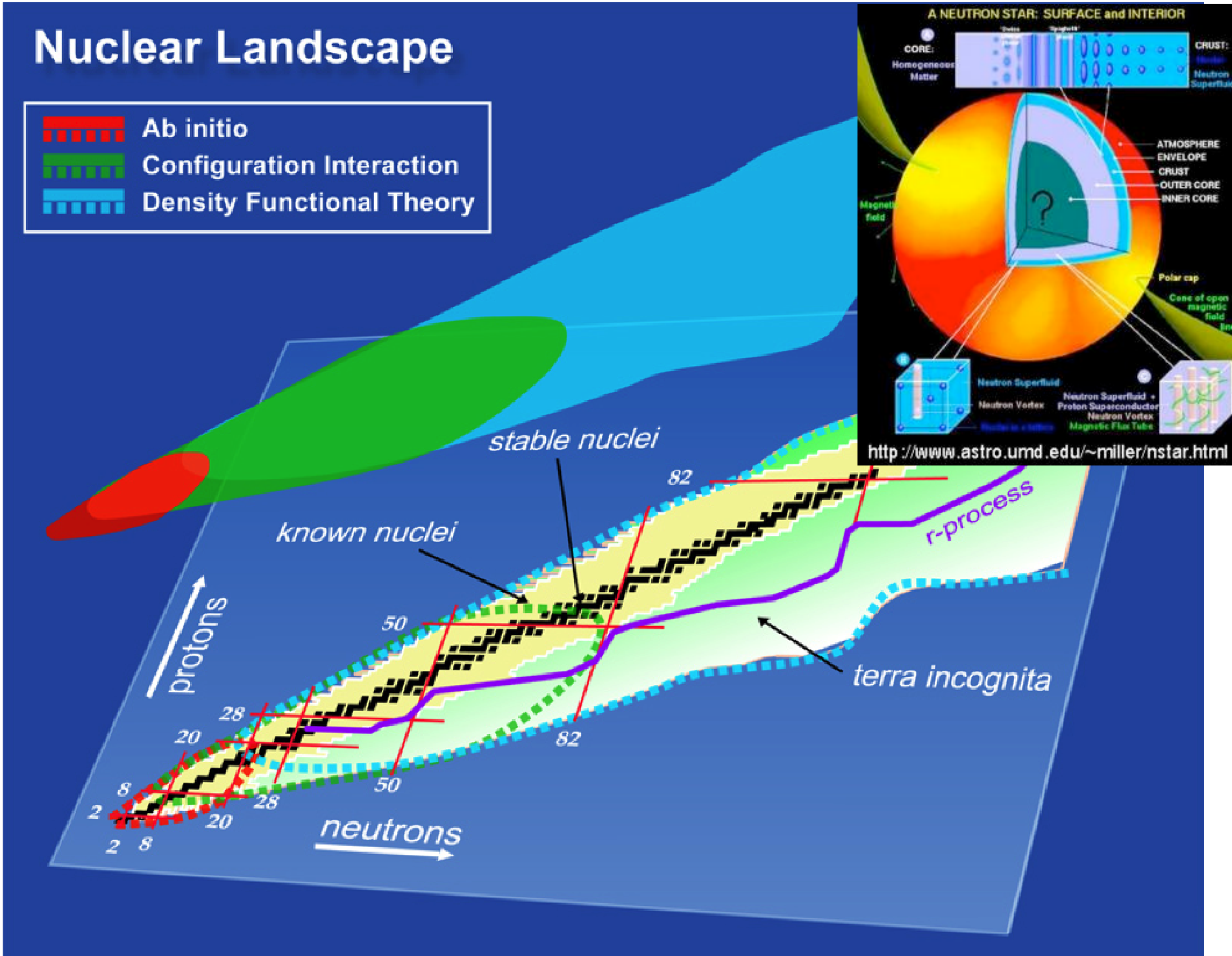
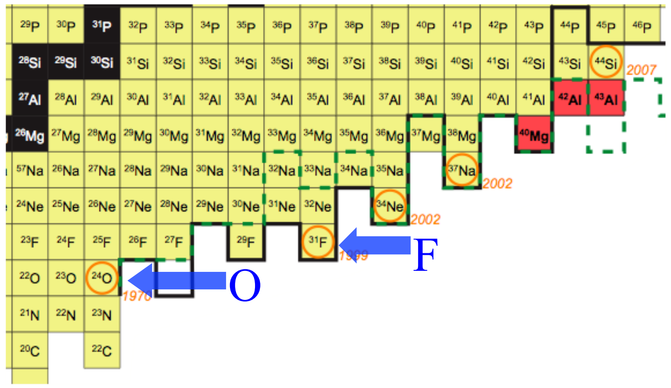
**MICRA 2015:**
**Workshop on Microphysics in Computational Relativistic Astrophysics**

# Nuclear Landscape



# Exciting recent developments on many fronts...




## LETTER

nature
doi:10.1038/nature11188

### The limits of the nuclear landscape

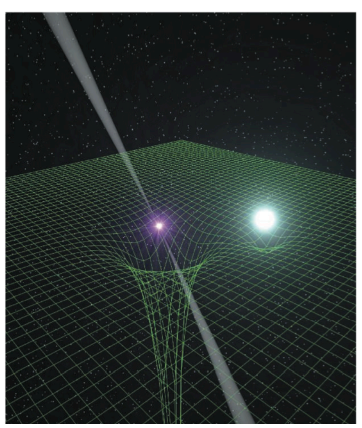
Jochen Erler[1,2], Noah Birge[1], Markus Kortelainen[1,2,3], Witold Nazarewicz[1,2,4], Erik Olsen[1,2], Alexander M. Perhac[1] & Mario Stoitsov[1,2]‡

## LETTER

nature
doi:10.1038/nature12522

### Evidence for a new nuclear 'magic number' from the level structure of $^{54}$Ca

D. Steppenbeck[1], S. Takeuchi[2], N. Aoi[3], P. Doornenbal[2], M. Matsushita[1], H. Wang[2], H. Baba[2], N. Fukuda[2], S. Go[1], M. Honma[4], J. Lee[2], K. Matsui[5], S. Michimasa[1], T. Motobayashi[2], D. Nishimura[6], T. Otsuka[1,5], H. Sakurai[2,5], Y. Shiga[7], P.-A. Söderström[2], T. Sumikama[8], H. Suzuki[2], R. Taniuchi[5], Y. Utsuno[9], J. J. Valiente-Dobón[10] & K. Yoneda[2]



PSR J0348+0432

## LETTER

nature
doi:10.1038/nature12226

### Masses of exotic calcium isotopes pin down nuclear forces

F. Wienholtz[1], D. Beck[2], K. Blaum[3], Ch. Borgmann[3], M. Breitenfeldt[4], R. B. Cakirli[3,5], S. George[1], F. Herfurth[2], J. D. Holt[6,7], M. Kowalska[8], S. Kreim[3,8], D. Lunney[9], V. Manea[9], J. Menéndez[6,7], D. Neidherr[2], M. Rosenbusch[1], L. Schweikhard[1], A. Schwenk[7,6], J. Simonis[6,7], J. Stanja[10], R. N. Wolf[1] & K. Zuber[10]

## LETTER

nature
doi:10.1038/nature09466

### A two-solar-mass neutron star measured using Shapiro delay

P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]

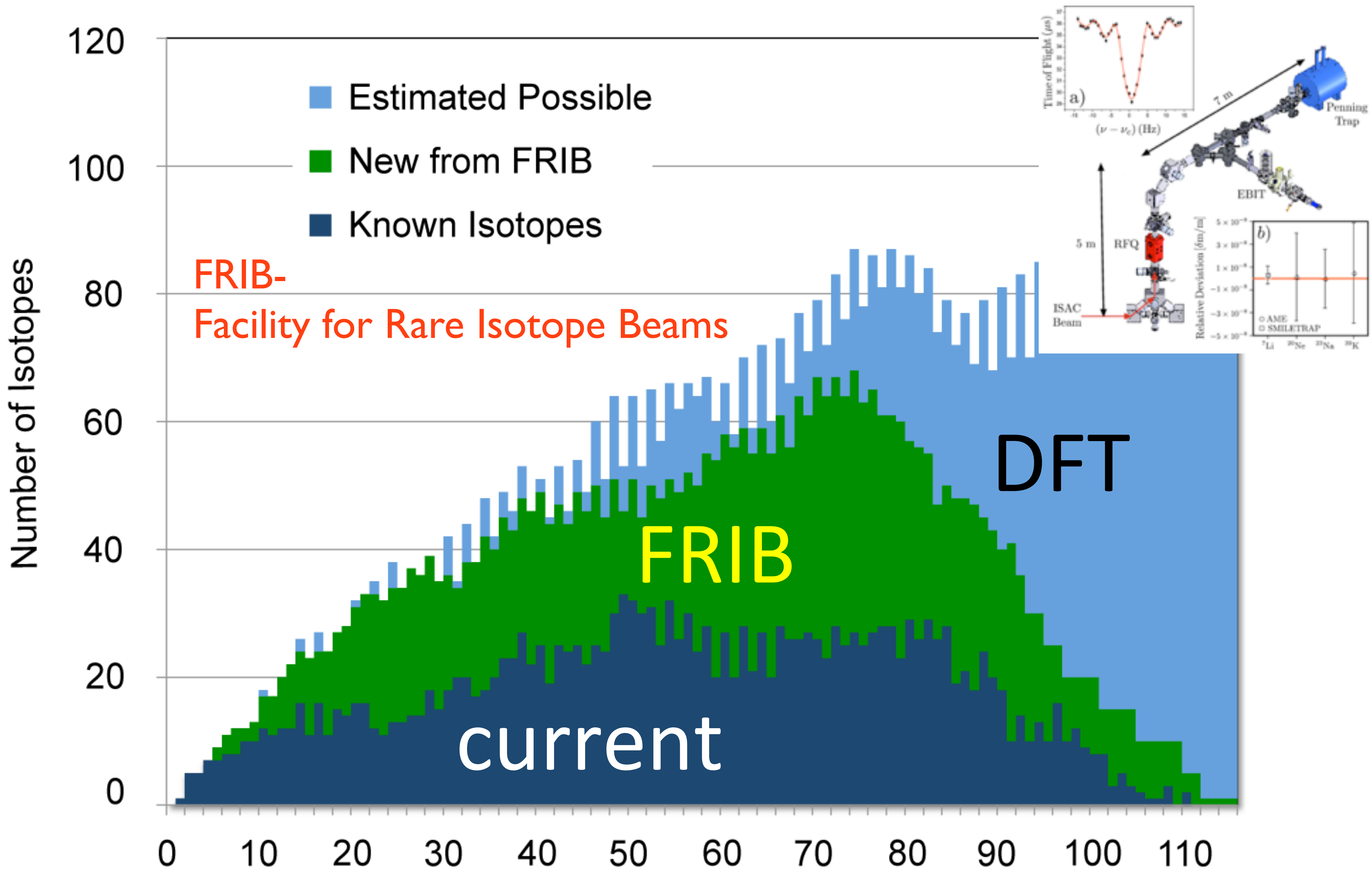
## RESEARCH ARTICLE SUMMARY

Science

### A Massive Pulsar in a Compact Relativistic Binary

John Antoniadis,* Paulo C. C. Freire, Norbert Wex, Thomas M. Tauris, Ryan S. Lynch, Marten H. van Kerkwijk, Michael Kramer, Cees Bassa, Vik S. Dhillon, Thomas Driebe, Jason W. T. Hessels, Victoria M. Kaspi, Vladislav I. Kondratiev, Norbert Langer, Thomas R. Marsh, Maura A. McLaughlin, Timothy T. Pennucci, Scott M. Ransom, Ingrid H. Stairs, Joeri van Leeuwen, Joris P. W. Verbiest, David G. Whelan

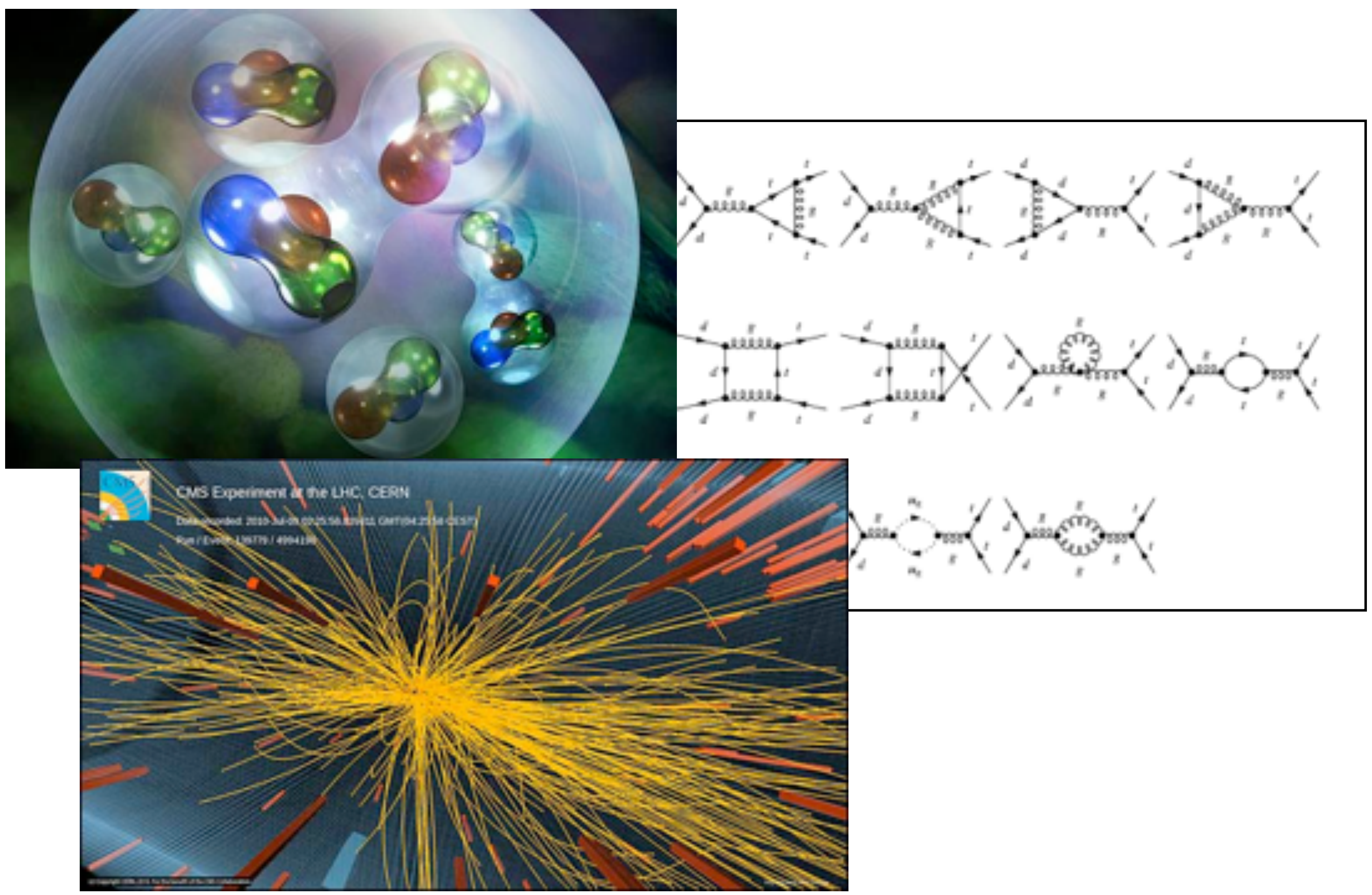
# New frontiers from rare isotope facilities

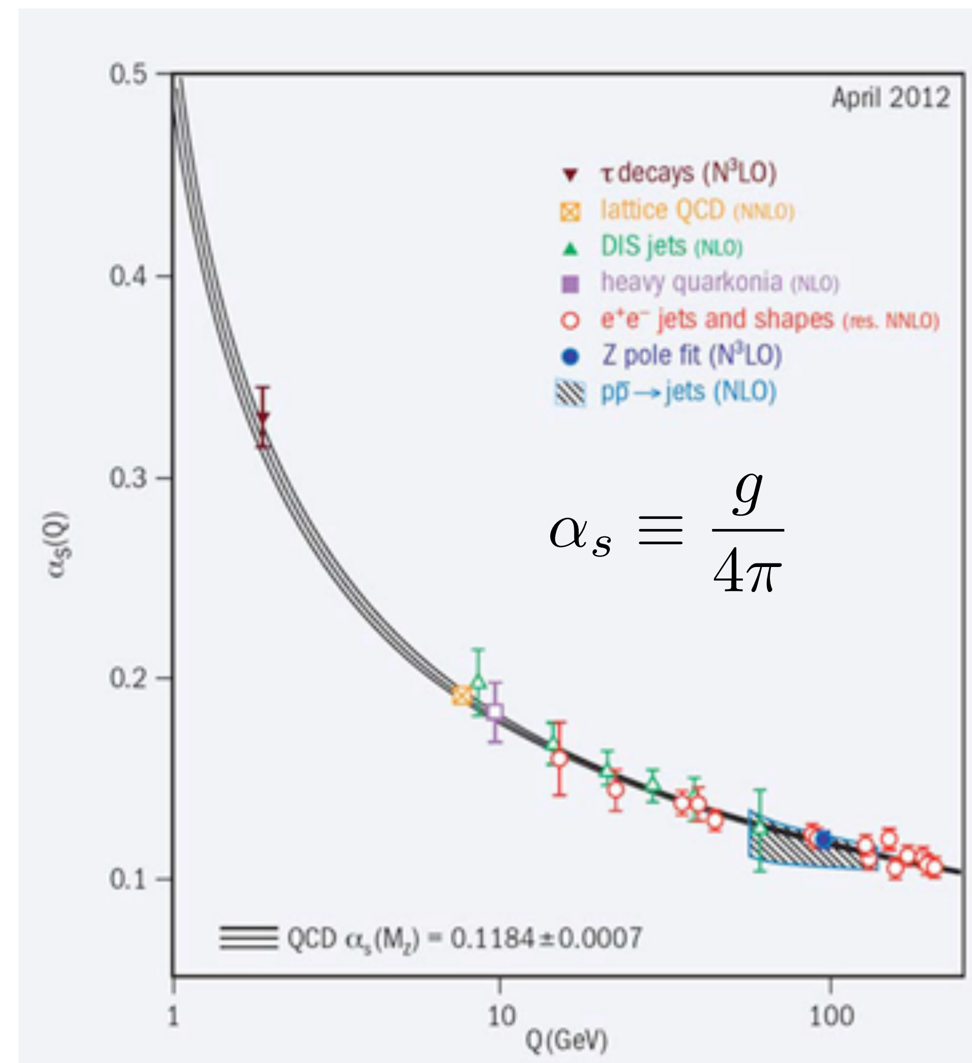



Balantekin et al. , arXiv:1401.6435

# Theory of the strong interaction: Quantum chromodynamics

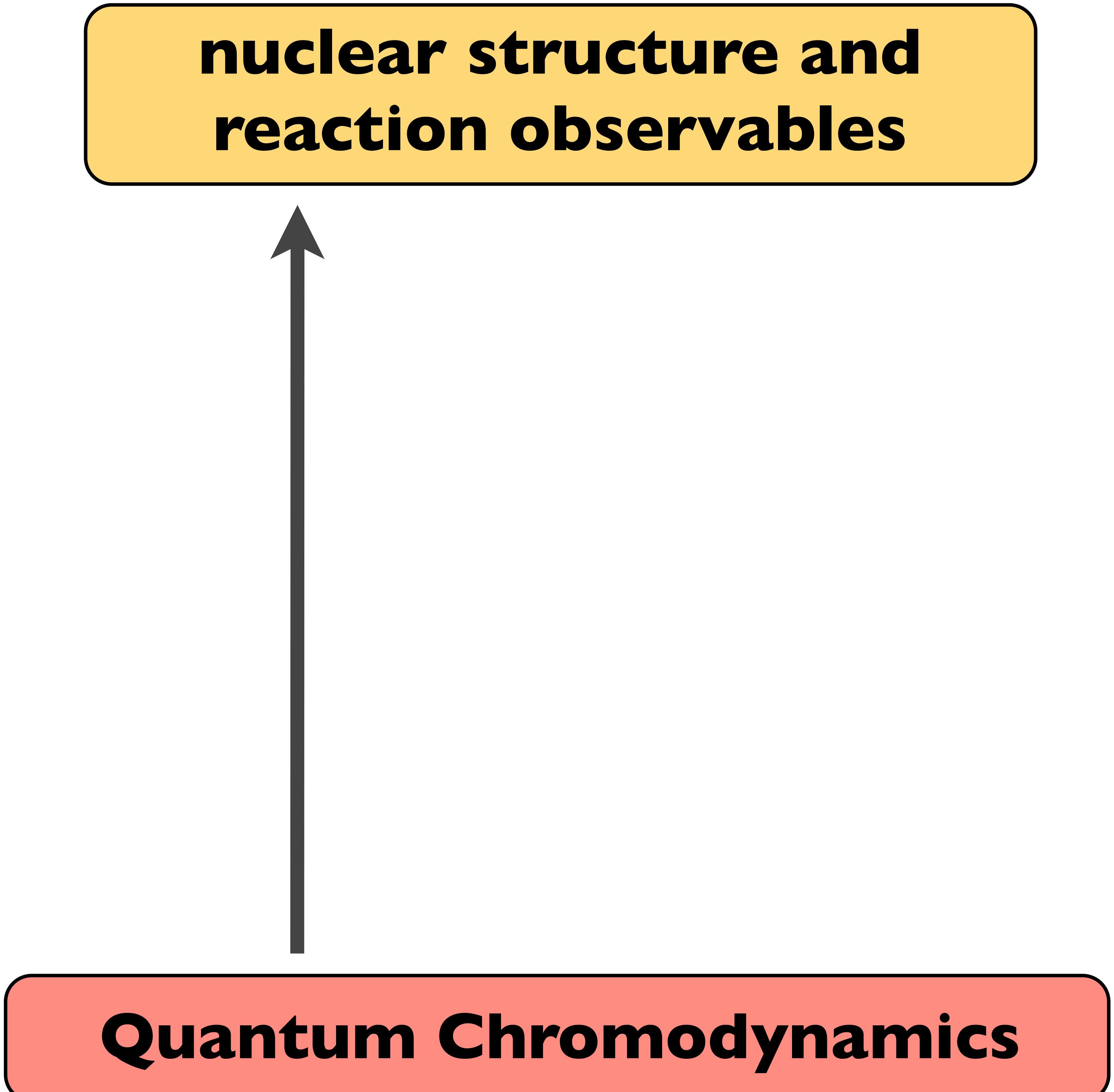
$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \overline{q}(i\gamma^\mu\partial_\mu - m)q + g\overline{q}\gamma^\mu T_a q A^a_\mu$$




- theory perturbative at high energies
- highly non-perturbative at low energies

# Ab initio nuclear structure theory



# Ab initio nuclear structure theory

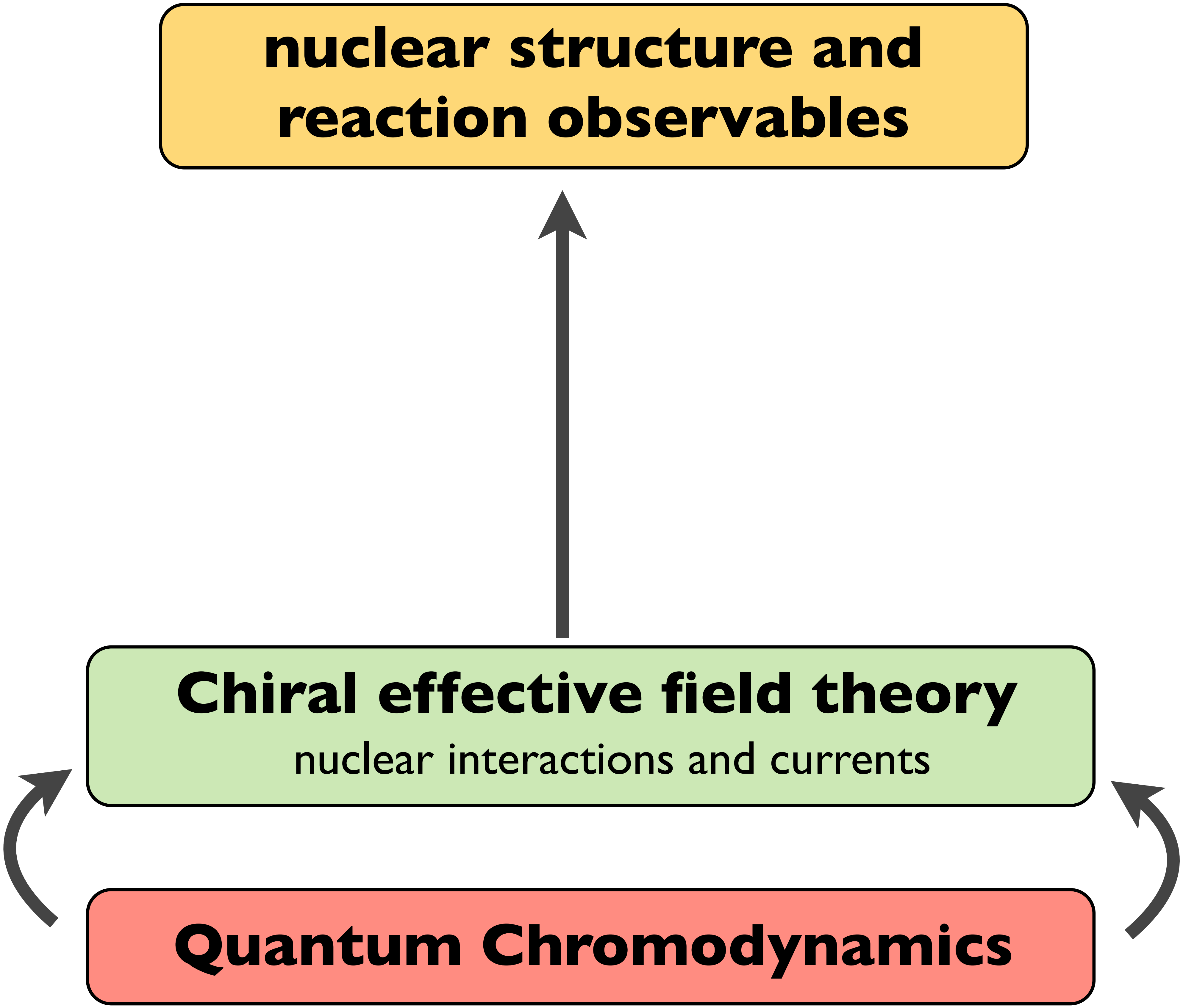
**nuclear structure and reaction observables**

## Lattice QCD

- requires extreme amounts of computational resources
- currently limited to 1- or 2-nucleon systems
- current accuracy insufficient for precision nuclear structure

**Quantum Chromodynamics**

# Ab initio nuclear structure theory



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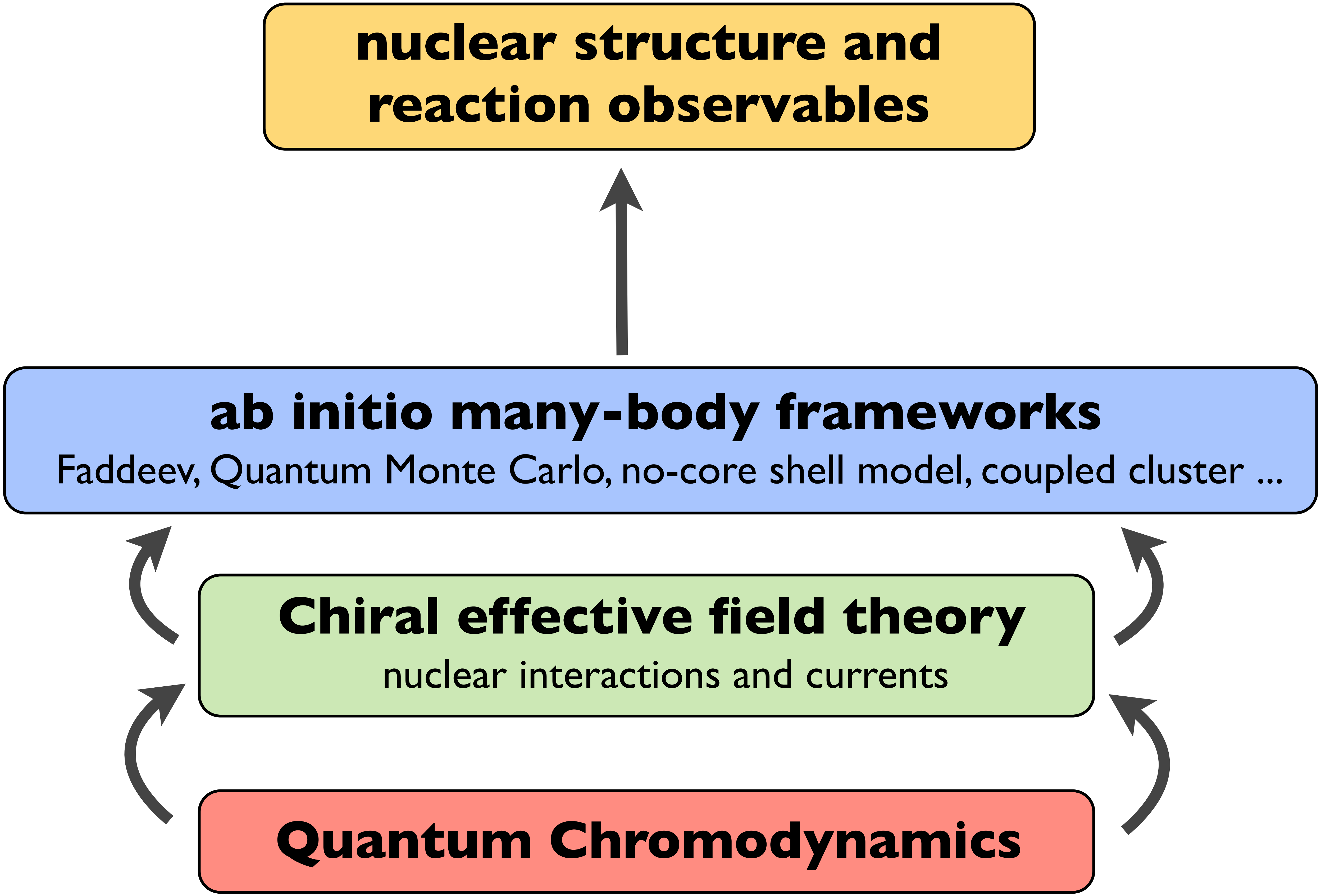

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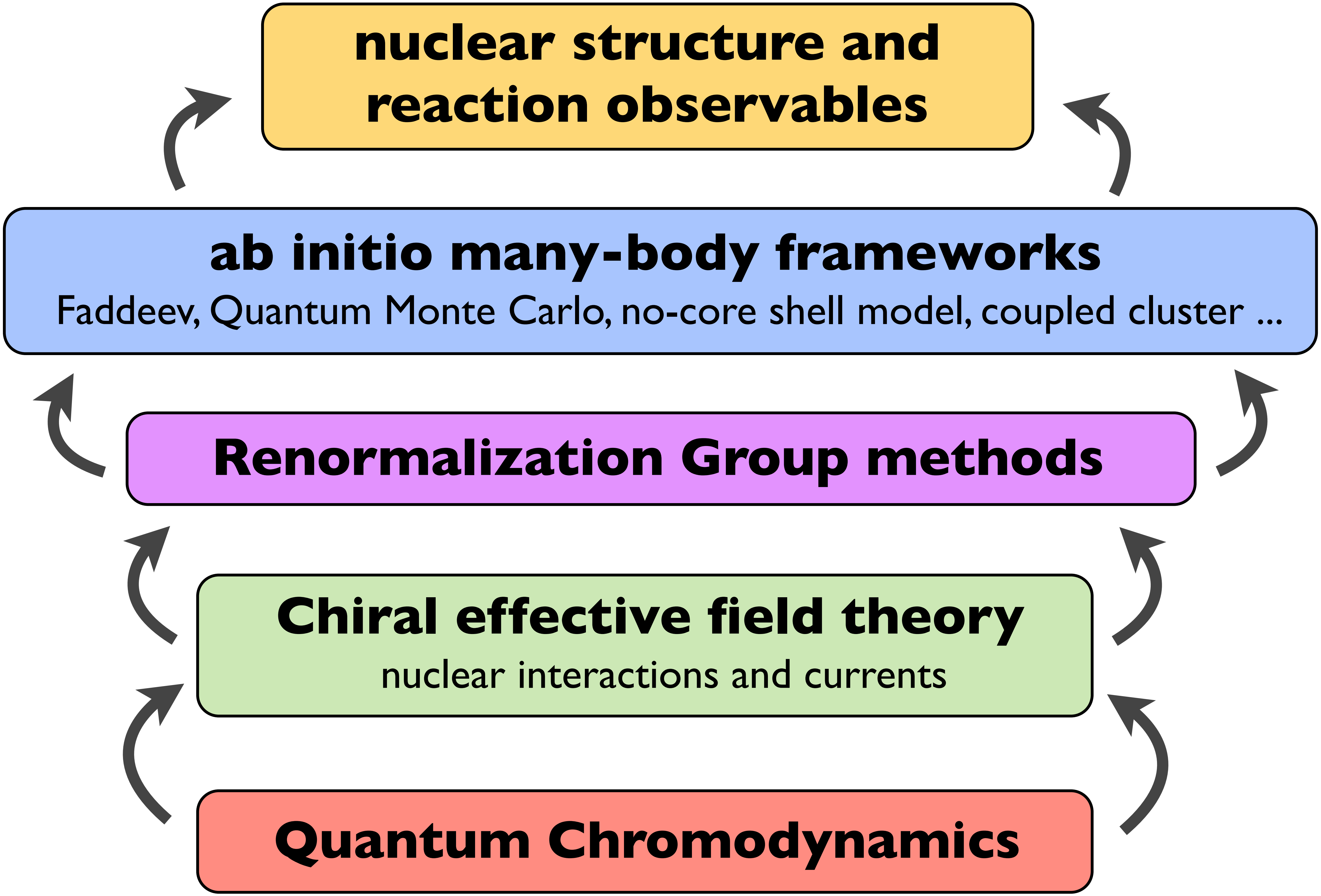
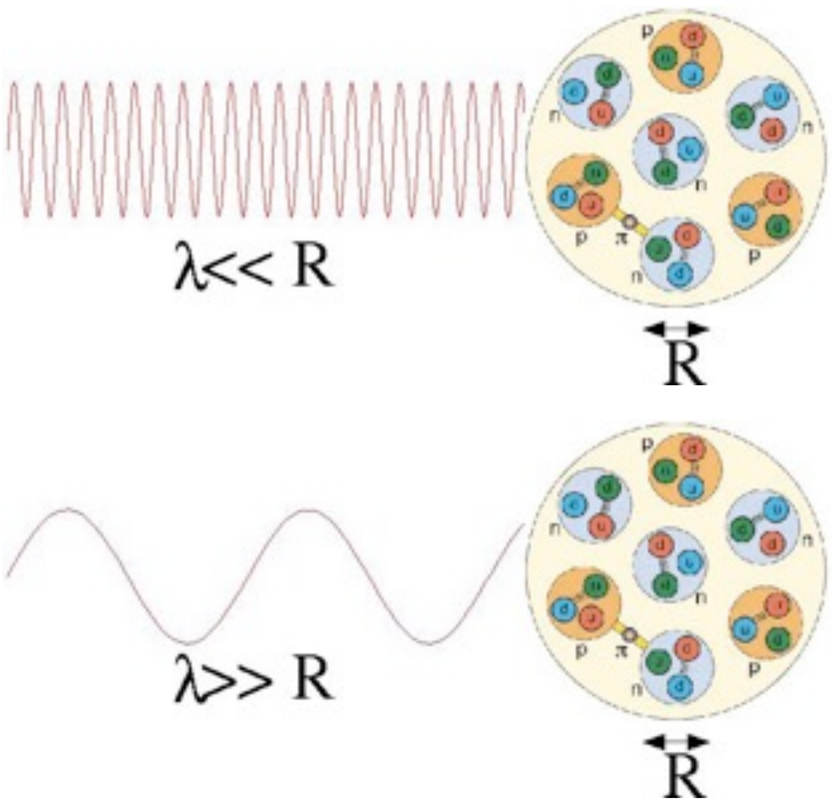


# Nuclear effective degrees of freedom




- if a nucleus is probed at high energies, nucleon substructure is resolved
- at low energies, details are not resolved

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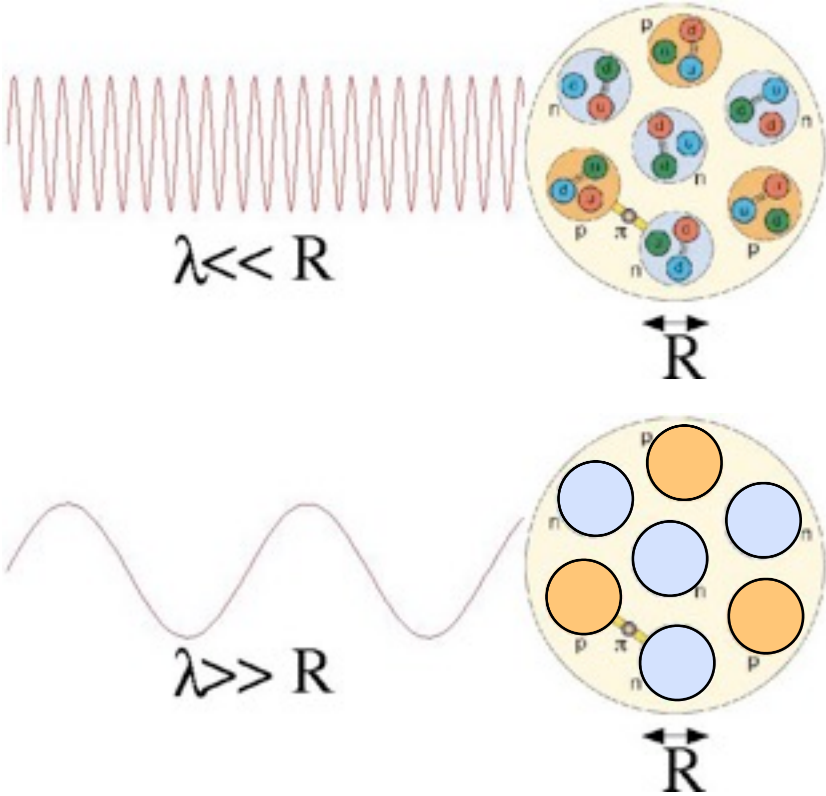
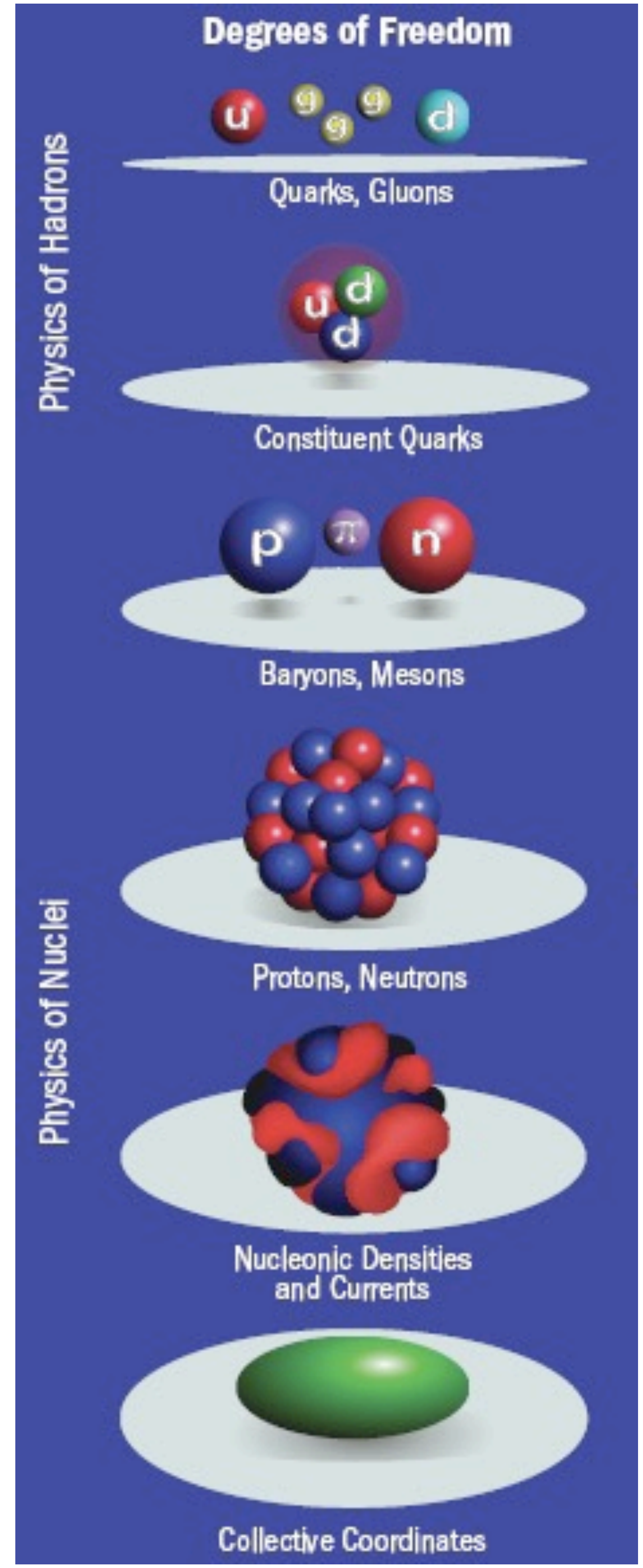


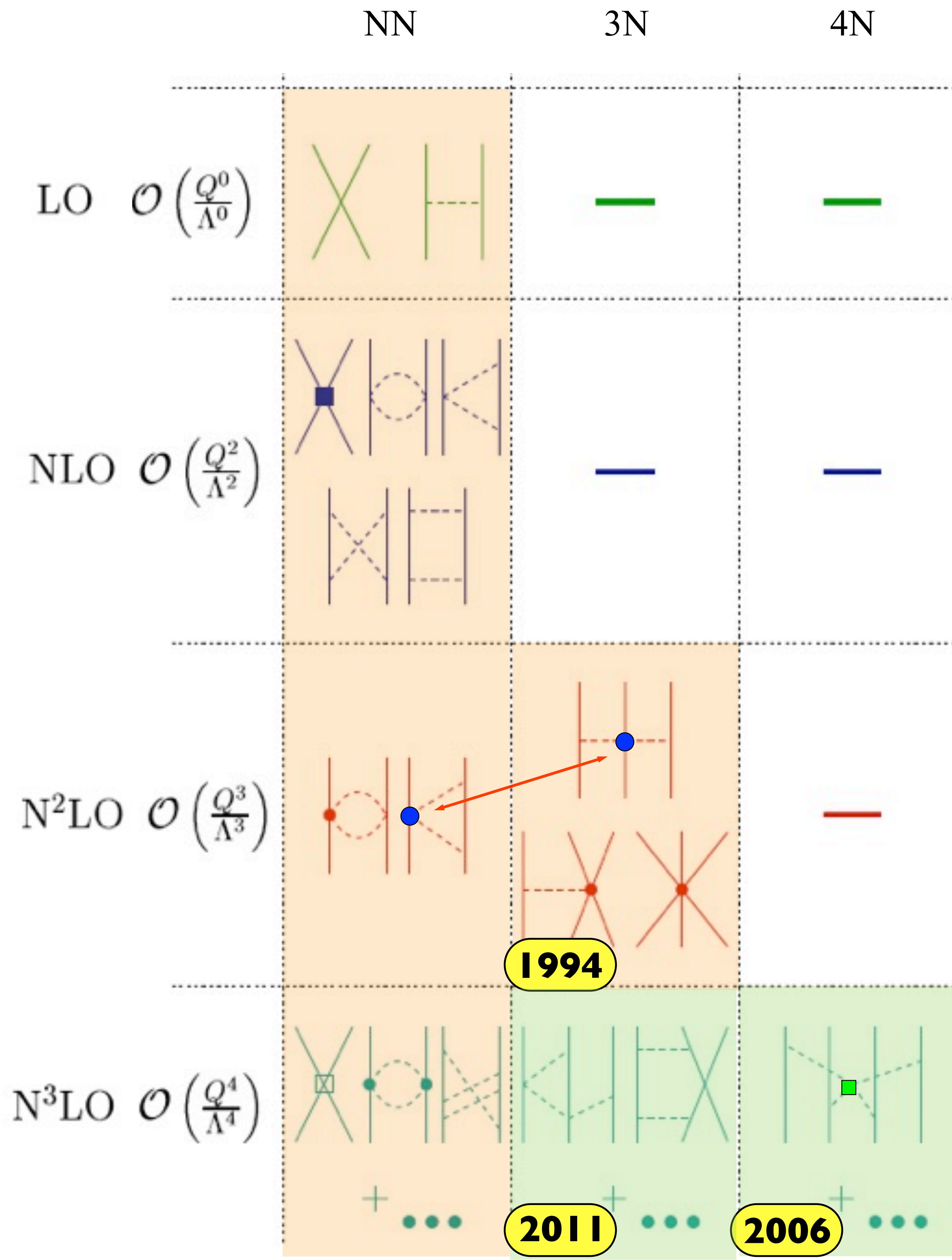

**Resolution** ↓

- if a nucleus is probed at high energies, nucleon substructure is resolved
- at low energies, details are not resolved
- replace fine structure by something simpler (like multipole expansion), low-energy observables unchanged

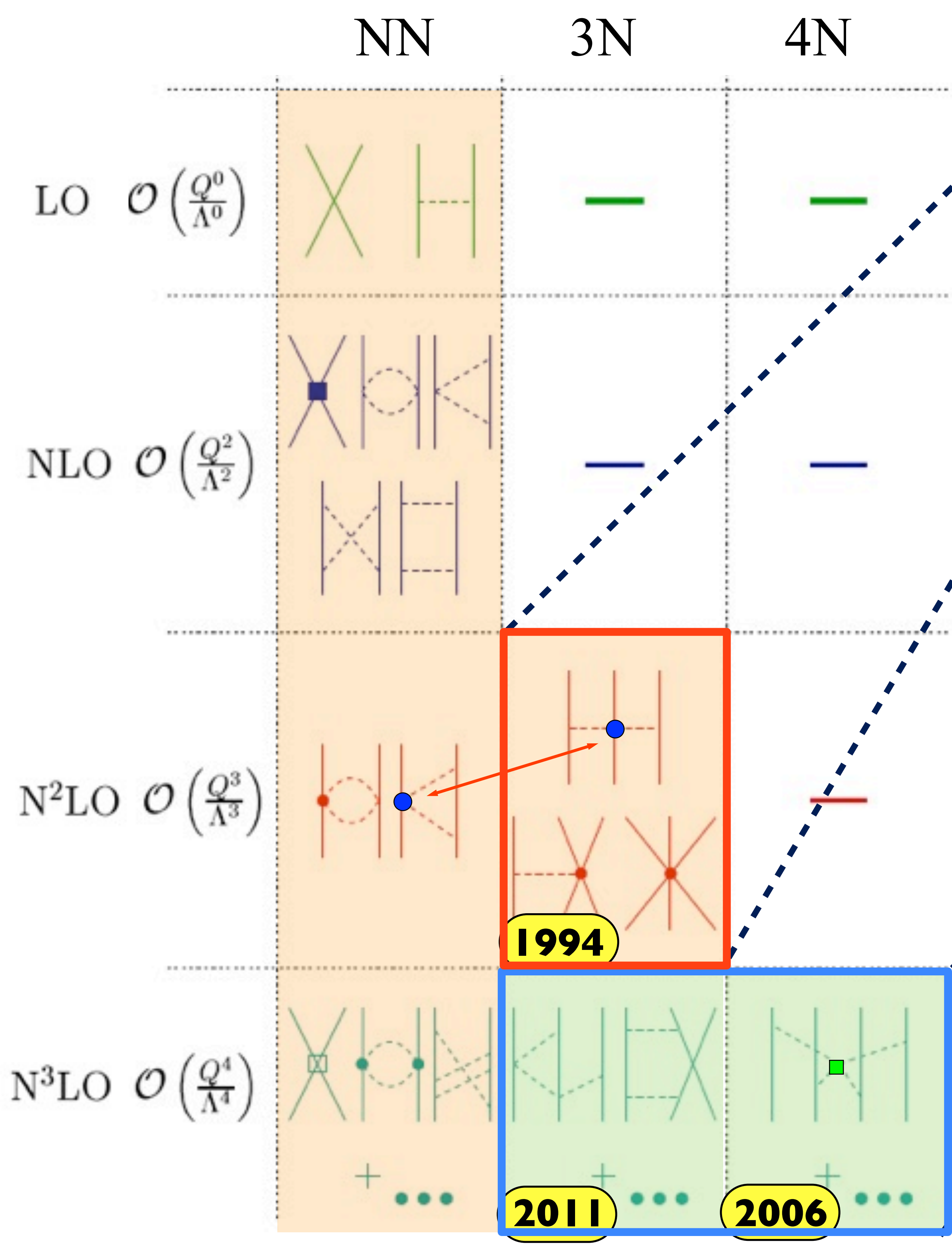
⟶ effective field theory

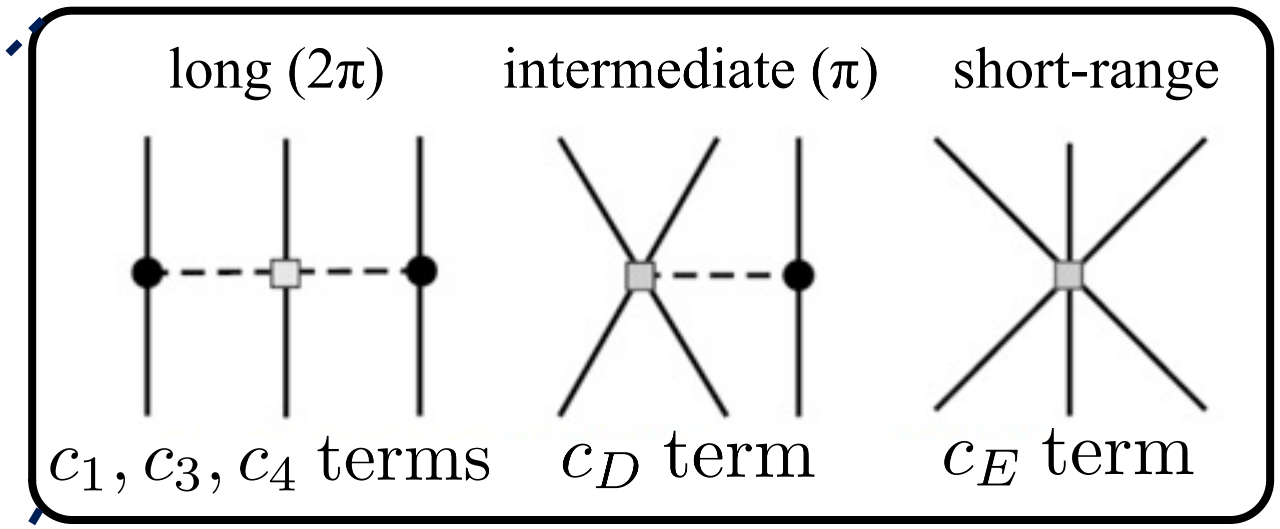
# Chiral effective field theory for nuclear forces

- choose relevant degrees of freedom: here nucleons and pions
- operators constrained by symmetries of QCD
- short-range physics captured in few short-range couplings
- separation of scales: Q << $\Lambda_b$, breakdown scale $\Lambda_b$~500 MeV
- power-counting: expand in powers Q/$\Lambda_b$
- systematic: work to desired accuracy, obtain error estimates
- 3NF at N3LO completely predicted, no new couplings



# Chiral EFT for nuclear forces




large uncertainties in coupling constants at present:

$$c_1 = -0.9^{+0.2}_{-0.5}\,,\ c_3 = -4.7^{+1.5}_{-1.0}\,,\ \ c_4 = 3.5^{+0.5}_{-0.2}$$

first incorporation in calculations of neutron and nuclear matter

Tews, Krueger, KH, Schwenk, PRL 110, 032504 (2013)
Krueger, Tews, KH, Schwenk, PRC 88, 025802 (2013)

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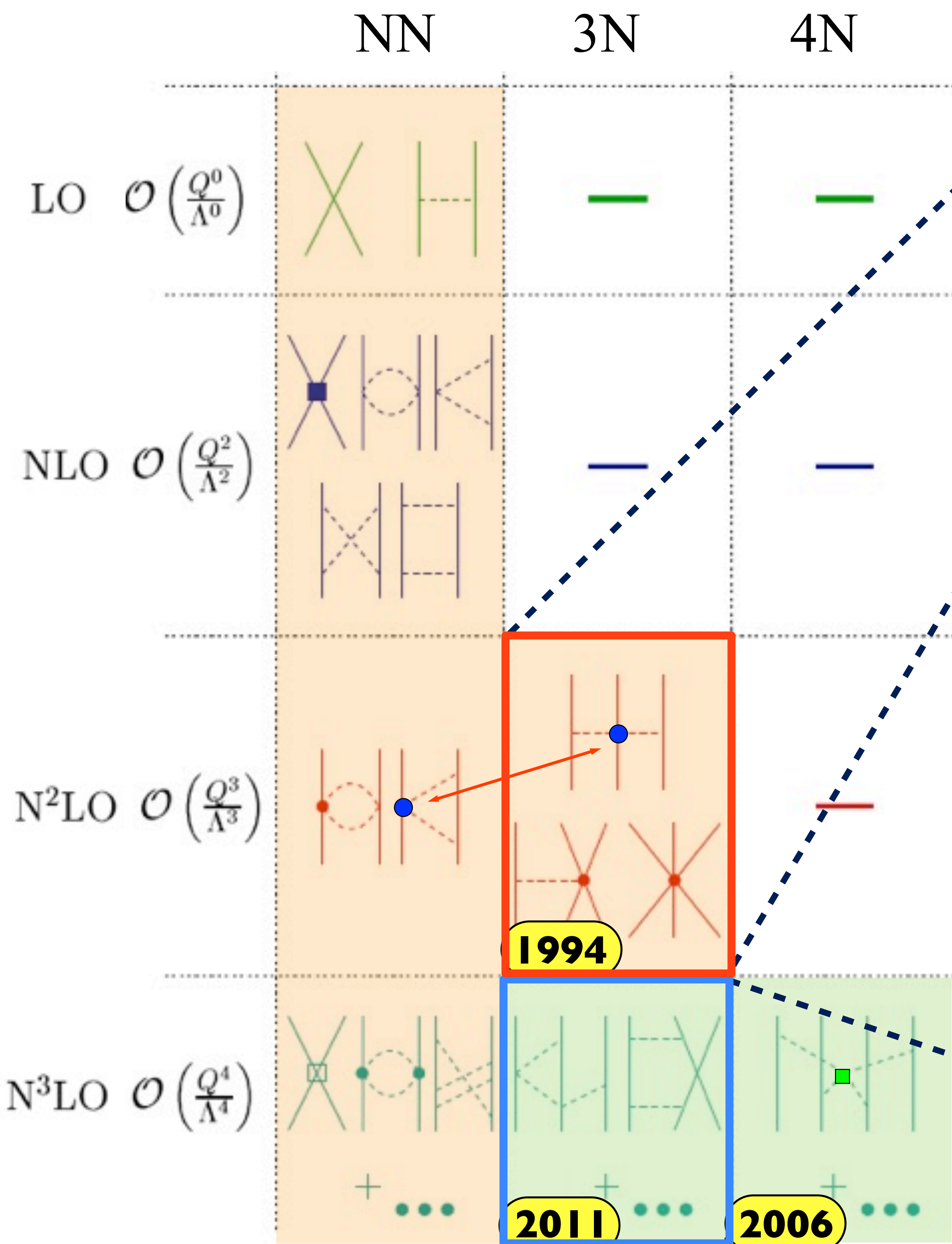


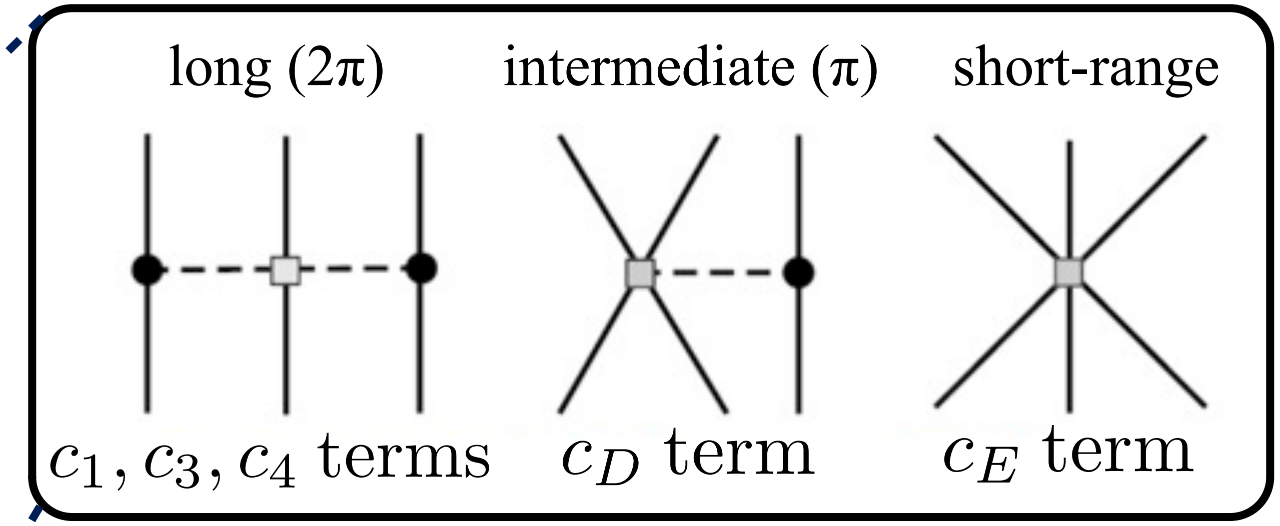


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Tews, Krueger, KH, Schwenk, PRL 110, 032504 (2013)
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first partial wave decomposition, opens the way to new ab initio studies

KH, Krebs, Epelbaum, Golak, Skibinski,
PRC 91, 044001 (2015)

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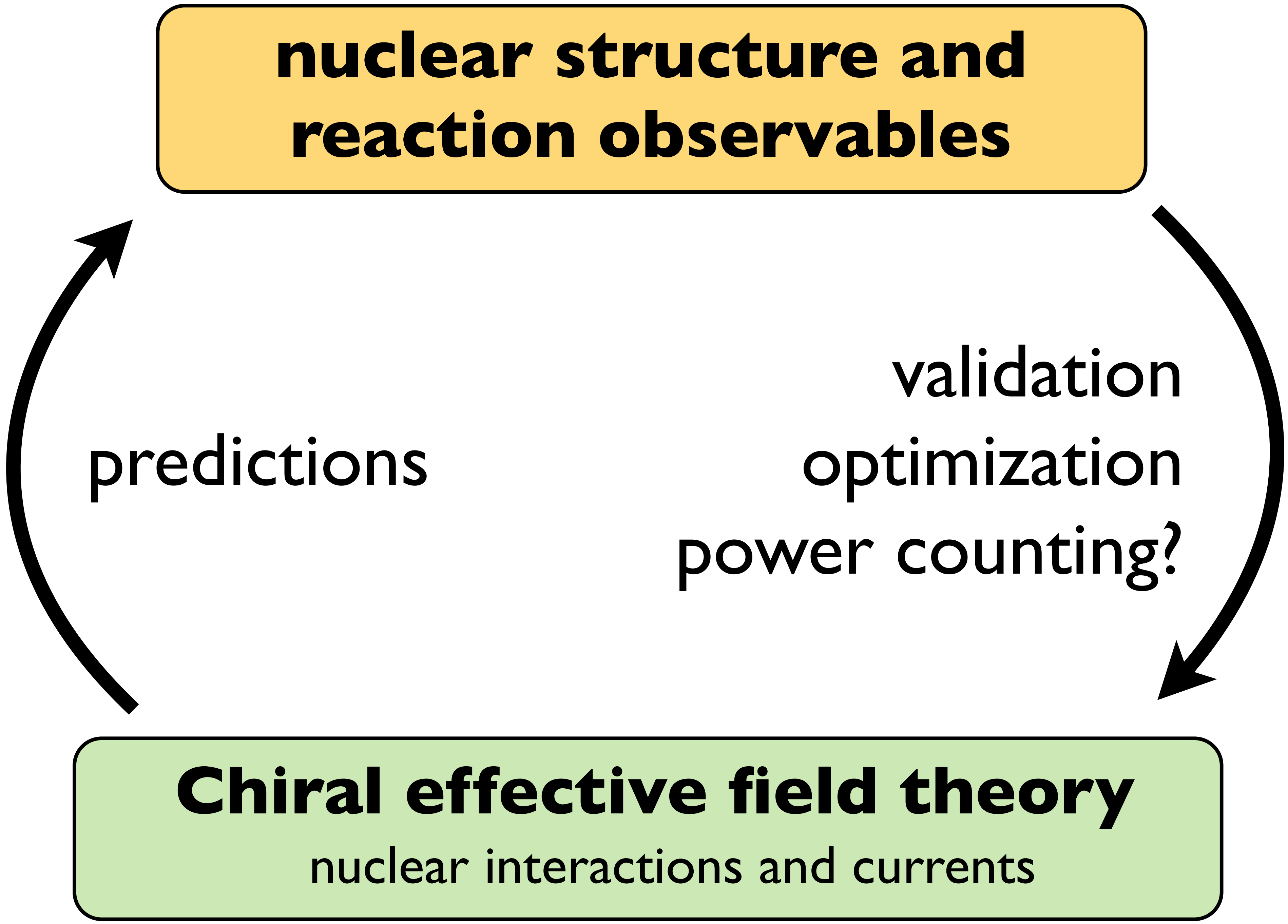
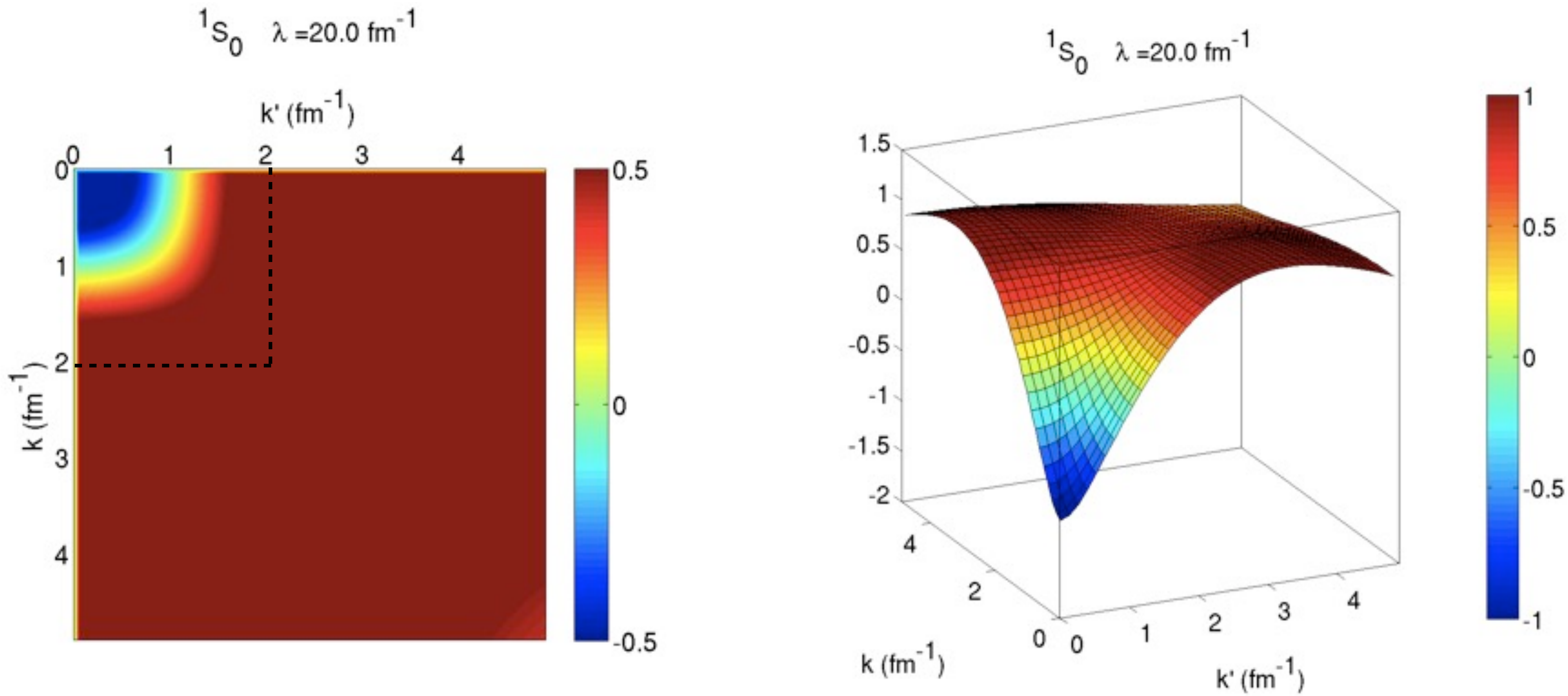


# Systematic decoupling of high-momentum physics: The Similarity Renormalization Group

- generate unitary transformation which decouples low- and high momenta

$$H_\lambda = U_\lambda H U_\lambda^\dagger \quad \text{with the resolution parameter } \lambda$$

- basic idea: change resolution successively in small steps: $\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$
- generator $\eta_\lambda$ can be chosen and tailored to different applications
- observables are preserved due to unitarity of transformation



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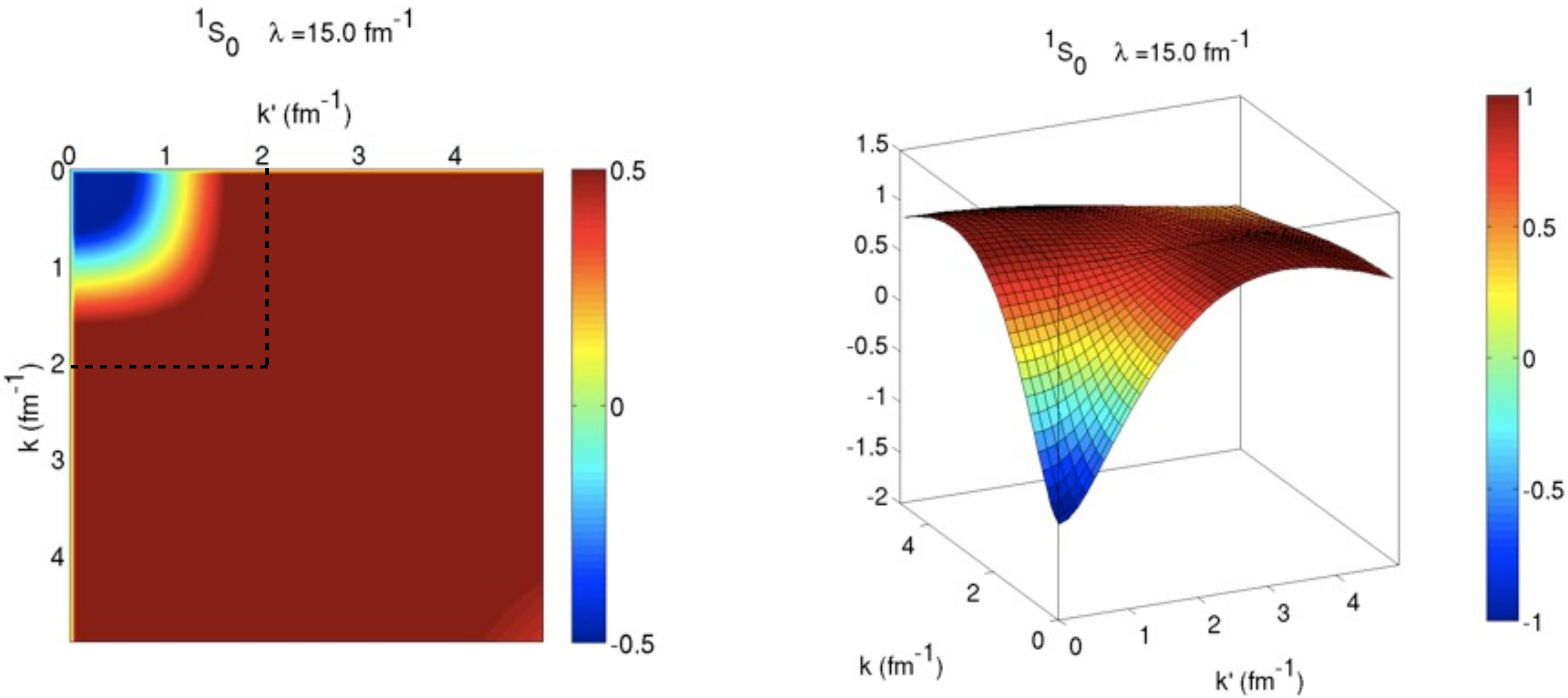

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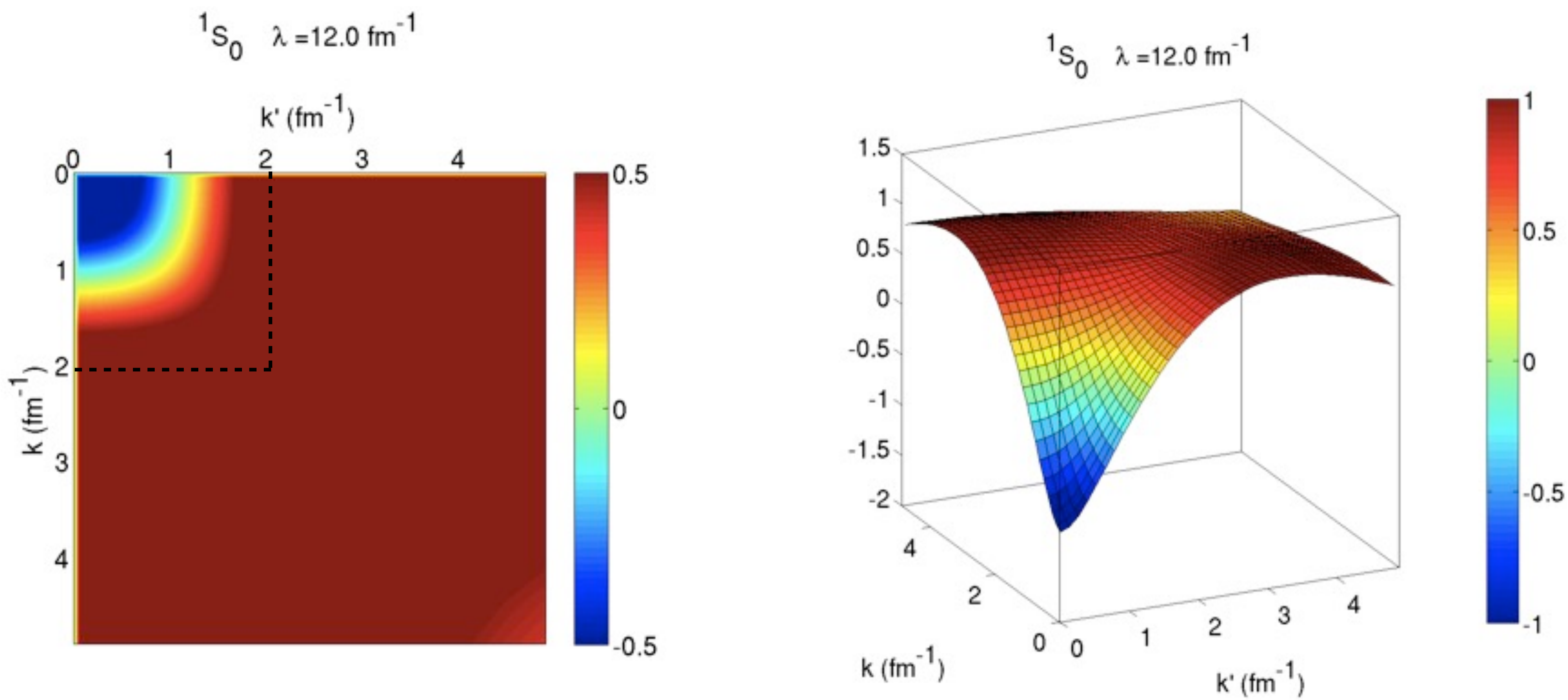

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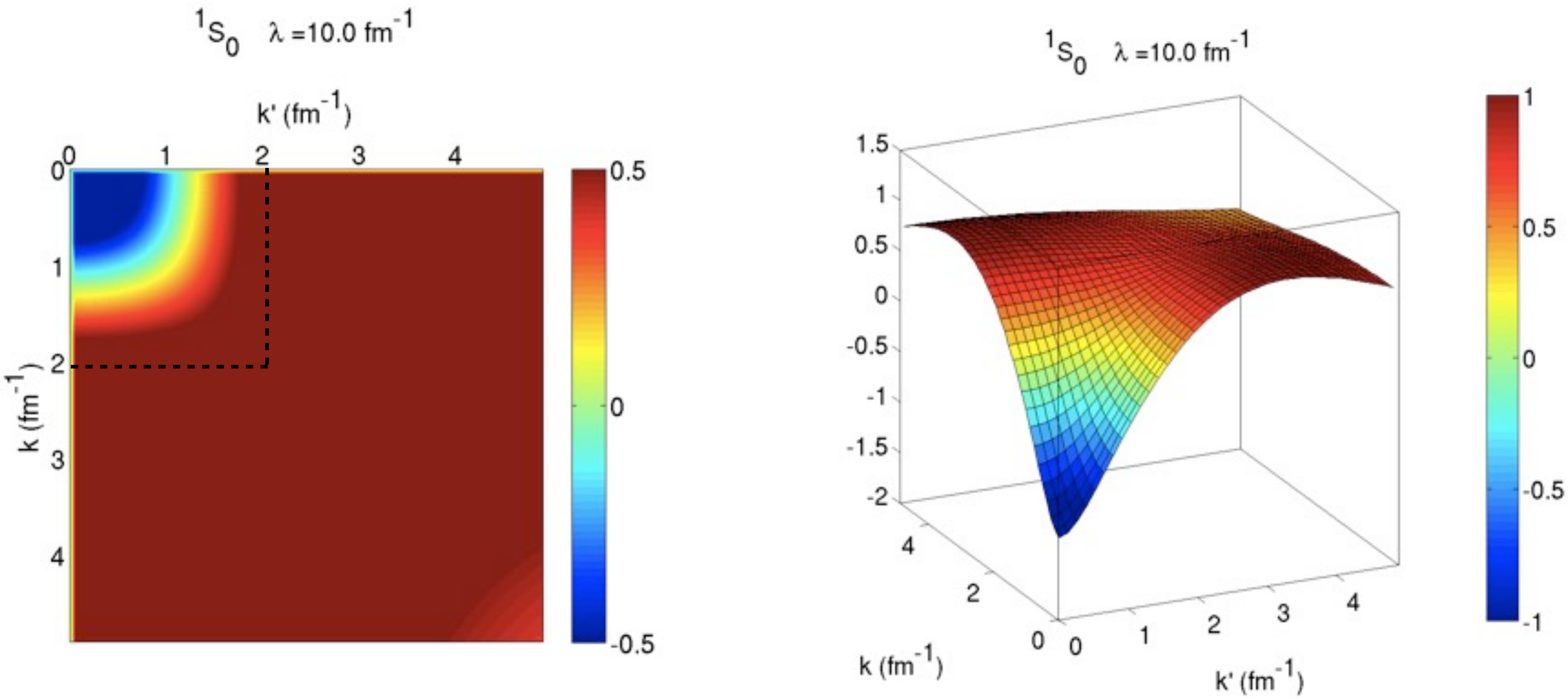

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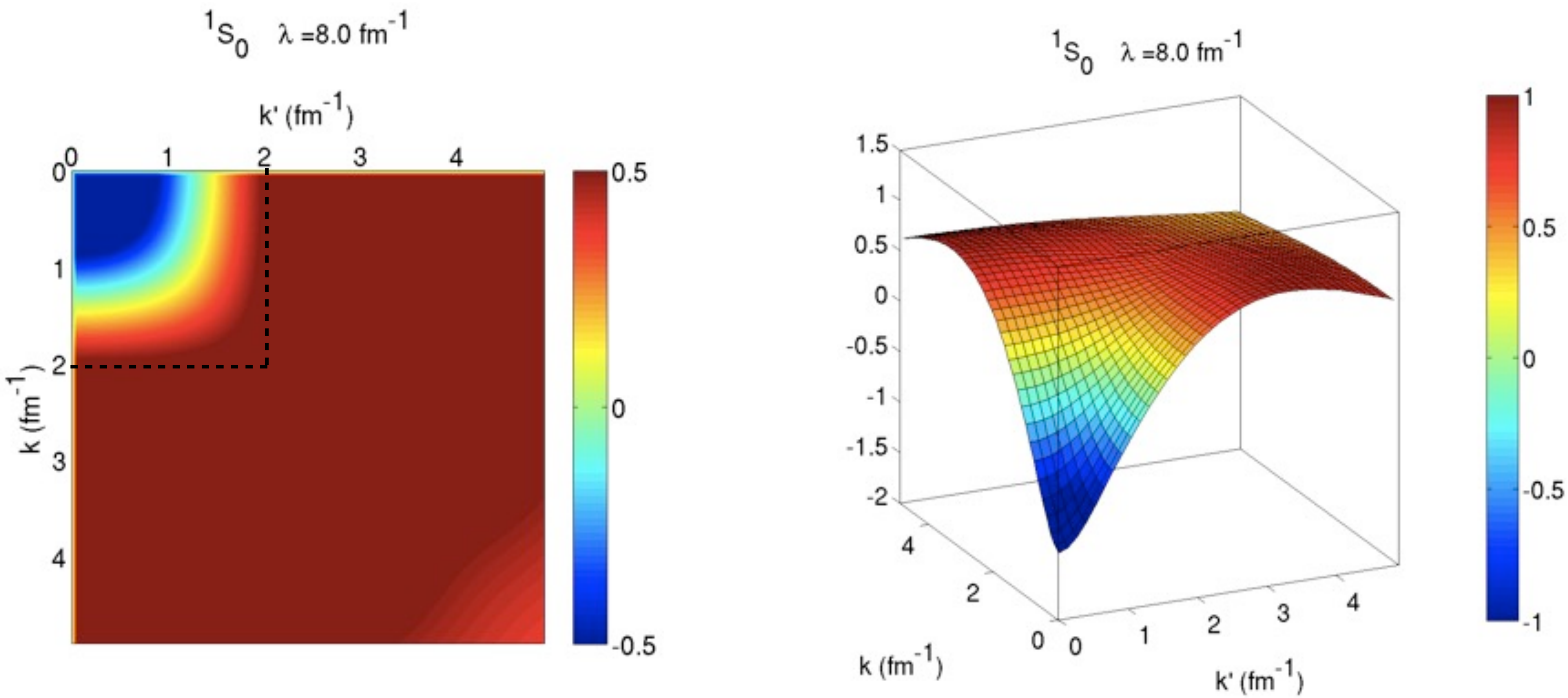

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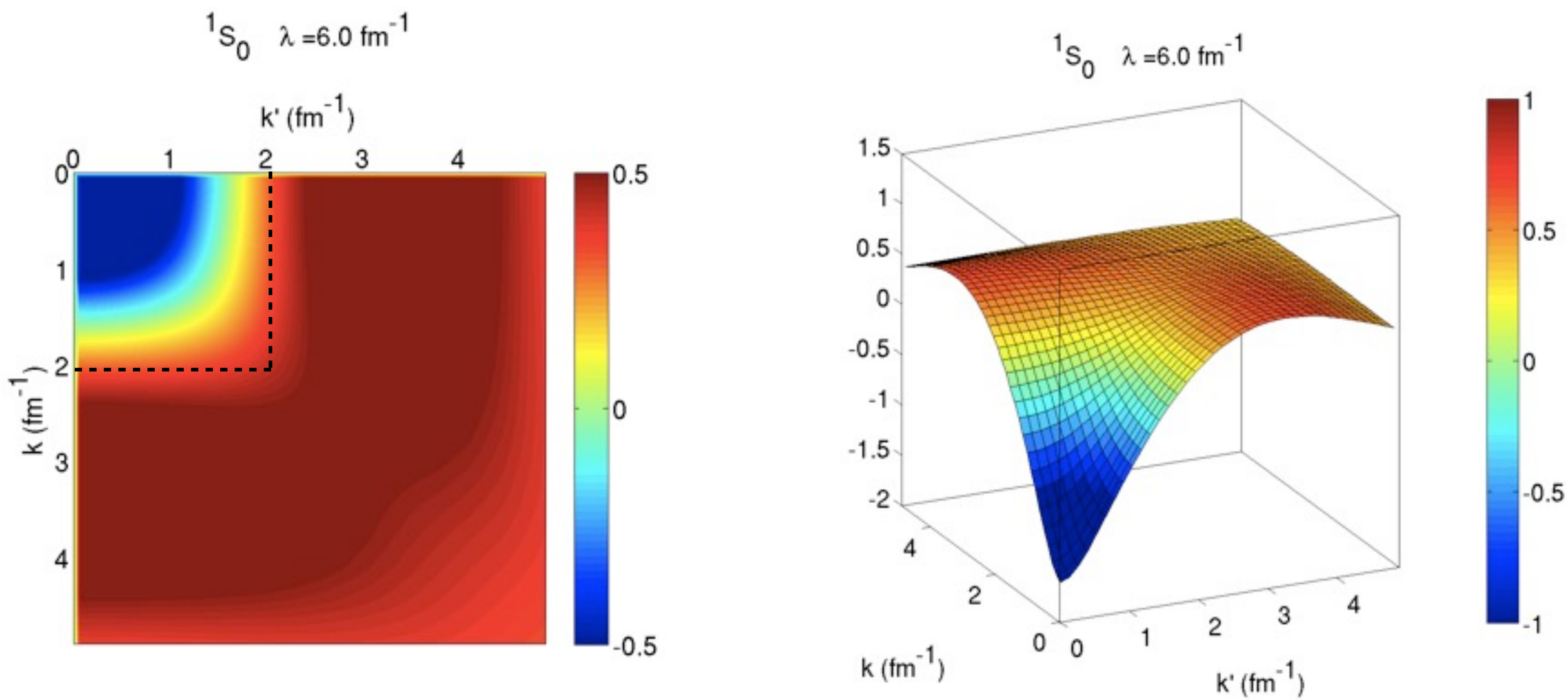

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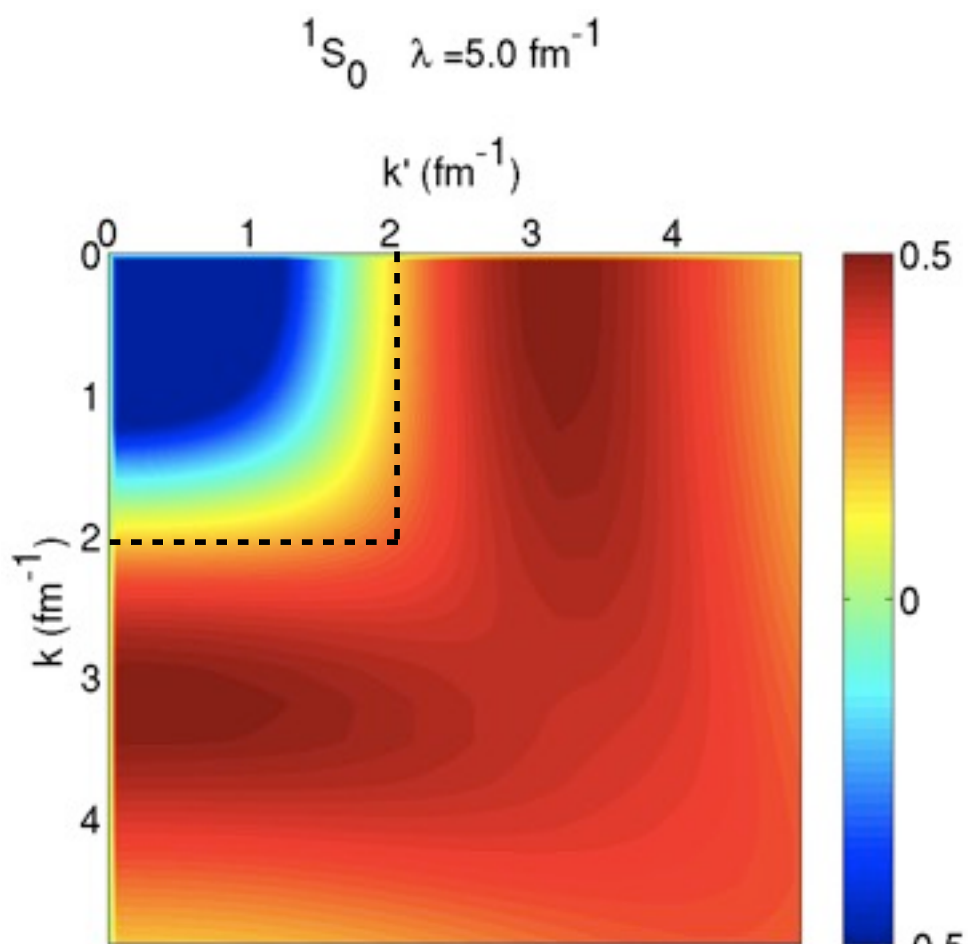


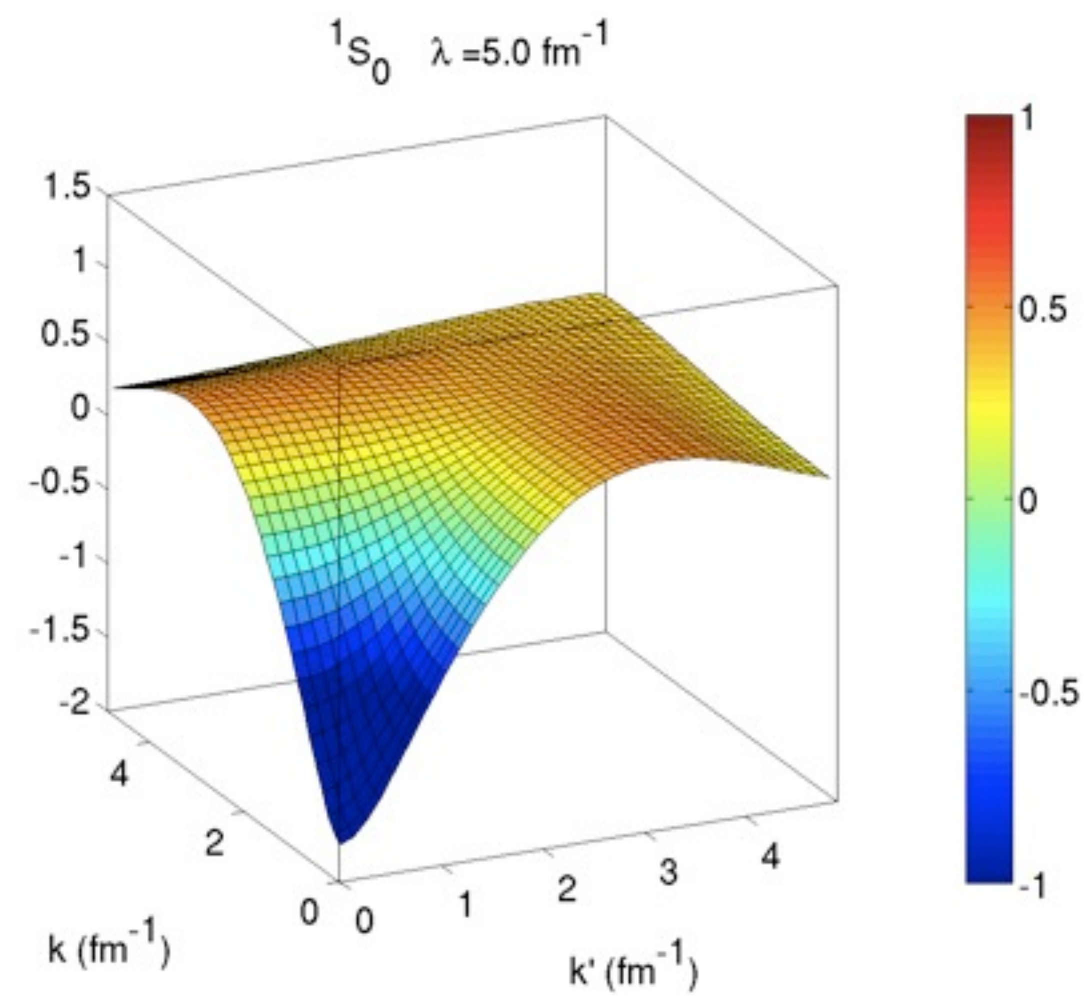

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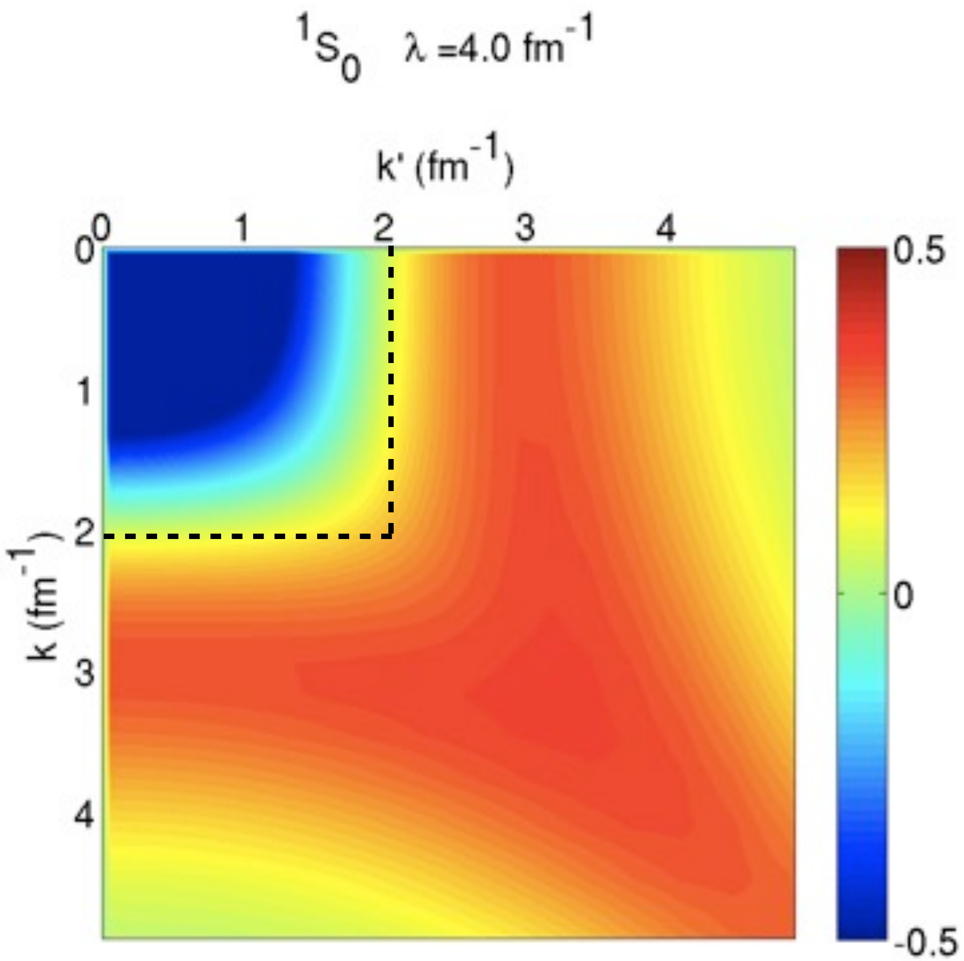


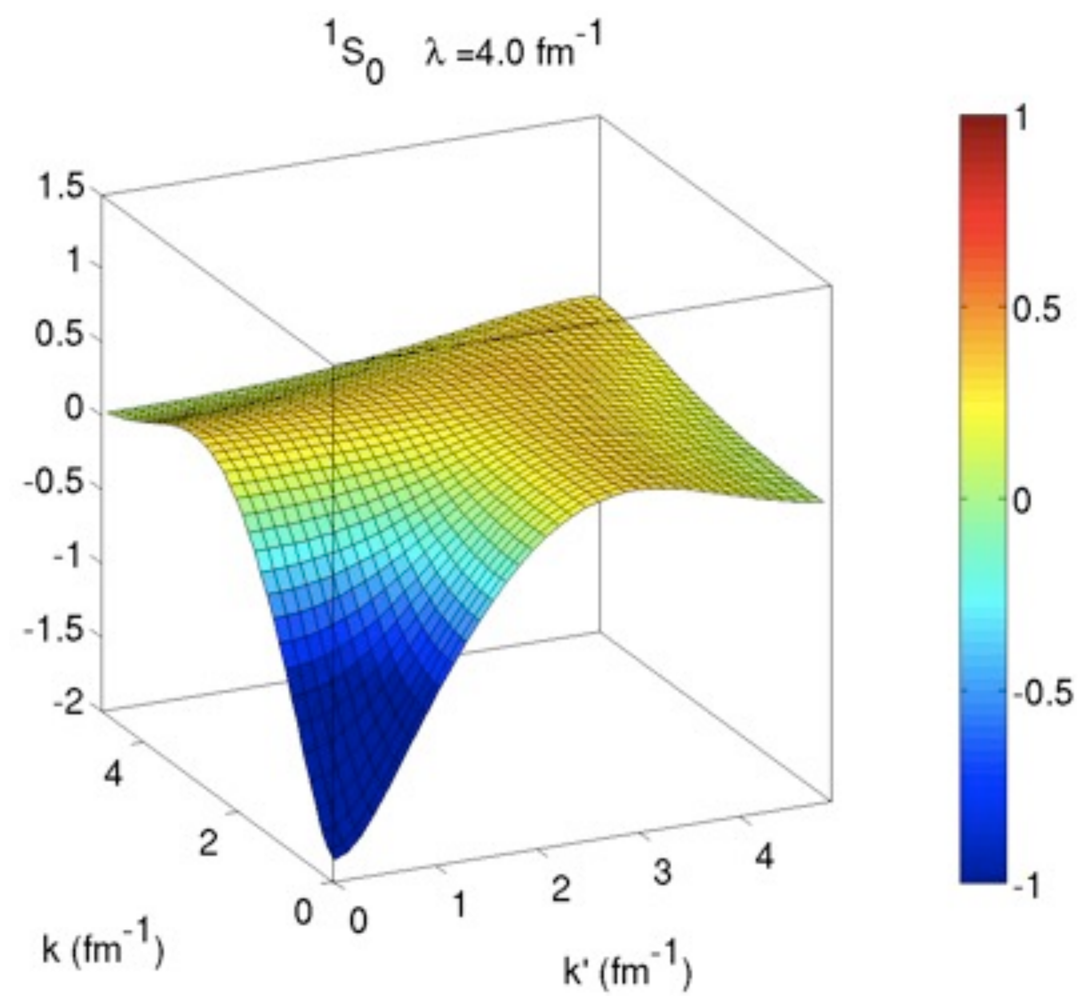

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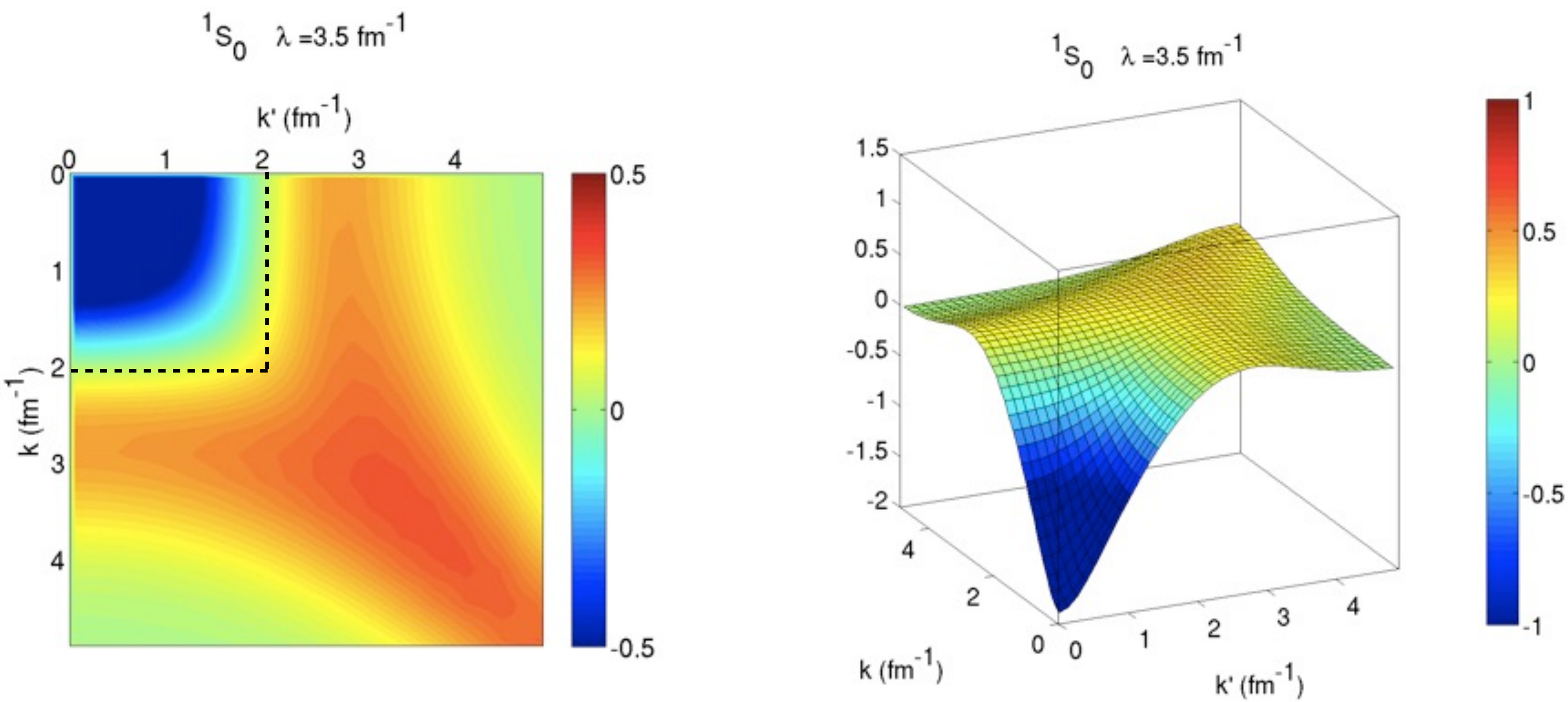

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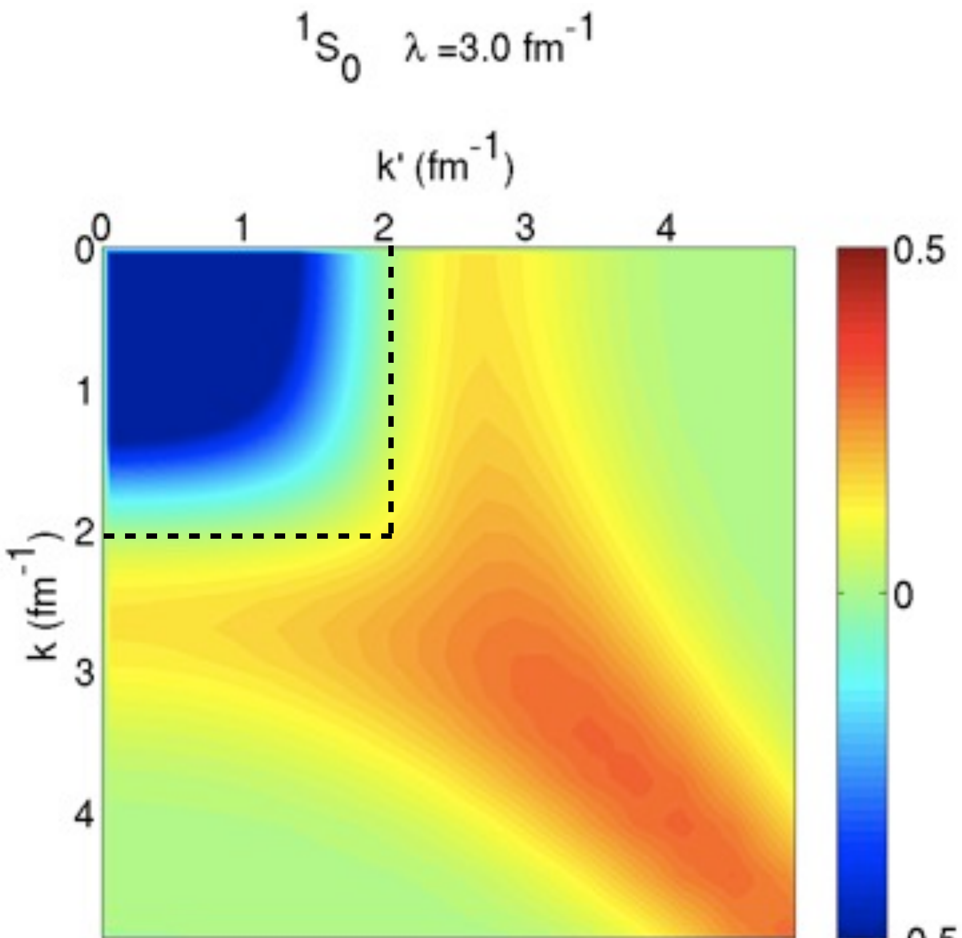


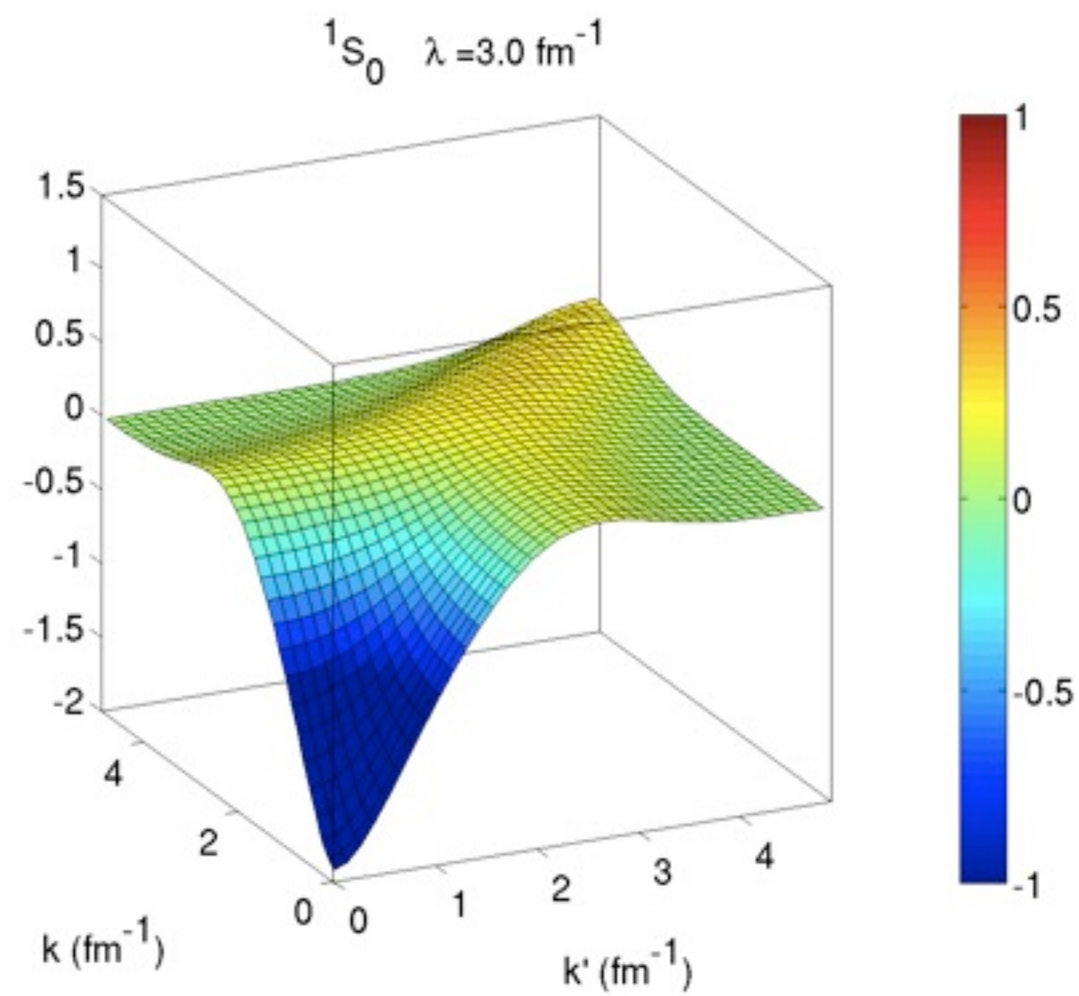

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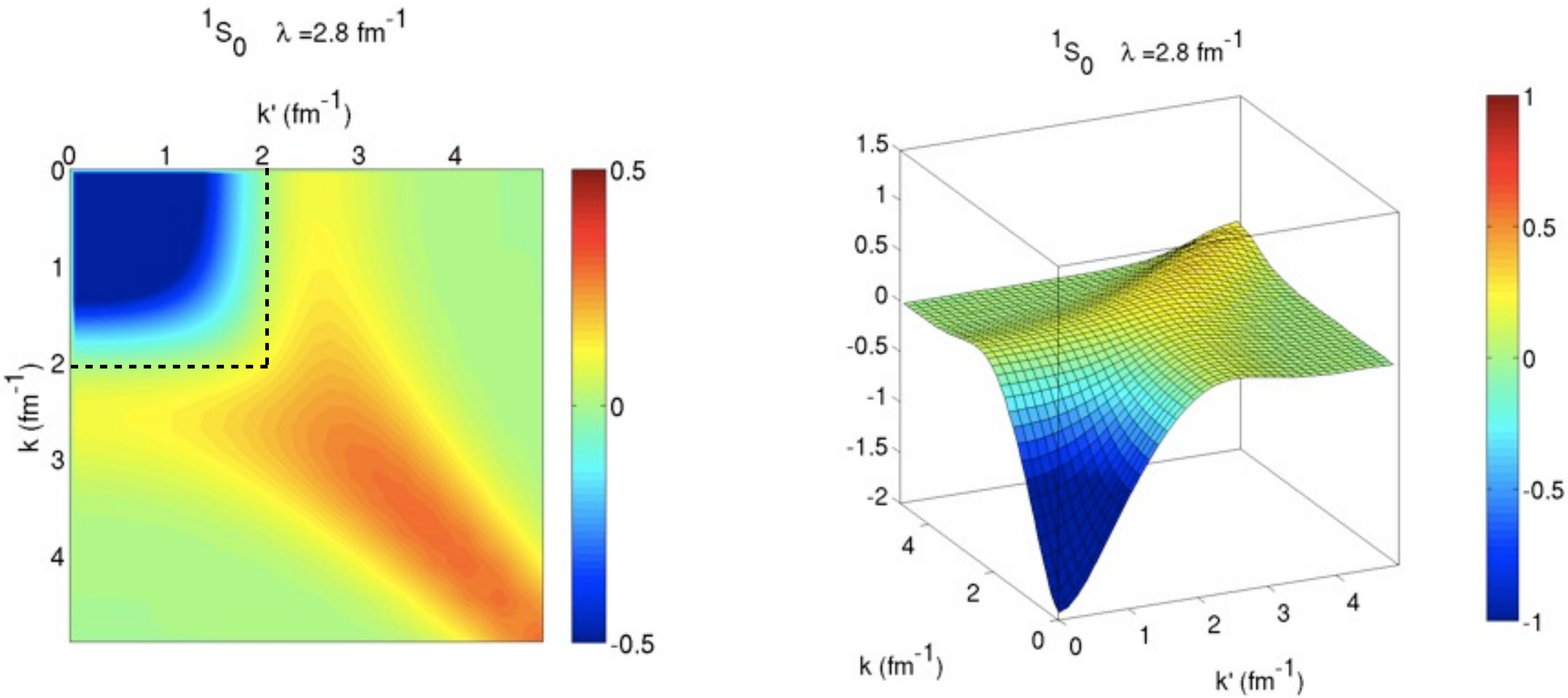

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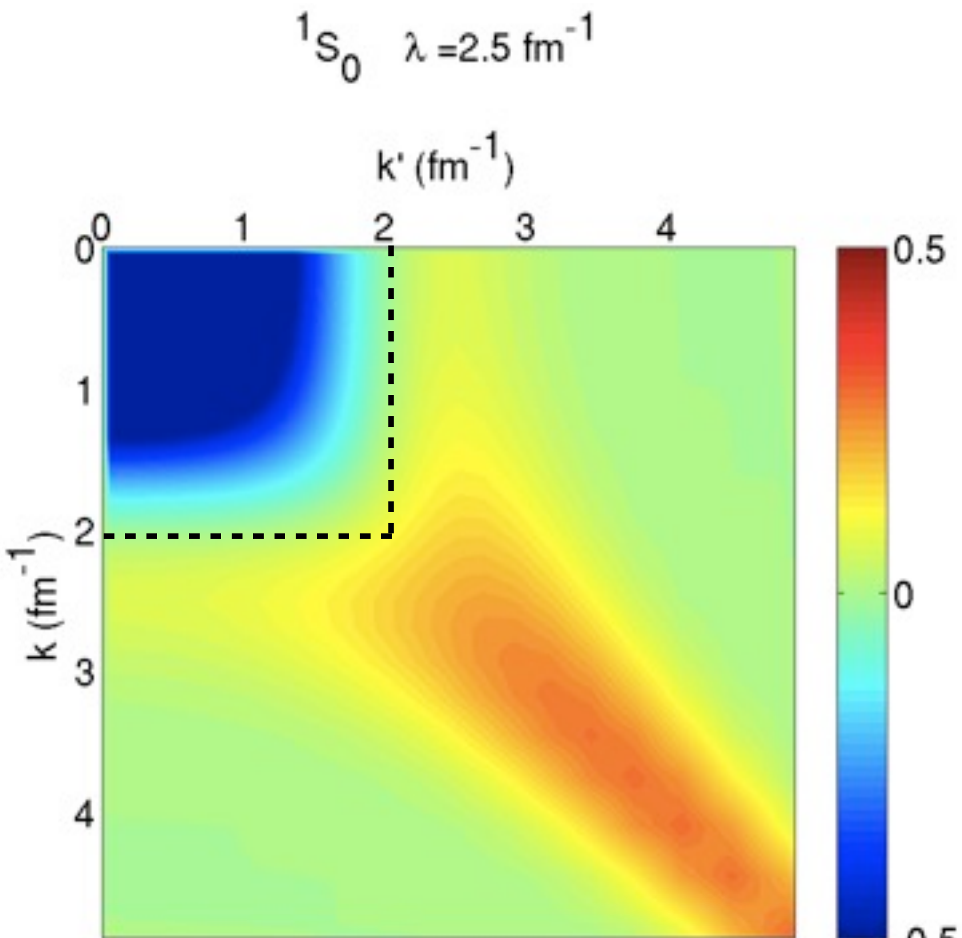


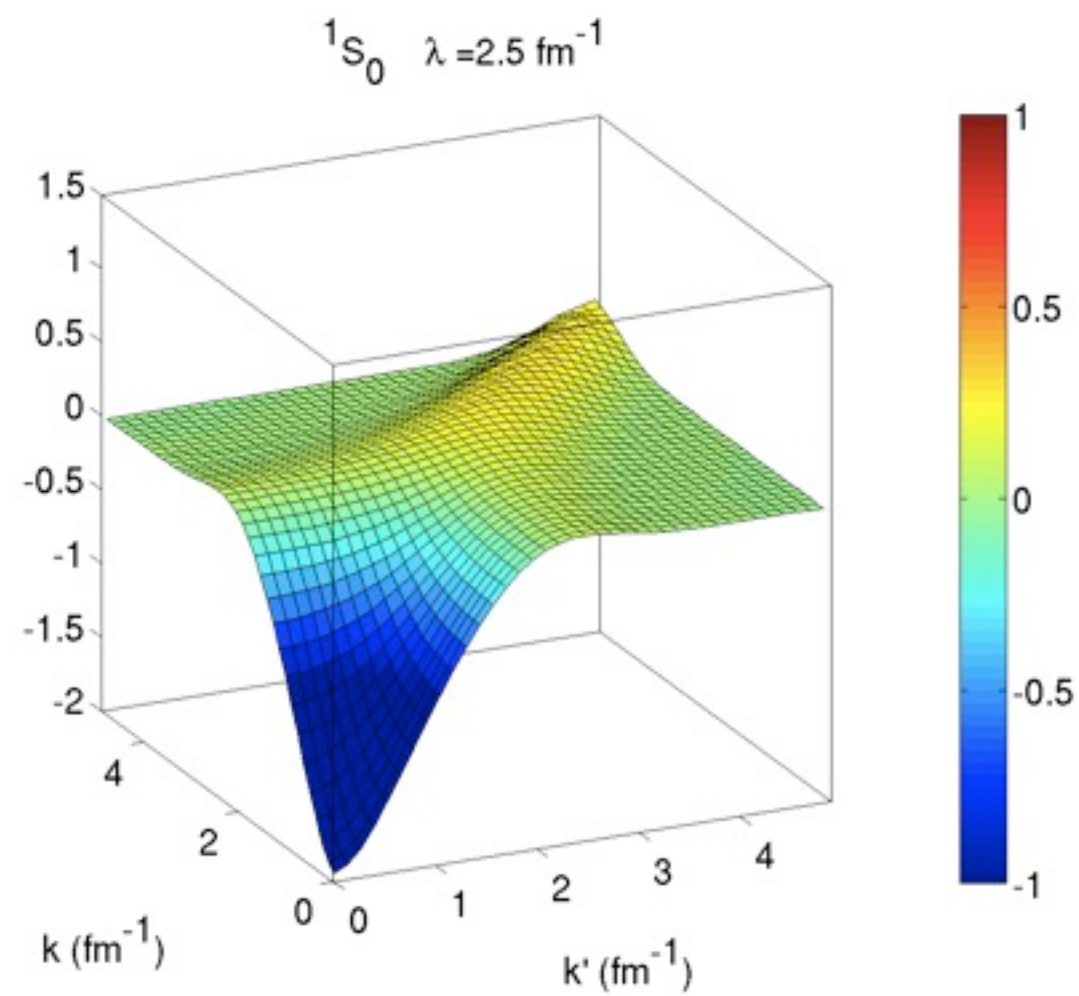

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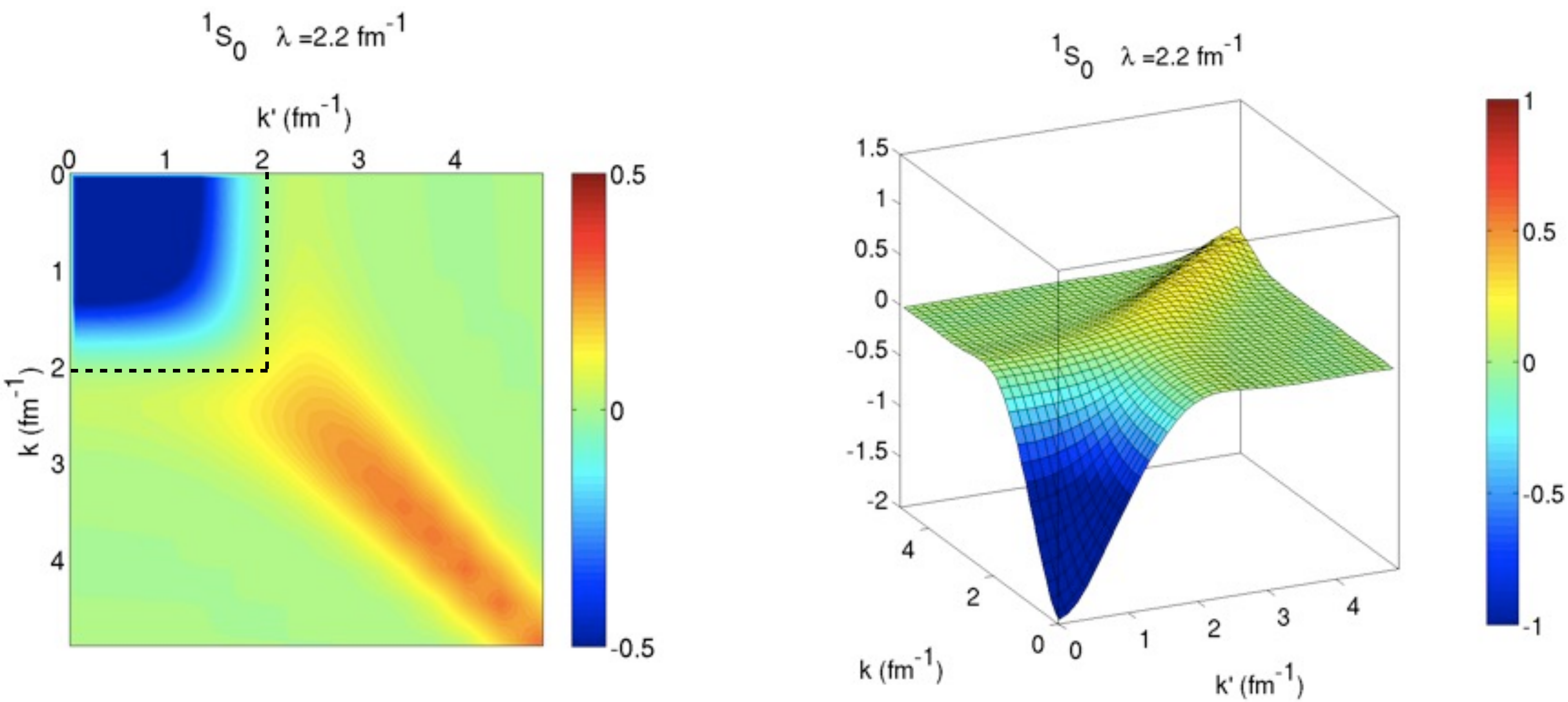

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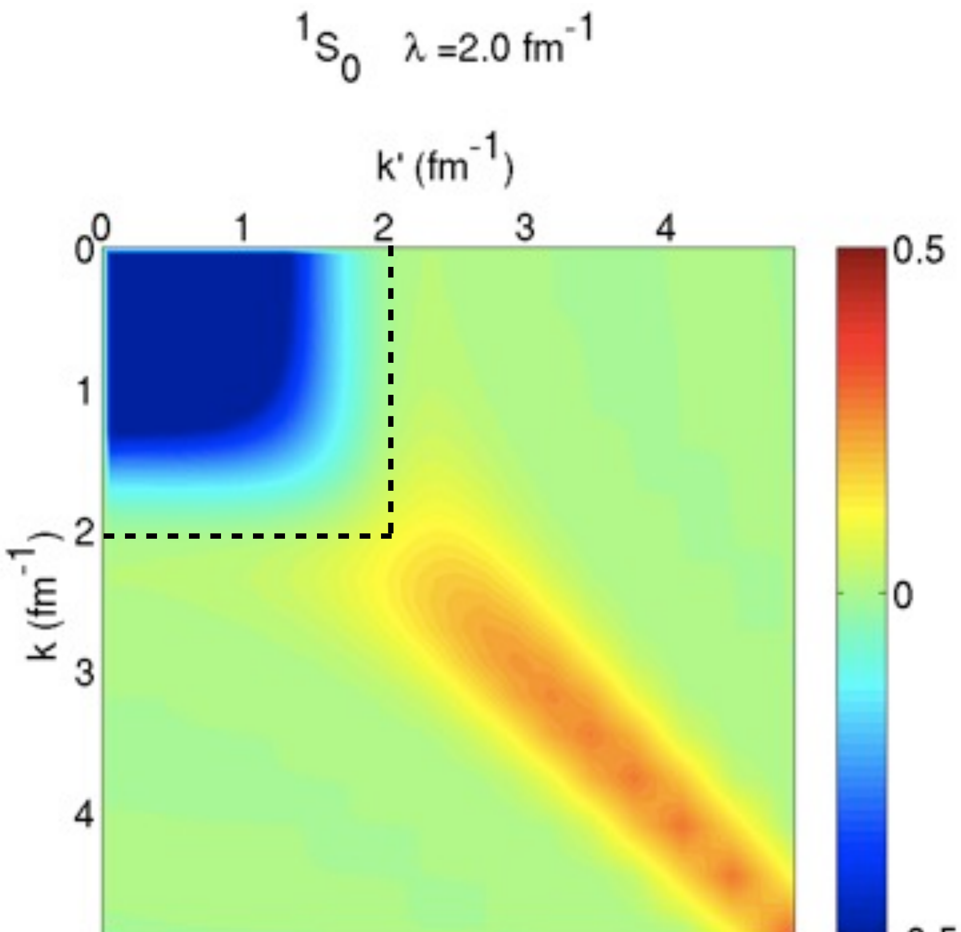


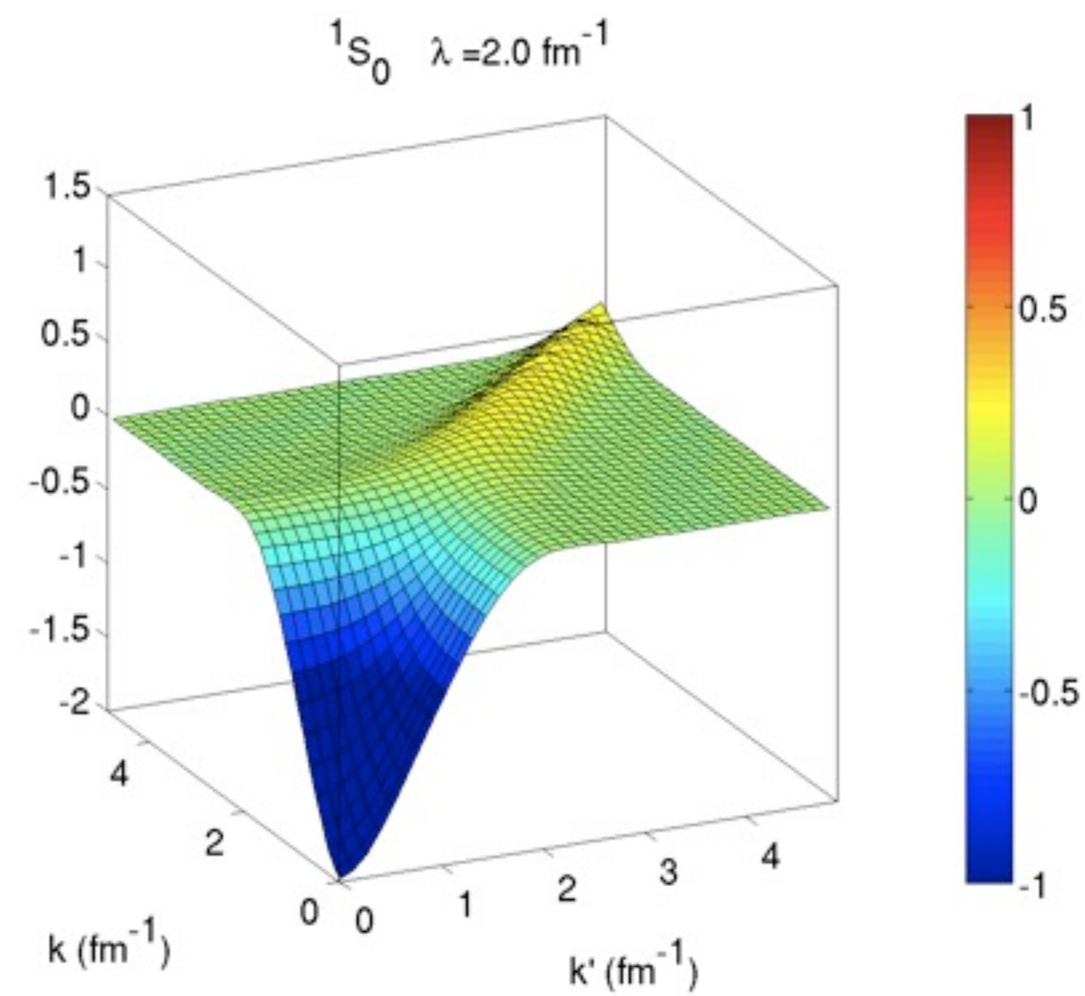

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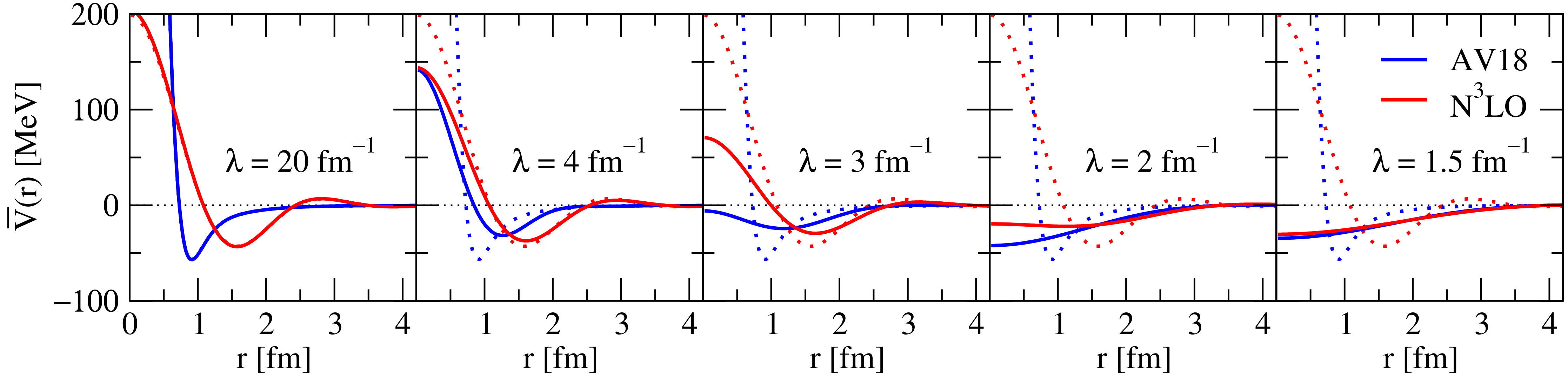
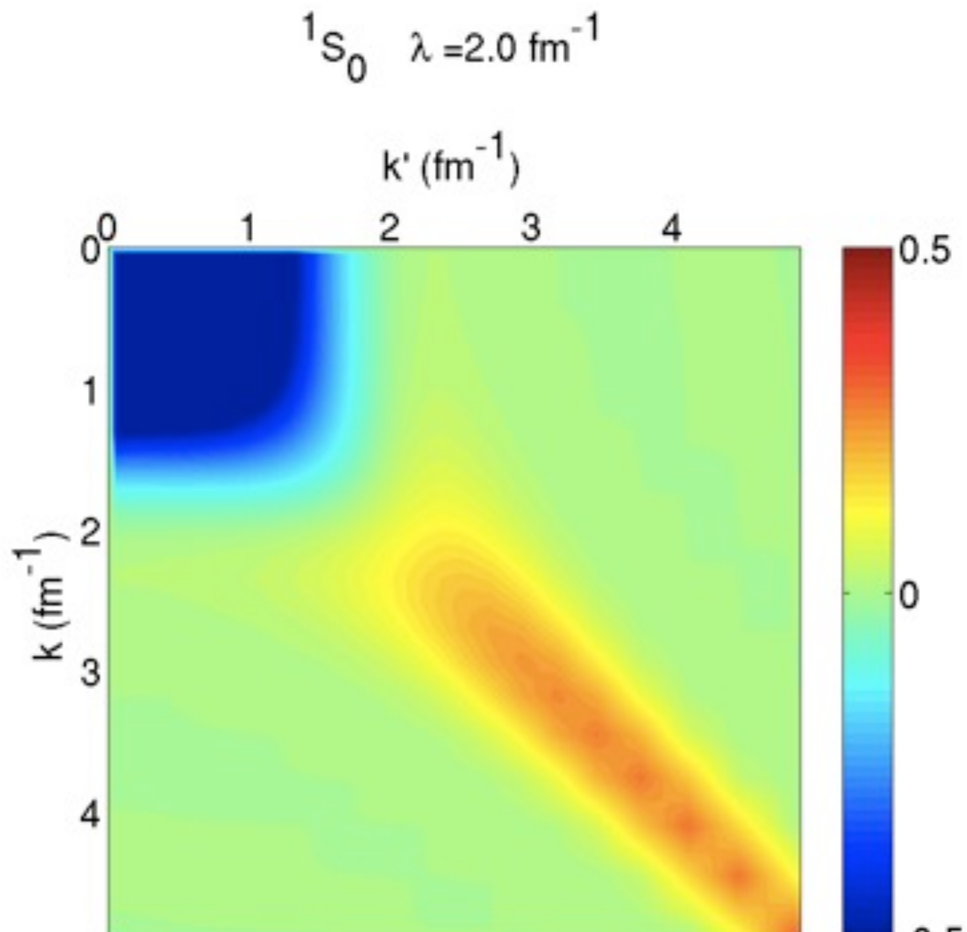
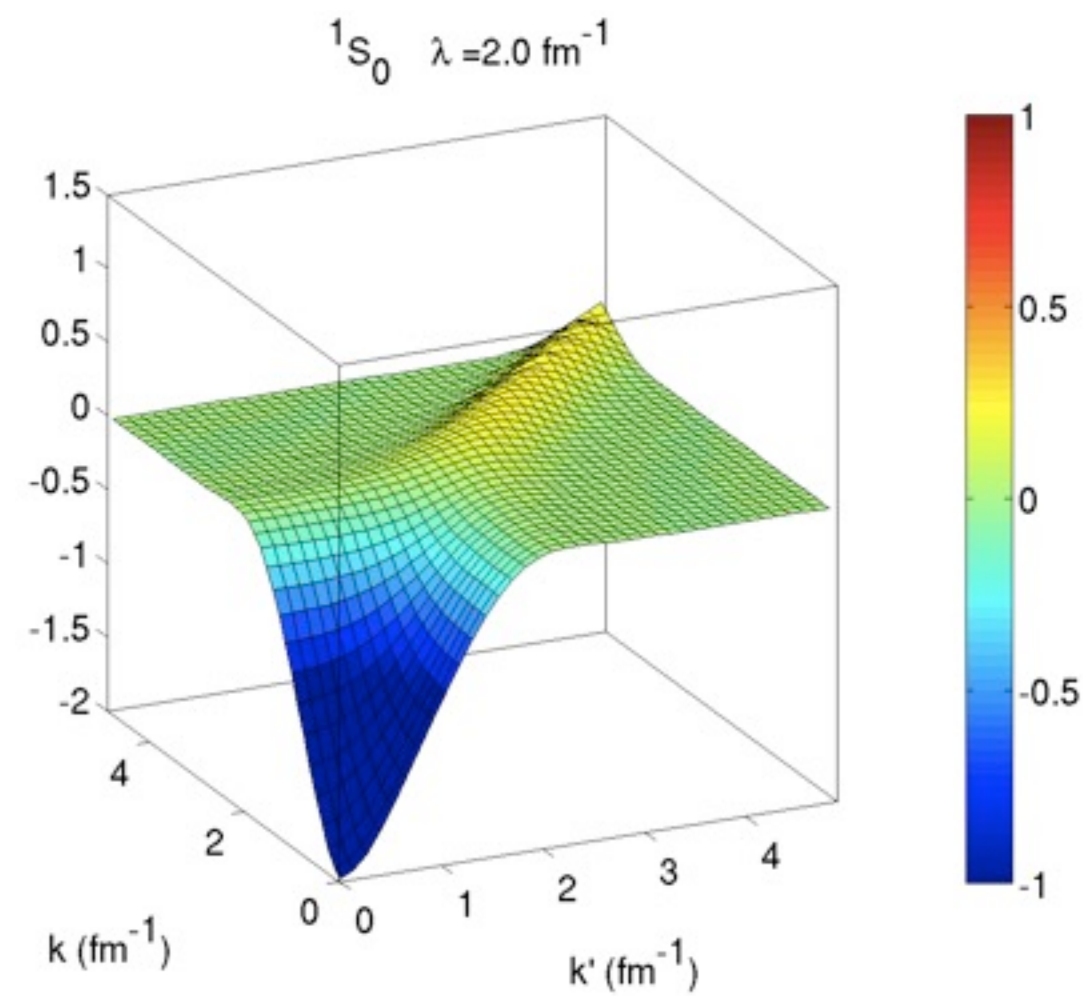
$$\overline{V}_\lambda(r) = \int dr' r'^2 V_\lambda(r, r')$$



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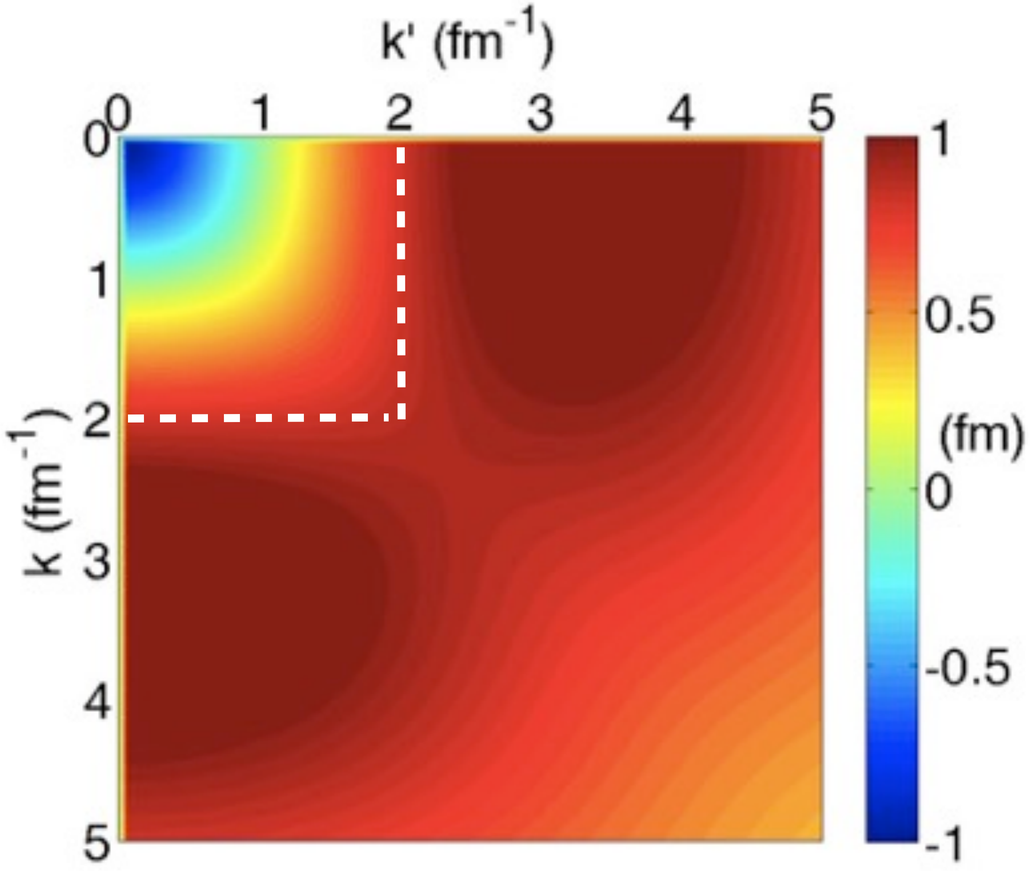
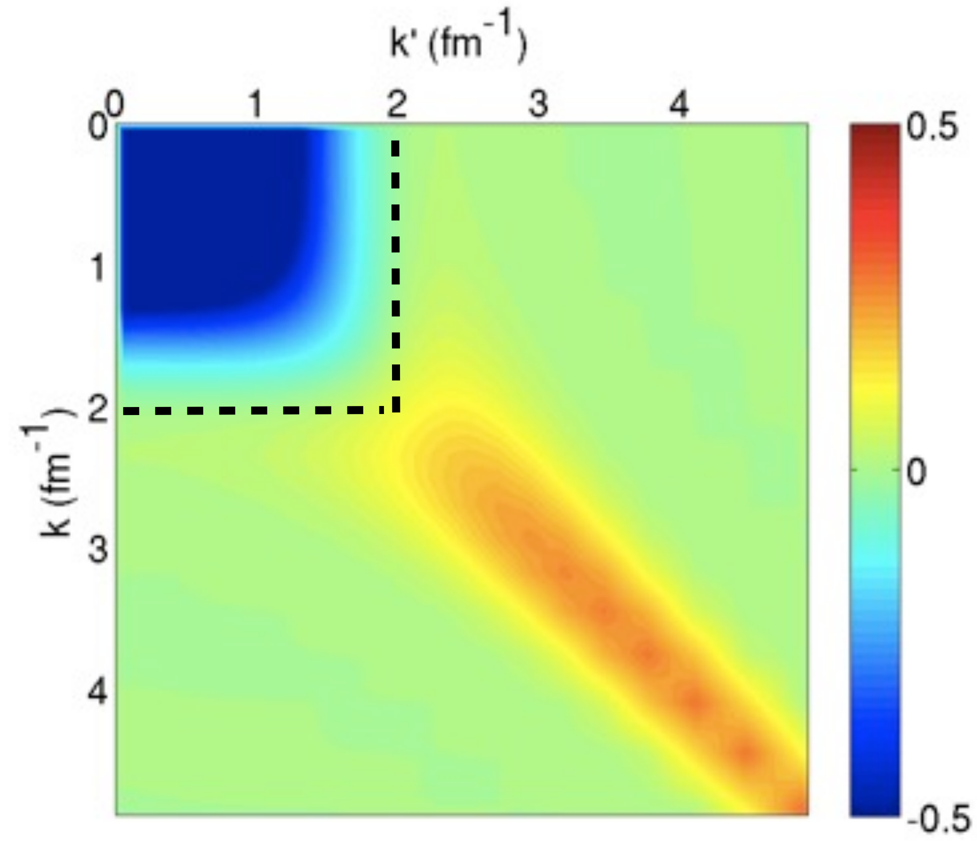


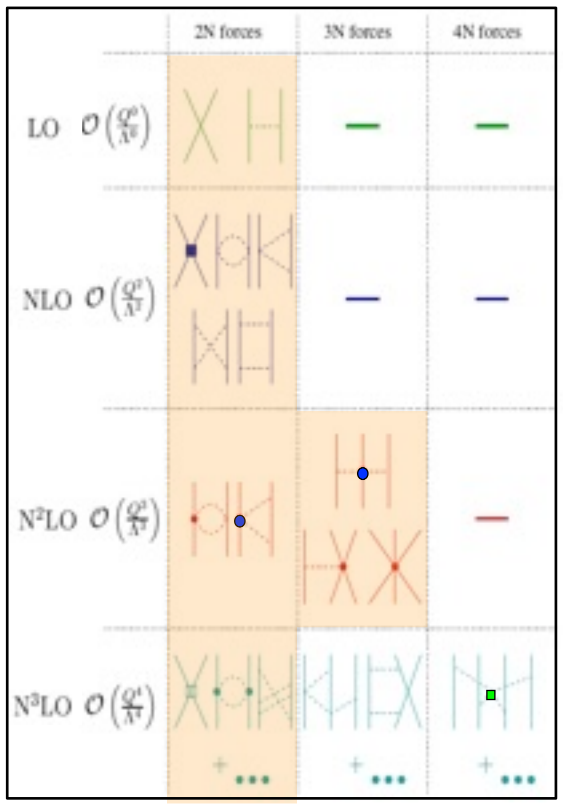



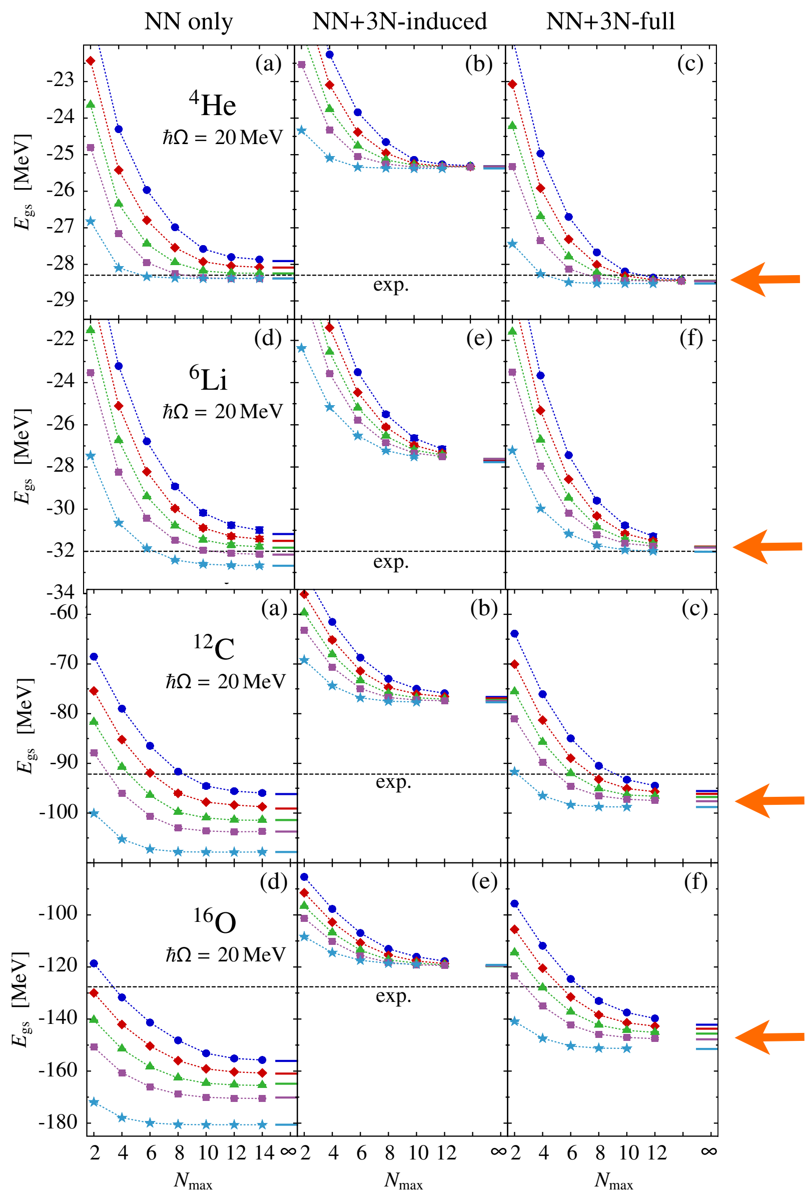

- elimination of coupling between low- and high momentum components, ⟶ simplified many-body calculations
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformation also changes three-body (and higher-body) interactions.

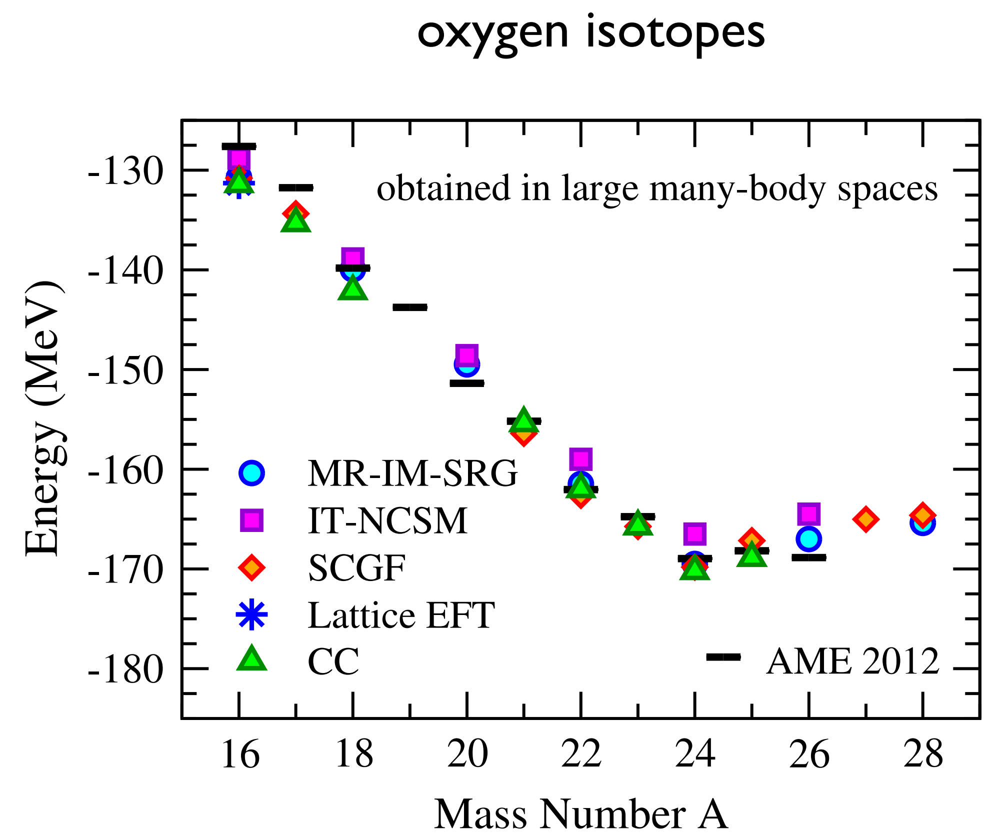
# Ground state energies of nuclei based on consistently evolved 3NF interactions

NN ($N^3LO$)
+ 3NF ($N^2LO$, 500 MeV)



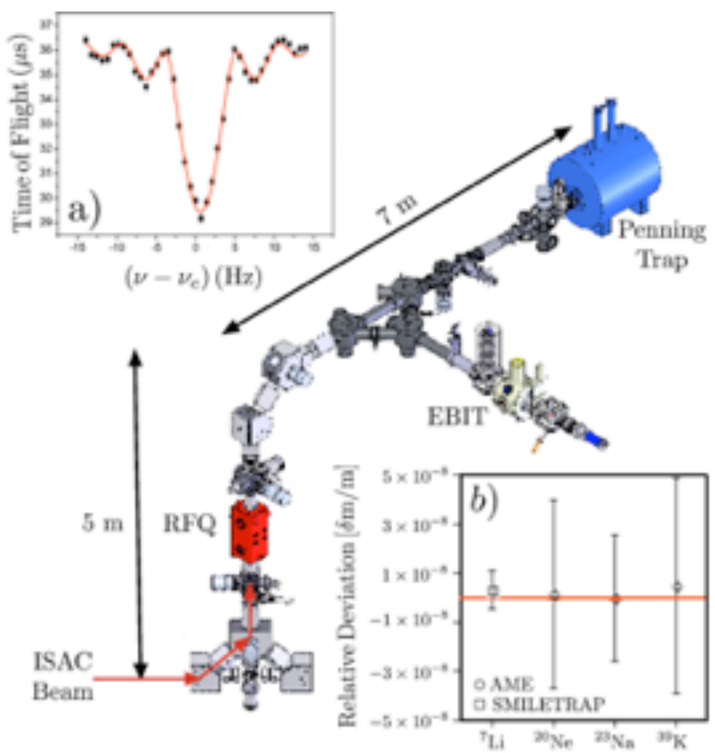



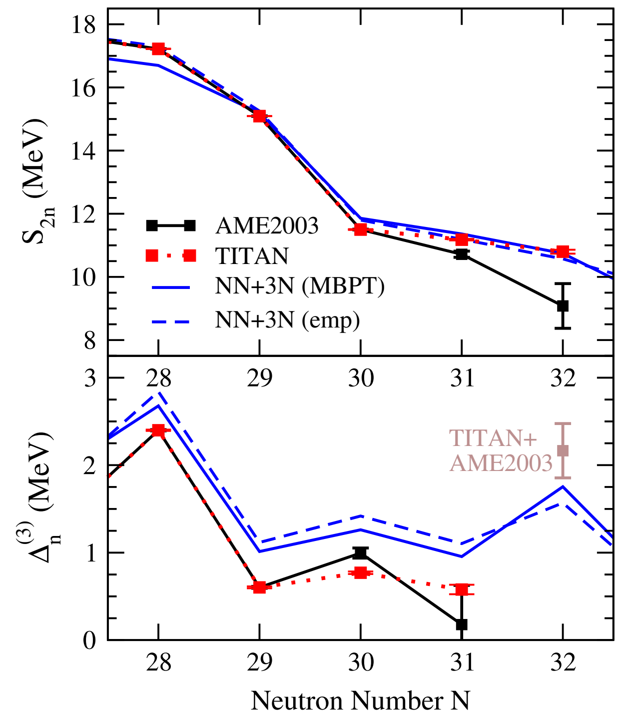

Roth, Langhammer, Calci, Binder, Navratil, PRL 107, 072501 (2011)



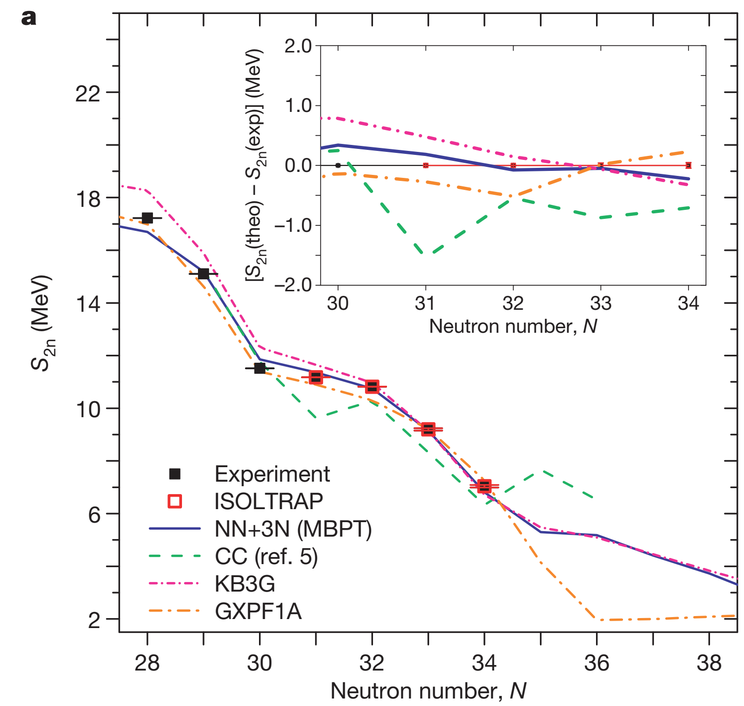

- very promising results for light nuclei, issues for heavier nuclei
- remarkable agreement of different MB calculations **for a given Hamiltonian**
- calculations are based on NN ($N^3LO$) and 3NF ($N^2LO$) forces
- need to quantify theoretical uncertainties

# Calculations and measurements of neutron-rich nuclei



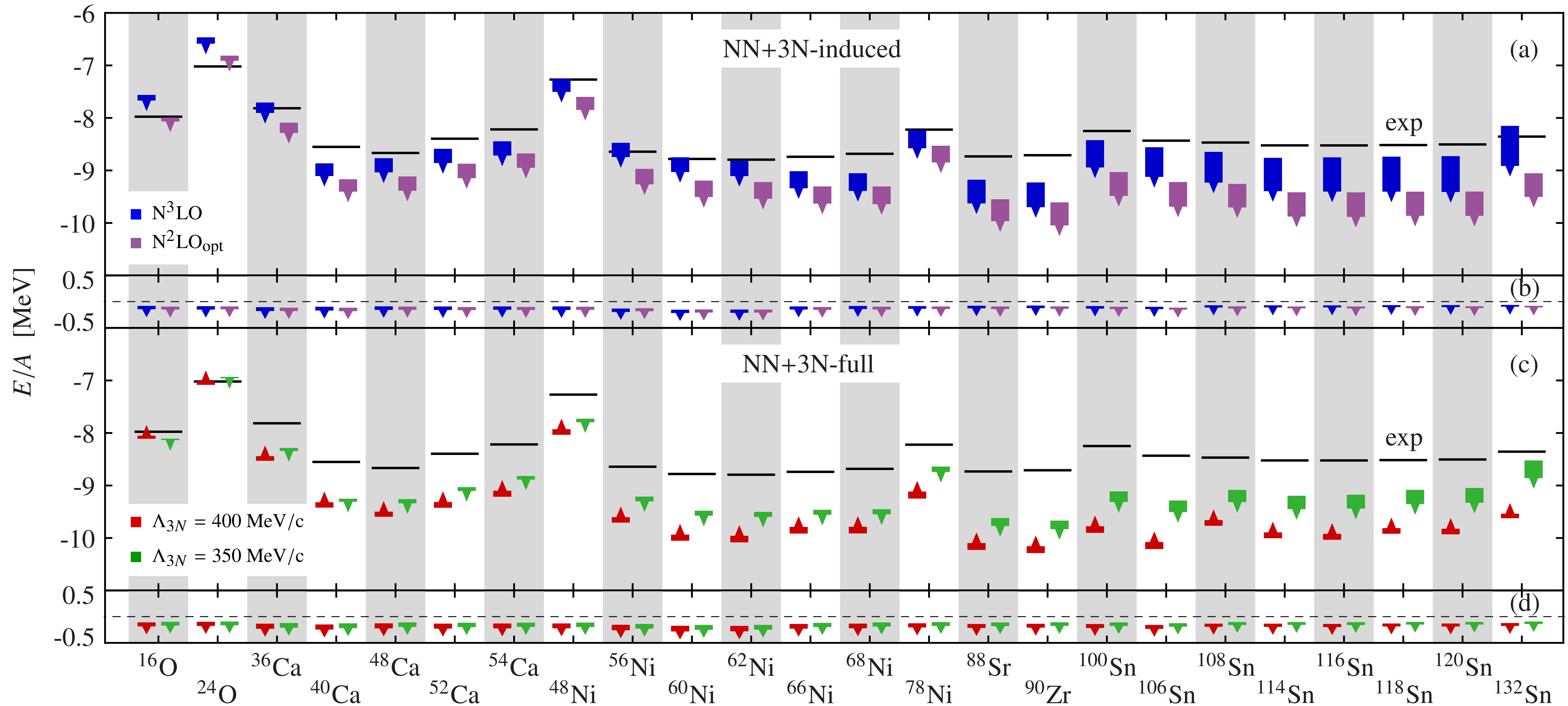

Gallant et al.
PRL 109, 032506 (2012)




Wienholtz et al.
Nature 498, 346 (2013)

- high precision mass measurements at TITAN showed that $^{52}$Ca is 1.74 MeV more bound compared to atomic mass evaluation
- neutron separation energies agree well with MBPT calculations based on NN+3NF chiral interactions
- need to quantify theoretical uncertainties

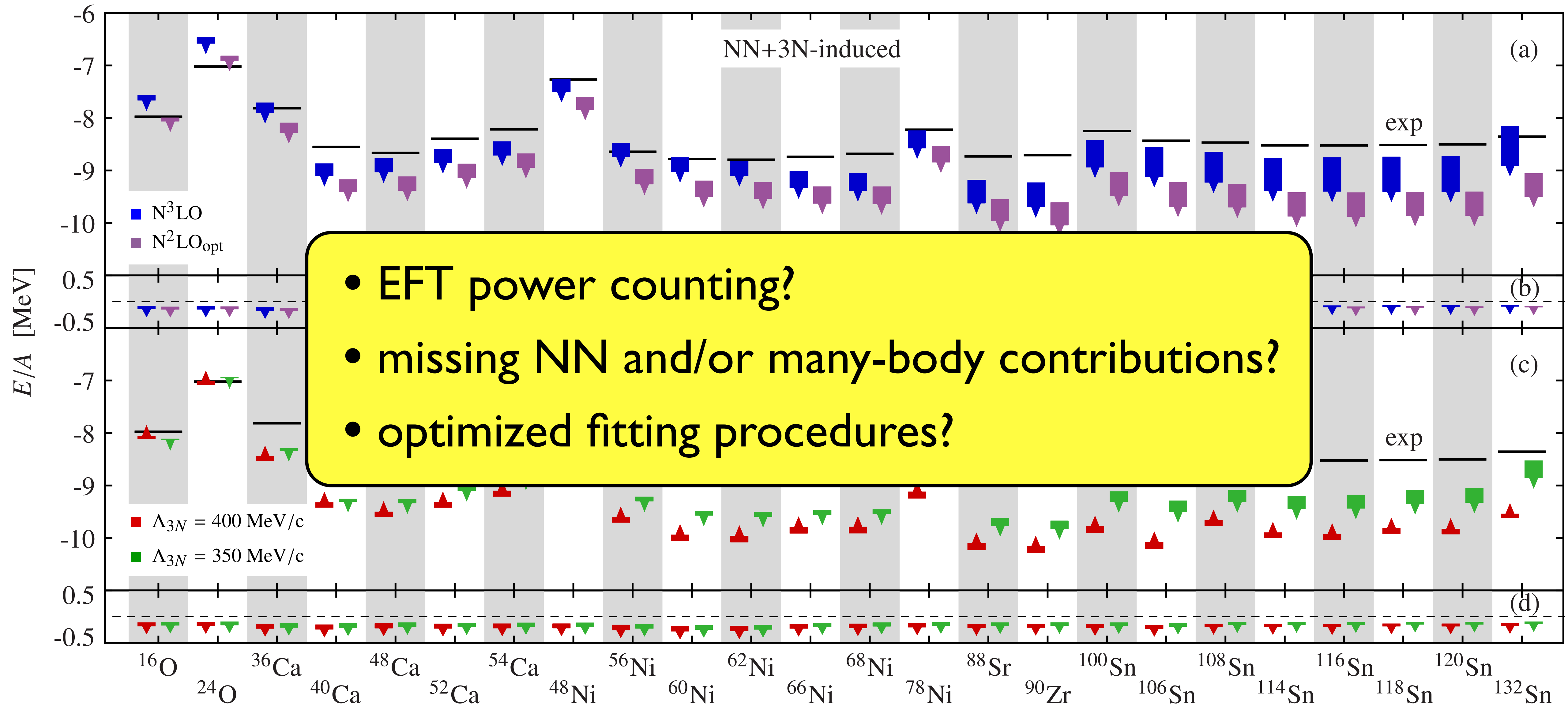
# Ground state energies of medium-mass and heavy nuclei




Binder, Calci, Langhammer, Roth
Phys. Lett B736, 119 (2014)

- significant overbinding of heavy nuclei
- need to quantify and reduce theoretical uncertainties

# Ground state energies of medium-mass and heavy nuclei



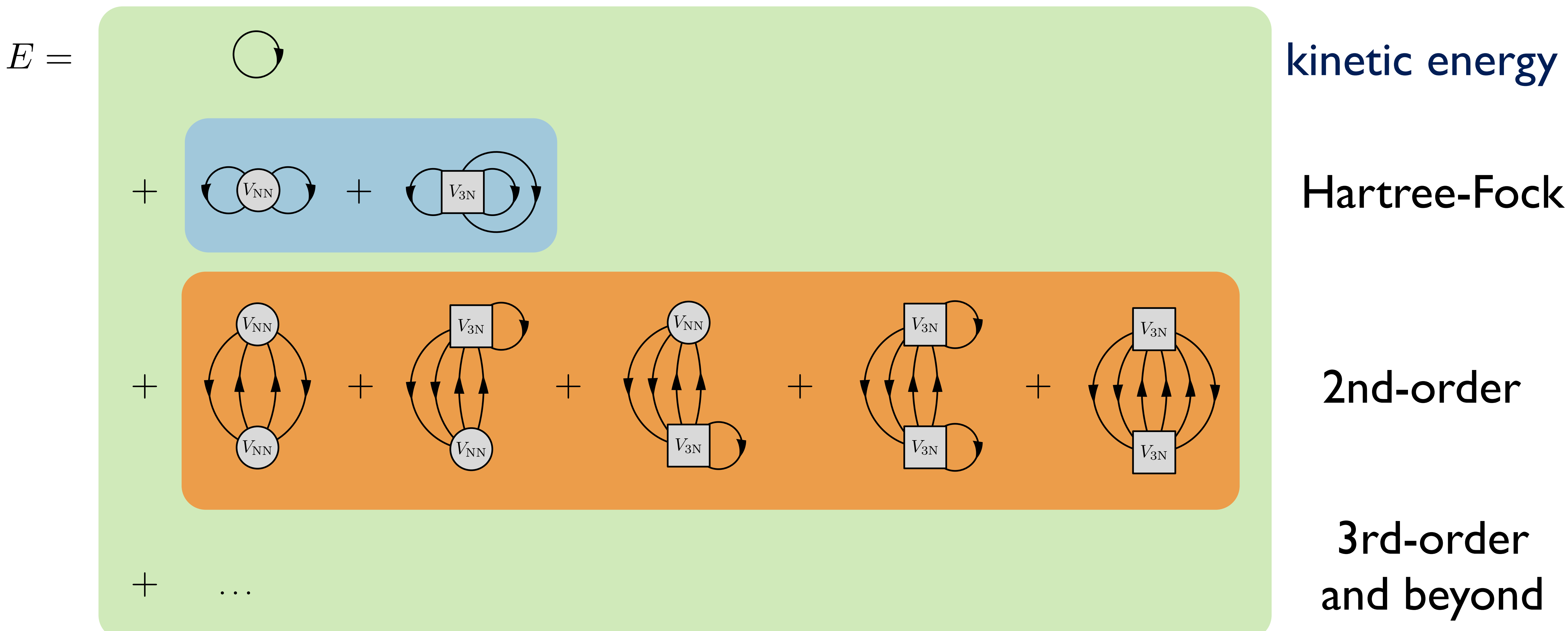

Binder, Calci, Langhammer, Roth
Phys. Lett B736, 119 (2014)

- significant overbinding of heavy nuclei
- need to quantify and reduce theoretical uncertainties

# Equation of state: Many-body perturbation theory

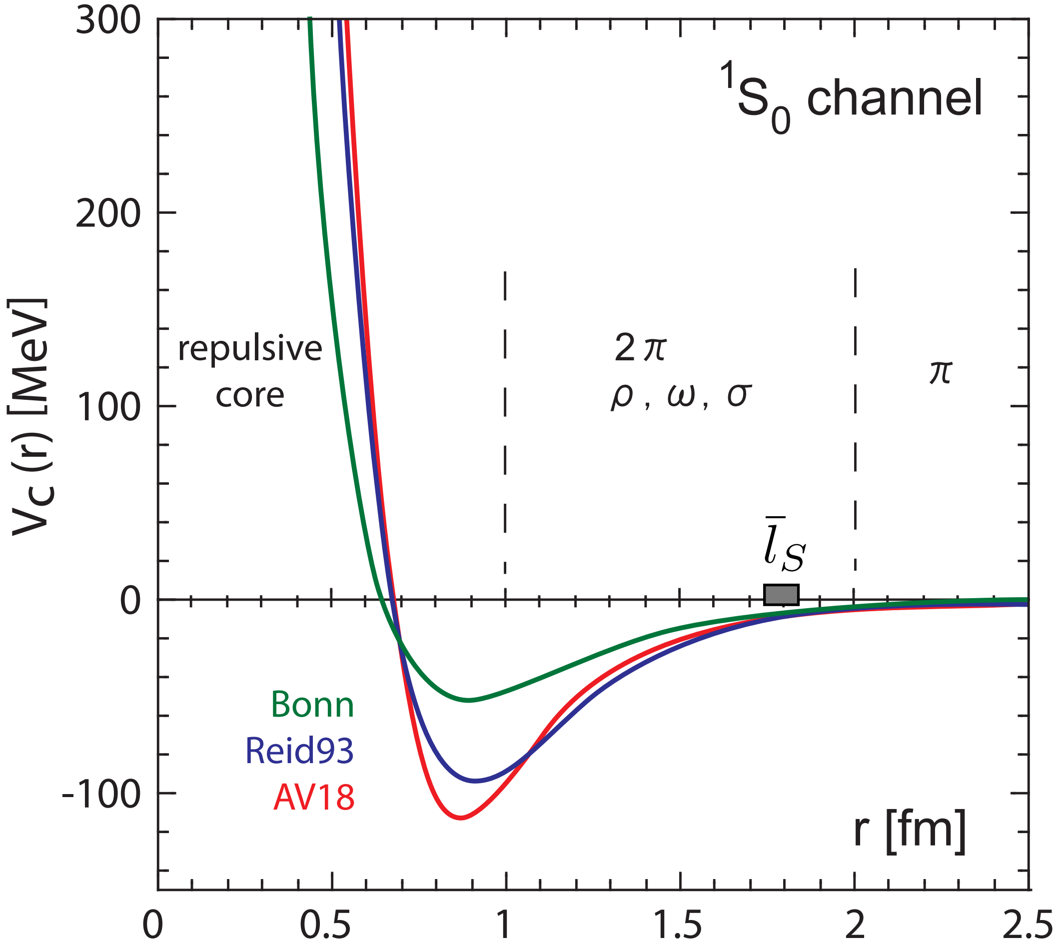
central quantity of interest: energy per particle $E/N$

$$H(\lambda) = T + V_{\rm NN}(\lambda) + V_{\rm 3N}(\lambda) + \ldots$$



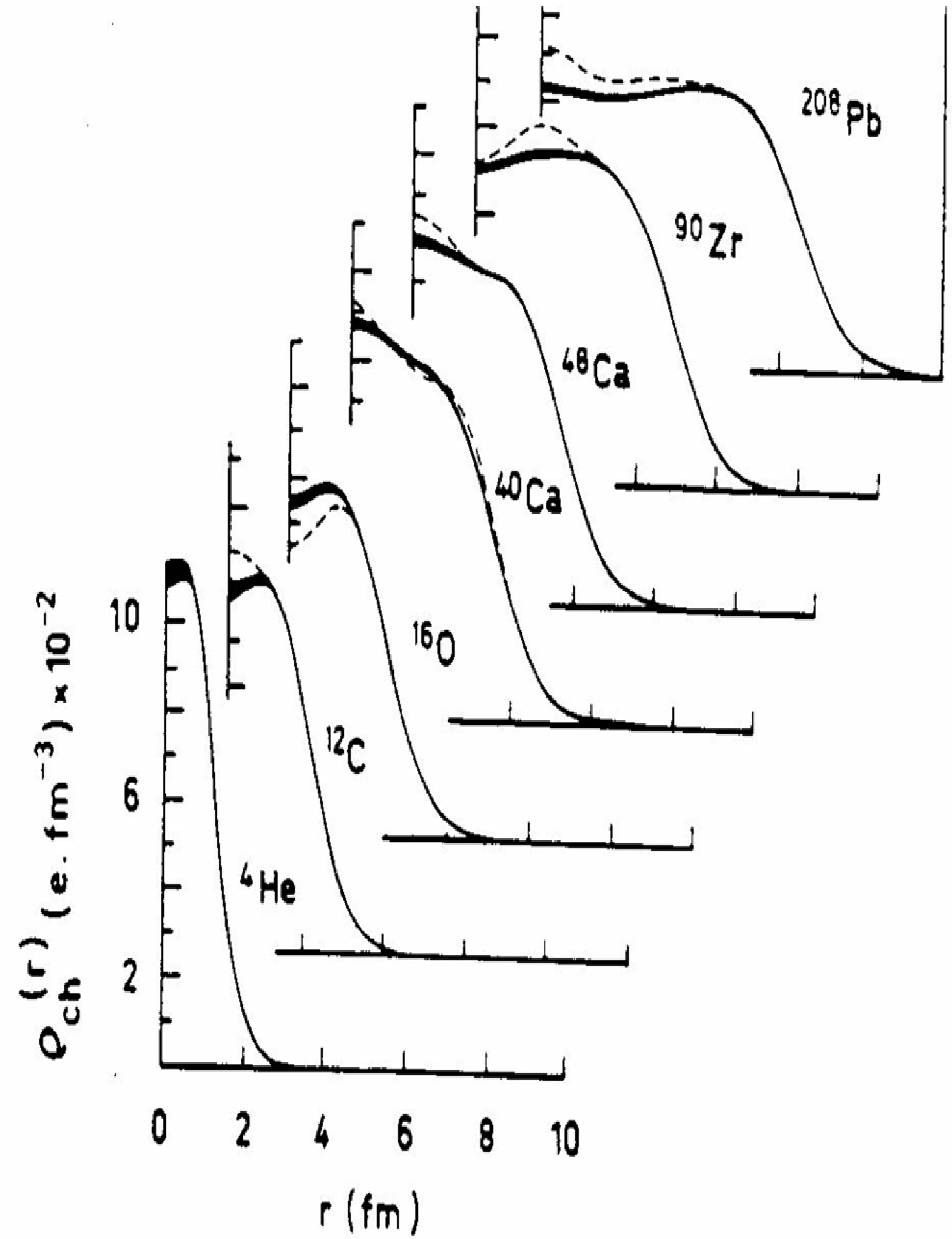

- "hard" interactions require non-perturbative summation of diagrams
- with low-momentum interactions much more perturbative
- inclusion of 3N interaction contributions crucial!

# Equation of state of symmetric nuclear matter, nuclear saturation

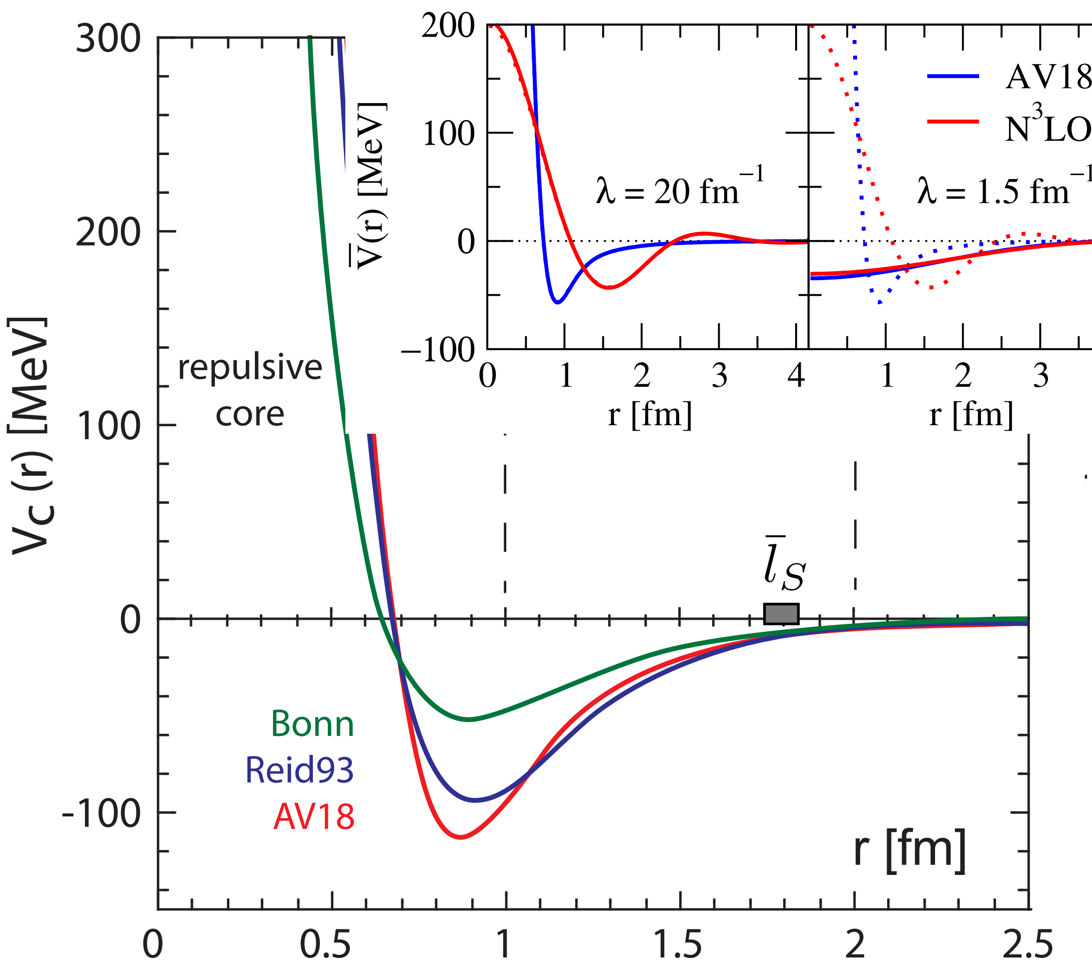


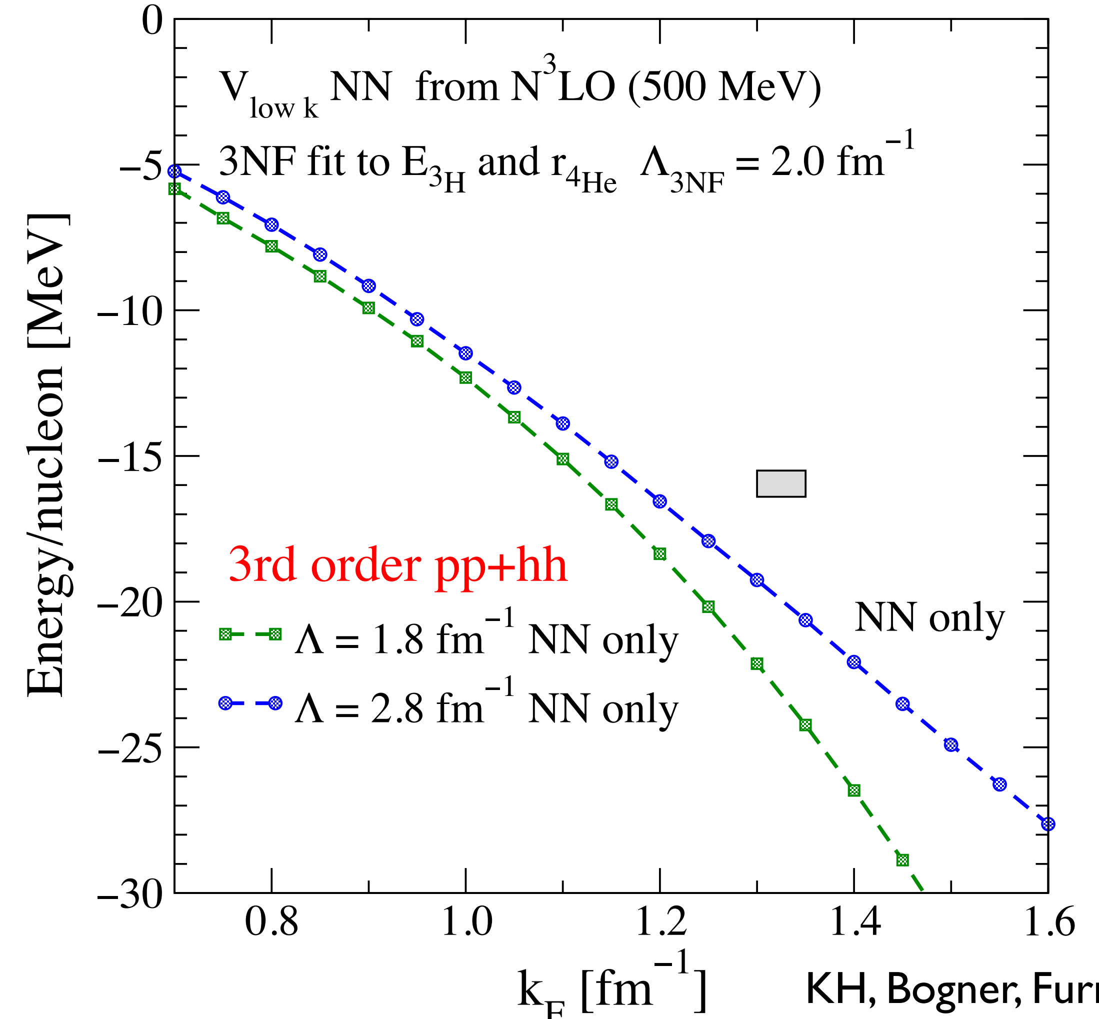

*"Very soft potentials must be excluded because they do not give saturation;*
*they give too much binding and too high density. In particular, a substantial tensor force is required."*

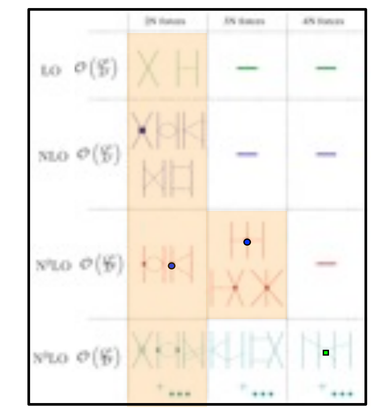
**Hans Bethe** (1971)



# Fitting the 3NF LECs at low resolution scales



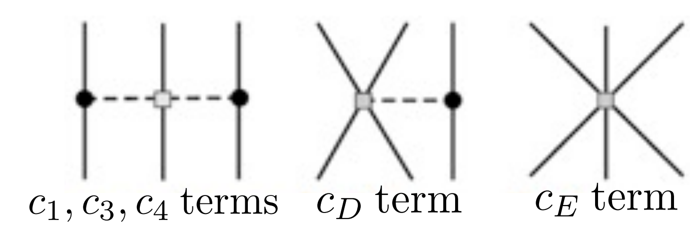
KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

*"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."*

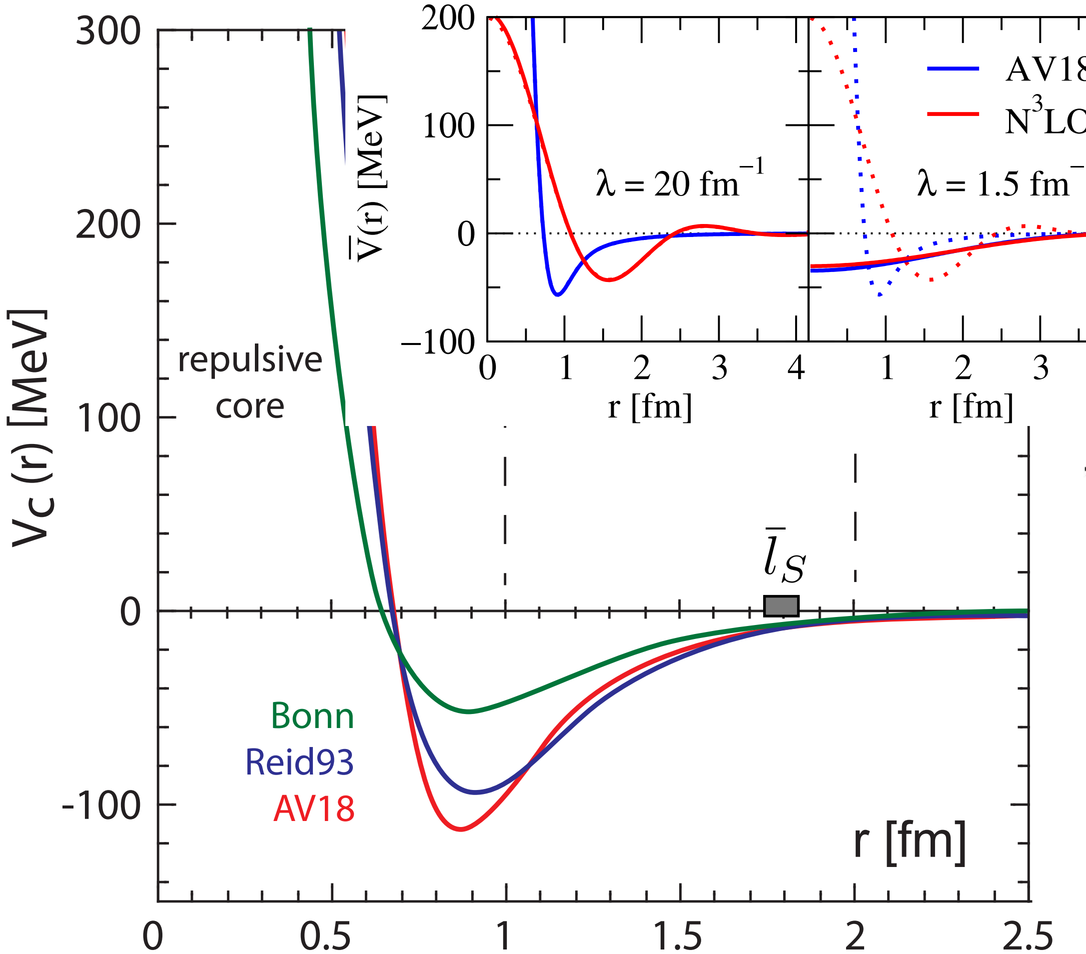
**Hans Bethe** (1971)

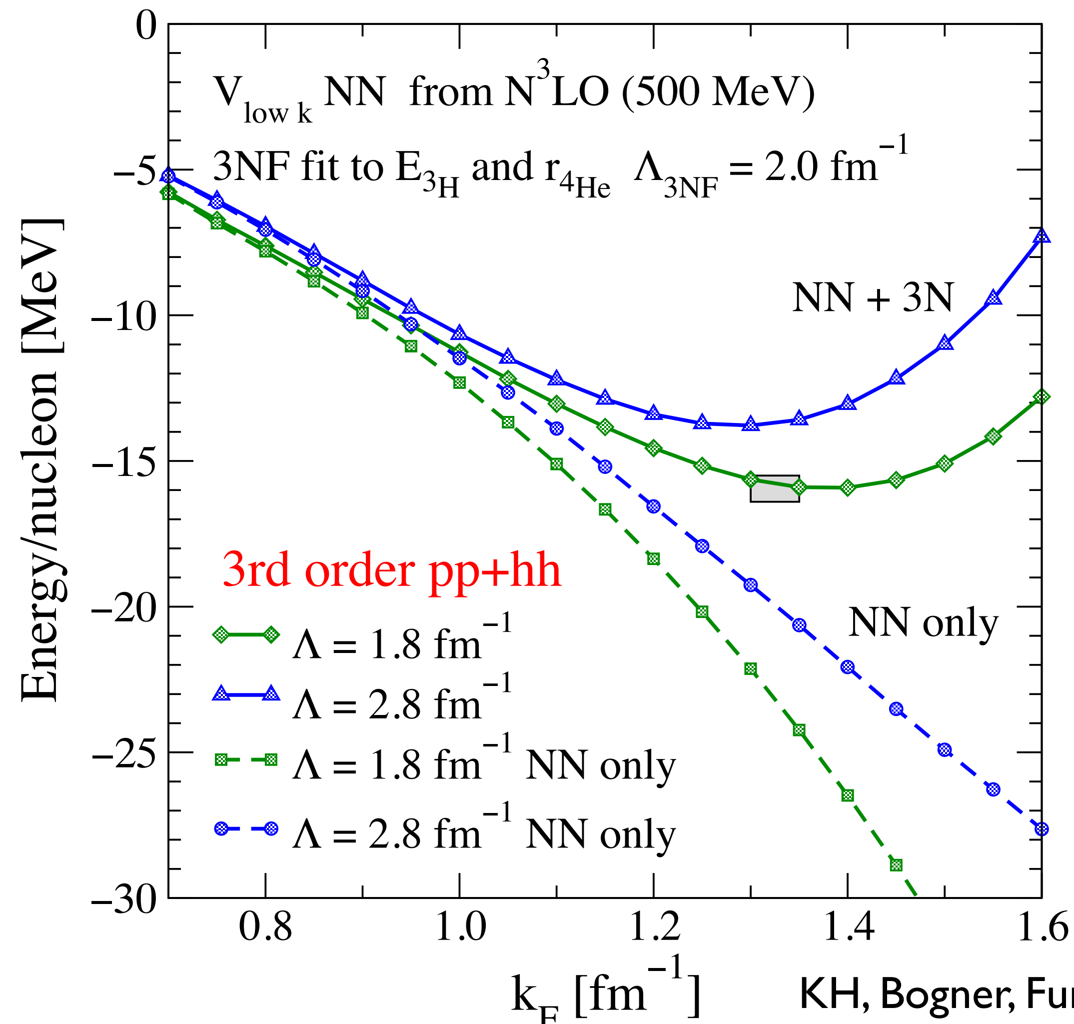
intermediate ($c_D$) and short-range ($c_E$) 3NF couplings fitted to few-body systems at different resolution scales:

$$E_{^3\mathrm{H}} = -8.482\,\mathrm{MeV} \qquad r_{^4\mathrm{He}} = 1.464\,\mathrm{fm}$$

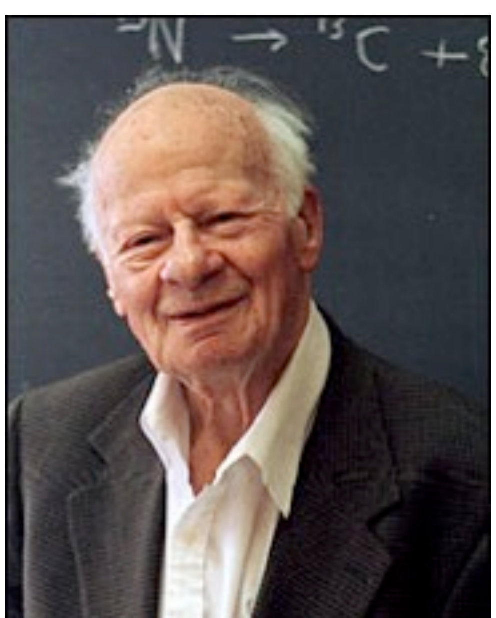


# Fitting the 3NF LECs at low resolution scales




KH, Bogner, Furnstahl, Nogga, PRC(R) 83, 031301 (2011)

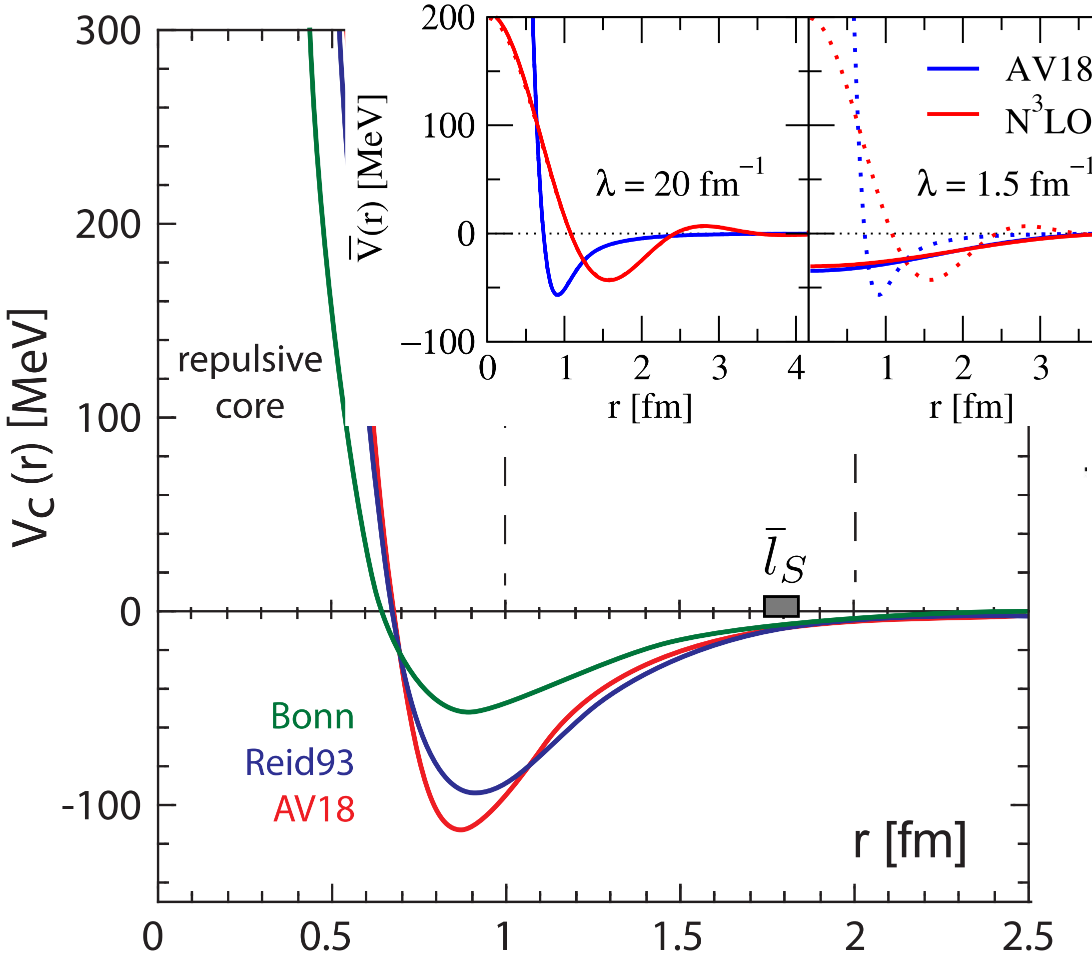


*"Very soft potentials must be excluded because they do not give saturation; they give too much binding and too high density. In particular, a substantial tensor force is required."*

**Hans Bethe** (1971)

Reproduction of saturation point
without readjusting parameters!

# Fitting the 3NF LECs at low resolution scales




*"Very soft potentials must be excluded because they do not give saturation;*
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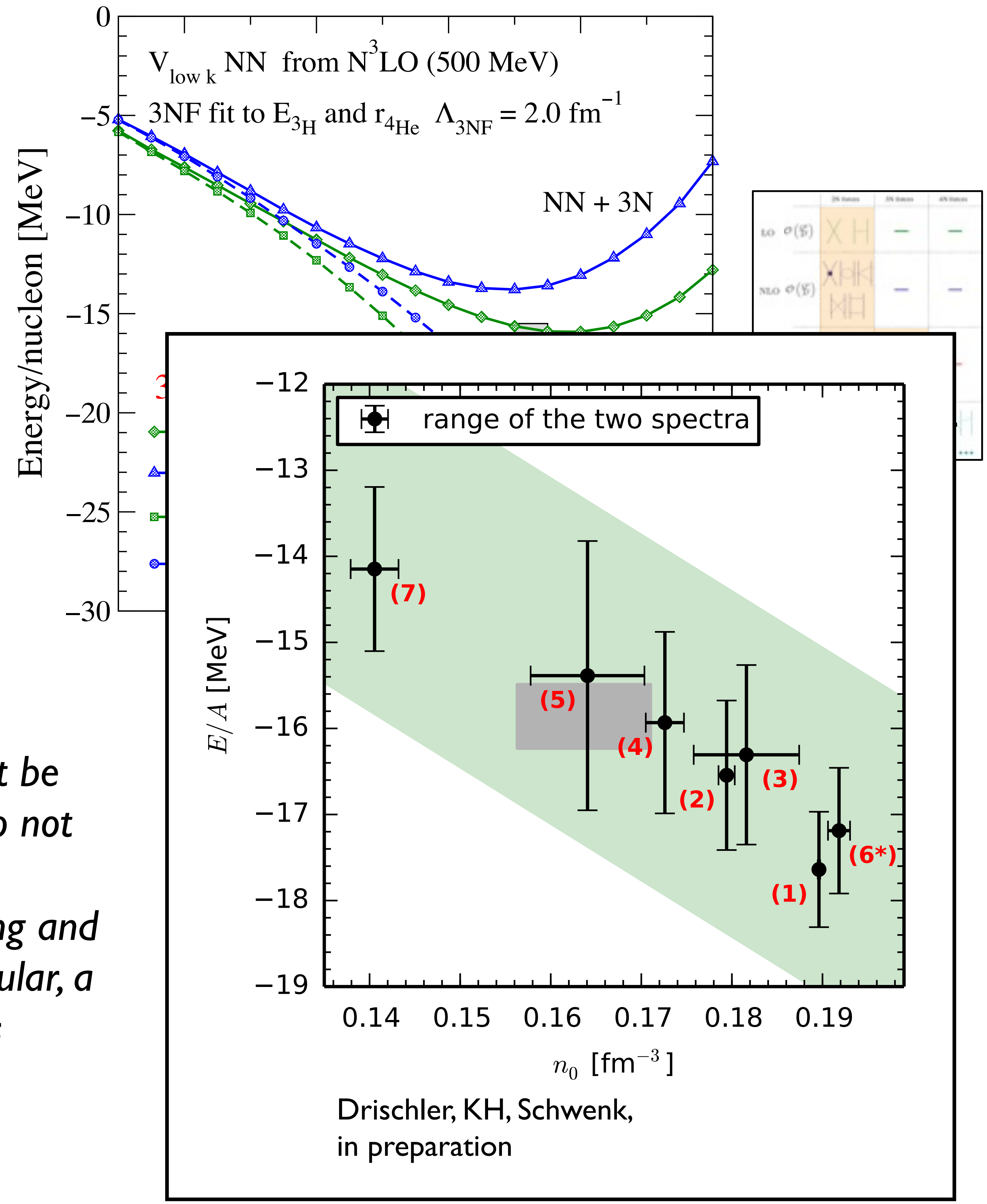
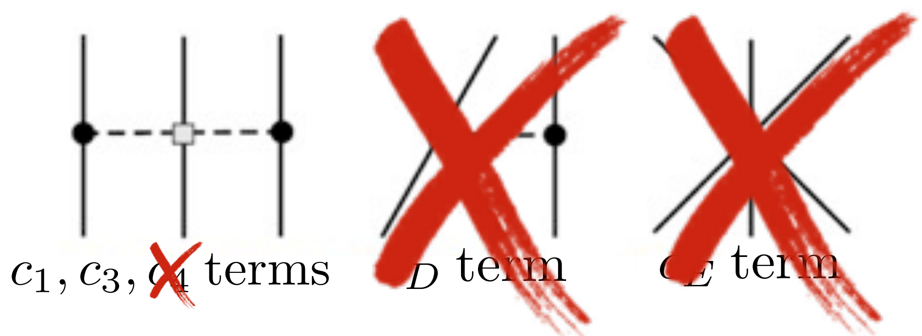
**Hans Bethe** (1971)

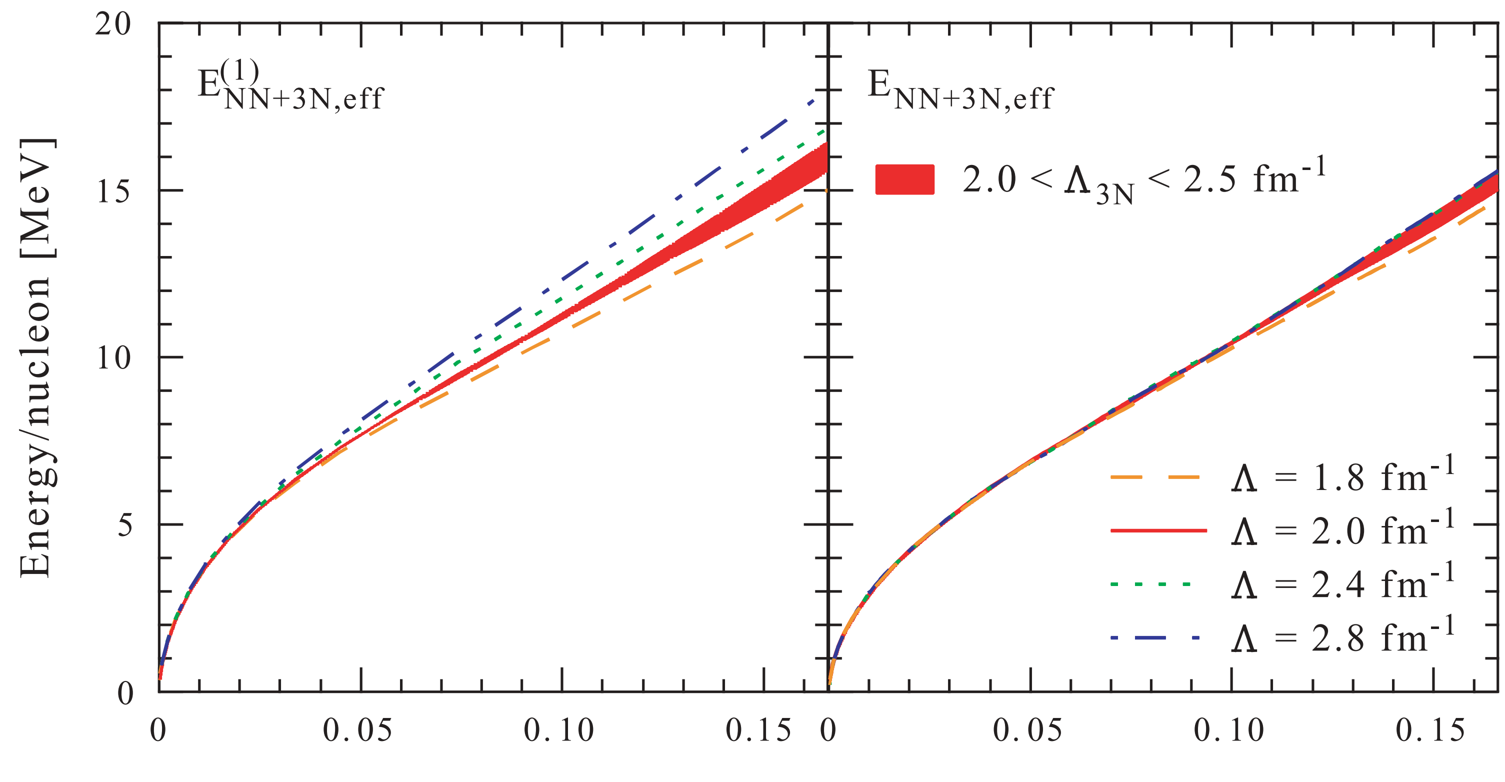


# Results for the neutron matter equation of state

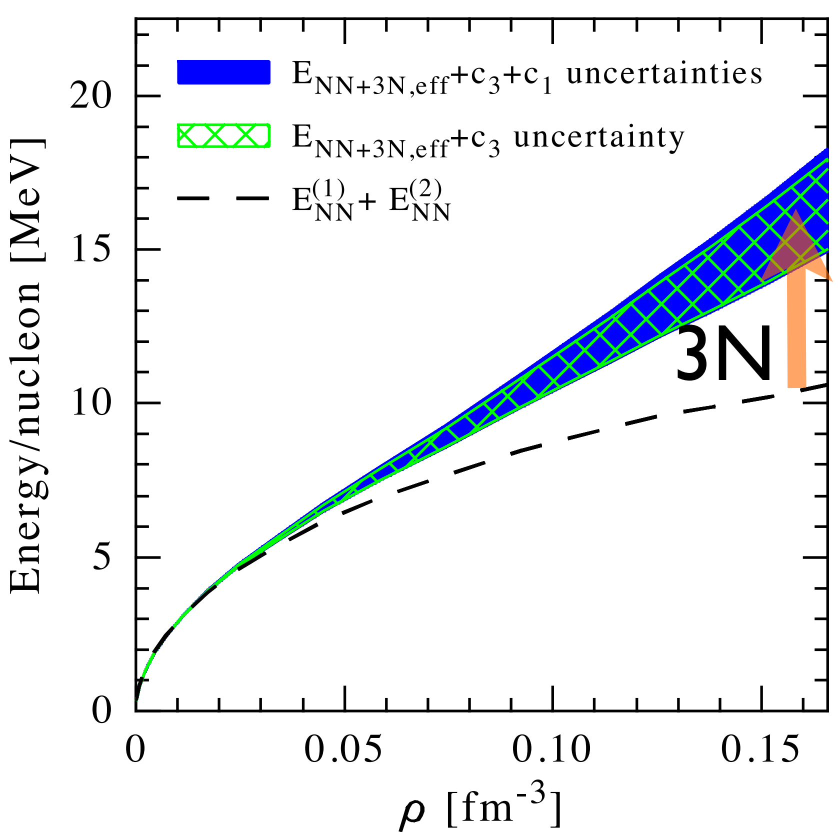
neutron matter is a **unique system** for chiral EFT:

only long-range 3NF contribute in leading order



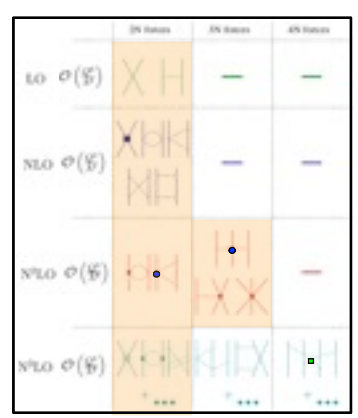




KH and Schwenk
PRC 82, 014314 (2010)



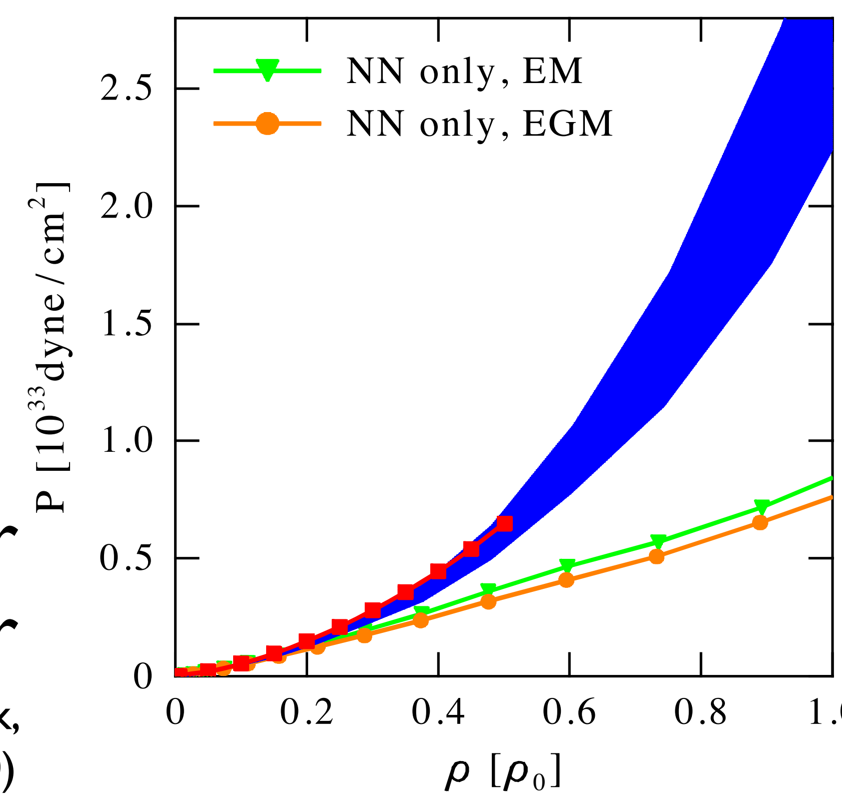

## pure neutron matter

KH and Schwenk PRC 82, 014314 (2010)

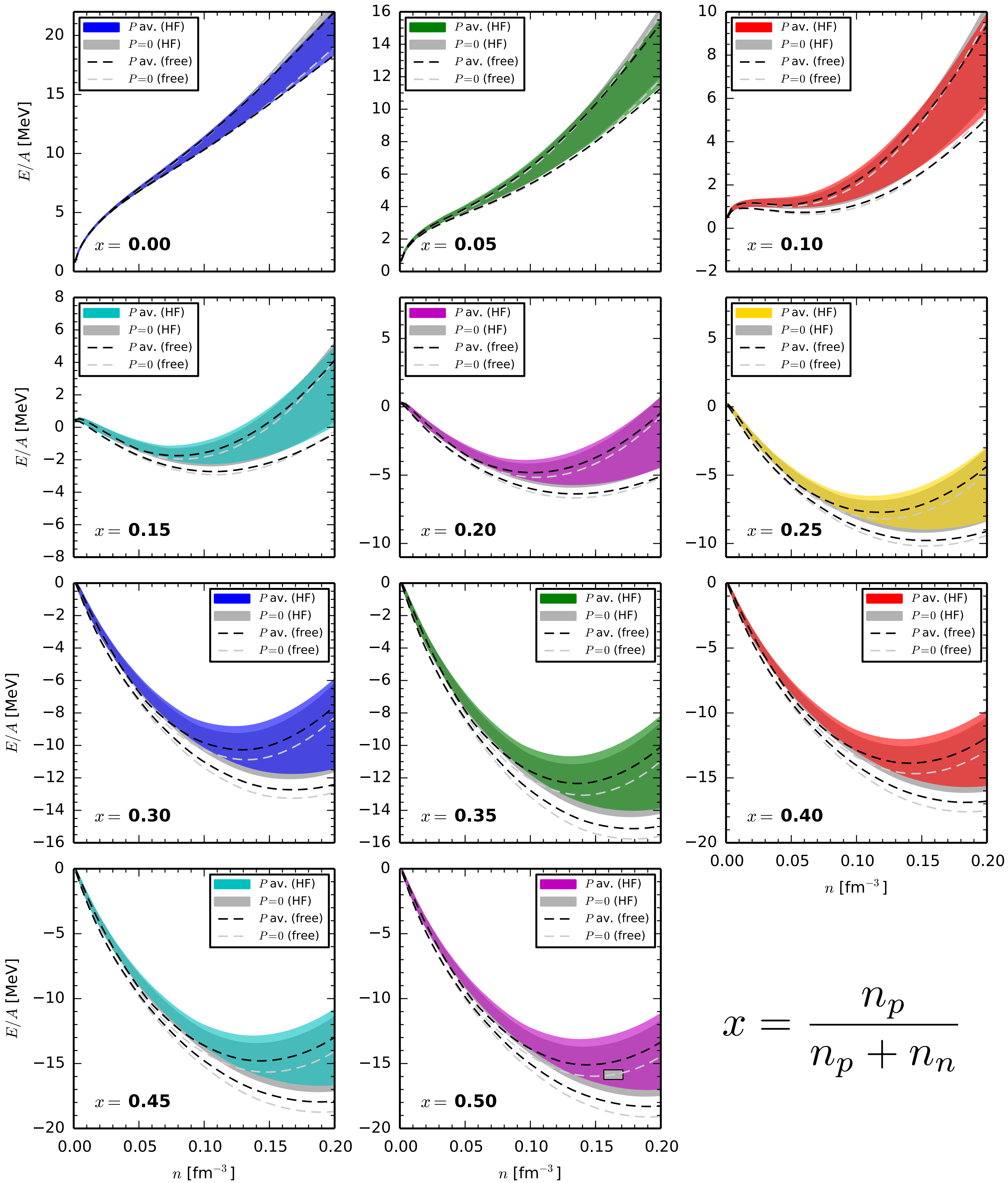


## neutron star matter

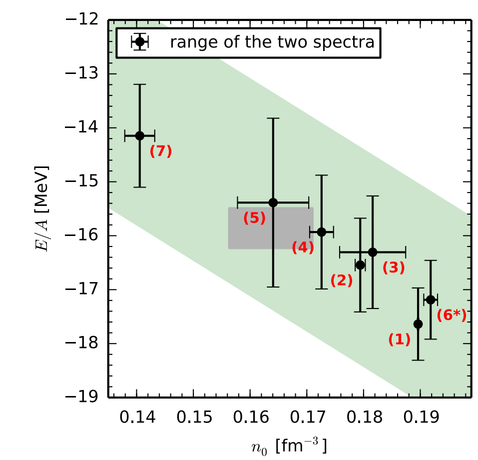
KH, Lattimer, Pethick, Schwenk,
PRL 105, 161102 (2010)



# First application to isospin asymmetric nuclear matter



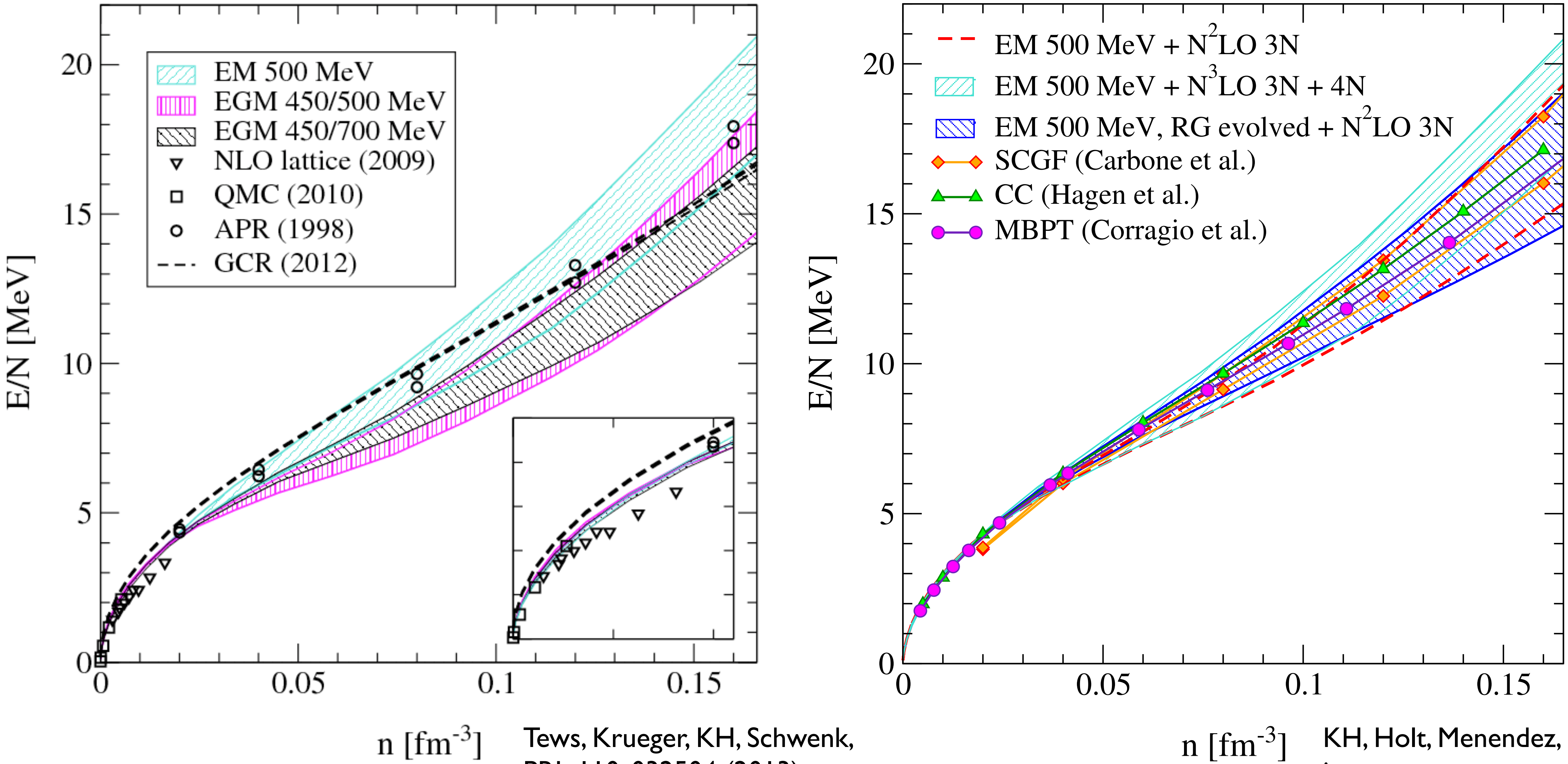

- uncertainty bands determined by set of 7 Hamitonians




$$x = \frac{n_p}{n_p + n_n}$$

Drischler, KH, Schwenk, in preparation

# First complete calculations of neutron matter at N³LO

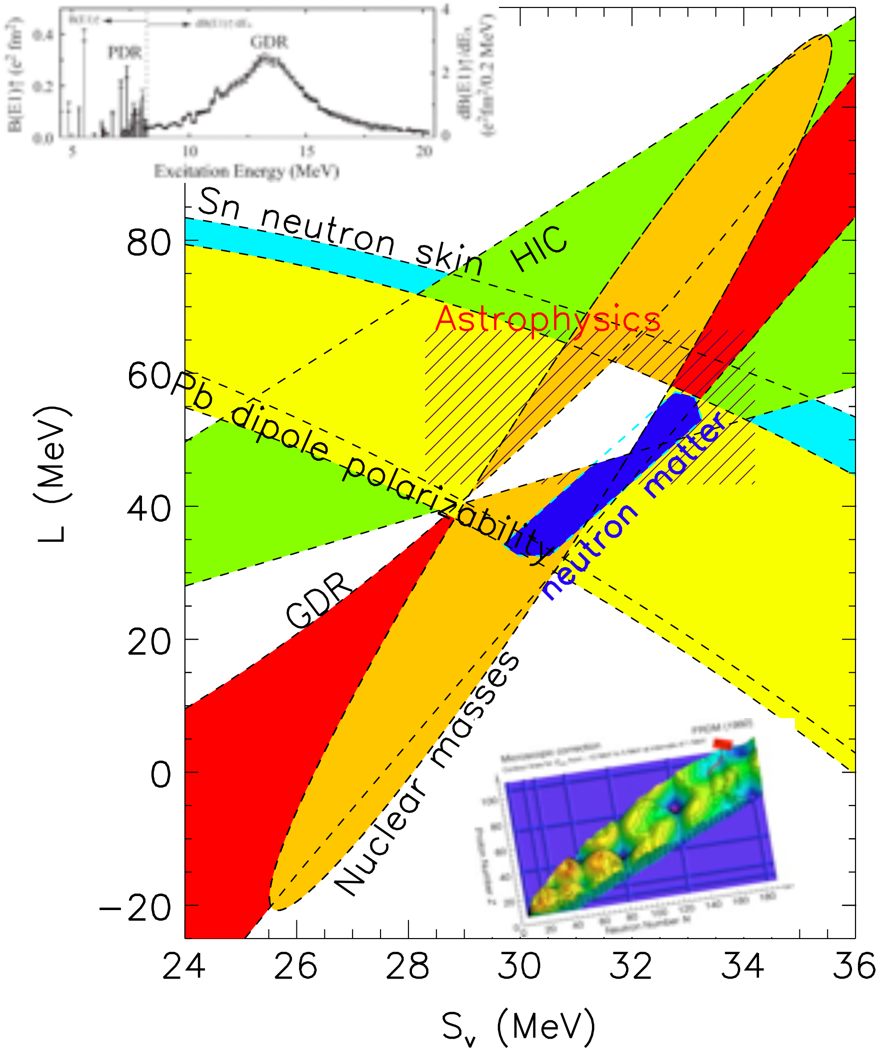



Tews, Krueger, KH, Schwenk, PRL 110, 032504 (2013)

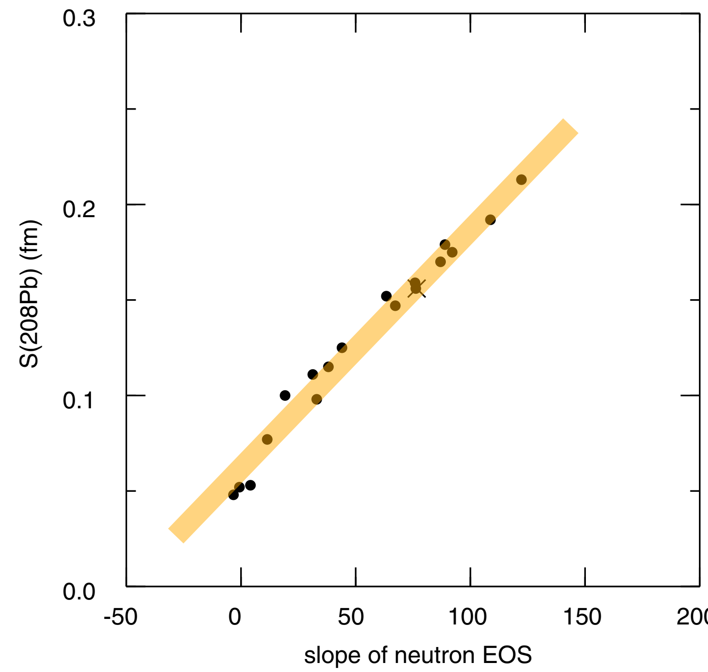
KH, Holt, Menendez, Schwenk, in press

- bands include uncertainties from many-body calculations and NN, 3NF and 4NF
- good agreement with other methods
- significant contributions from 3NF at N3LO

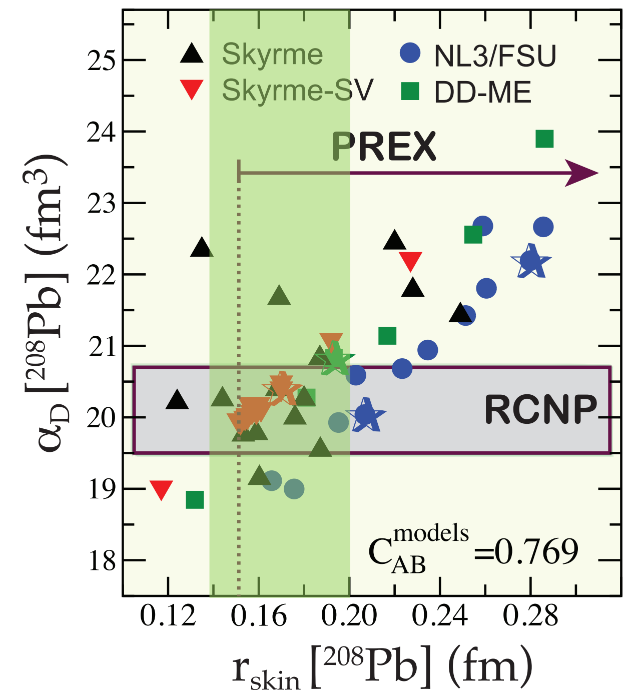
# Symmetry energy and neutron skin constraints




KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)




Brown,
PRL 85, 5296 (2000)




Piekarewicz,
PRC 85, 041302 (2012)

$$S_v = \left. \frac{\partial^2 E/N}{\partial^2 x} \right|_{\rho=\rho_0, x=1/2}$$

$$L = \frac{3}{8} \left. \frac{\partial^3 E/N}{\partial \rho \partial^2 x} \right|_{\rho=\rho_0, x=1/2}$$

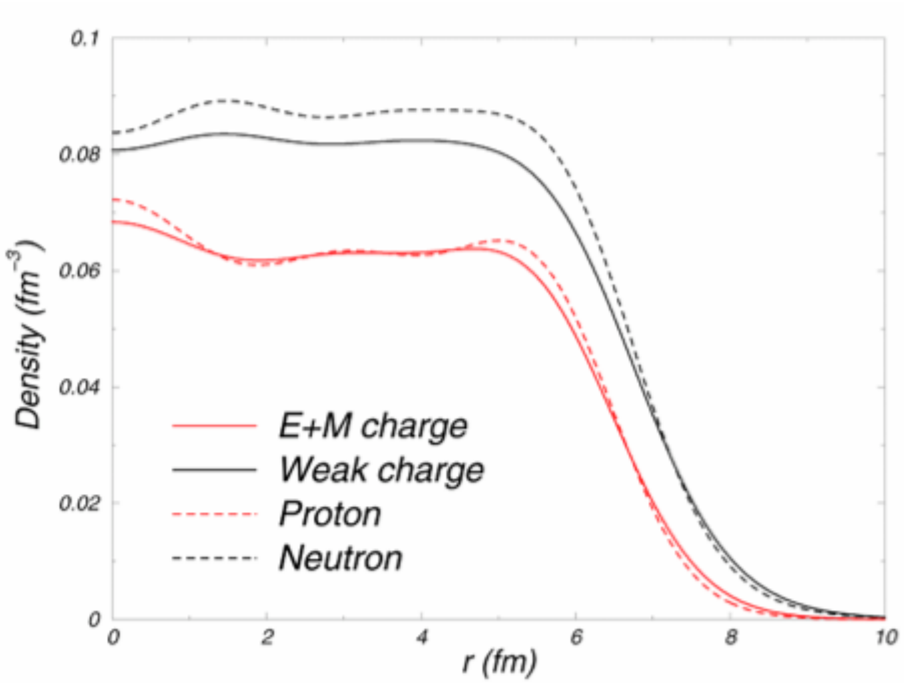
neutron skin constraint from
neutron matter results:

$$r_{\rm skin}[^{208}{\rm Pb}] = 0.14 - 0.2 \text{ fm}$$

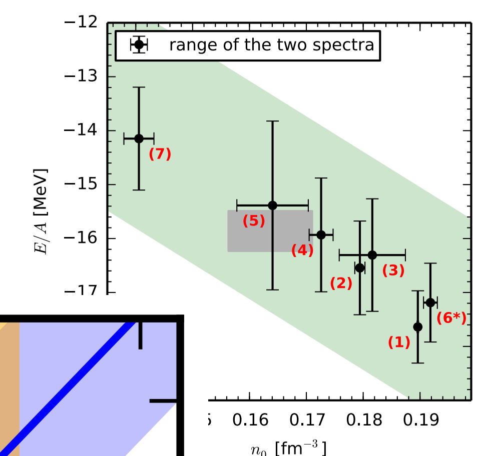
KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

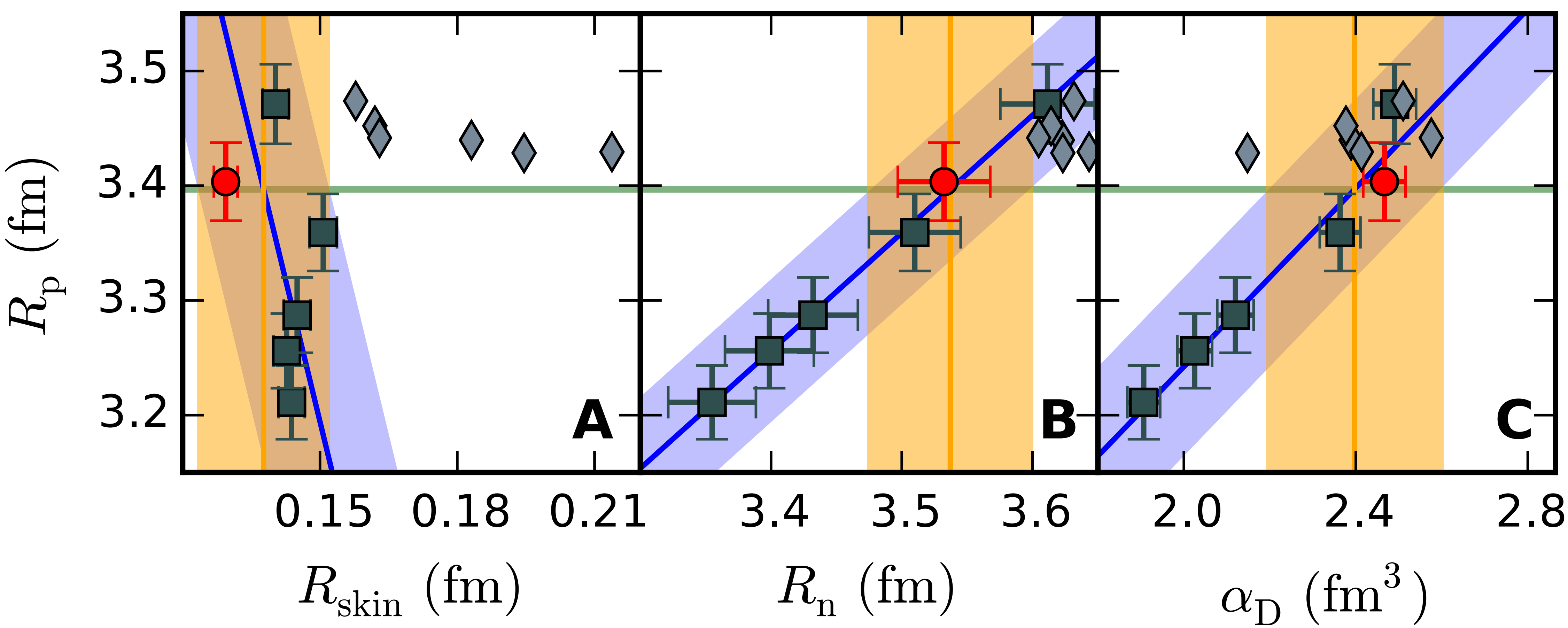
- neutron matter give tightest constraints
- in agreement with all other constraints

# Symmetry energy and neutron skin constraints




ab intio coupled cluster calculations of neutron skin and dipole polarizability of $^{48}$Ca



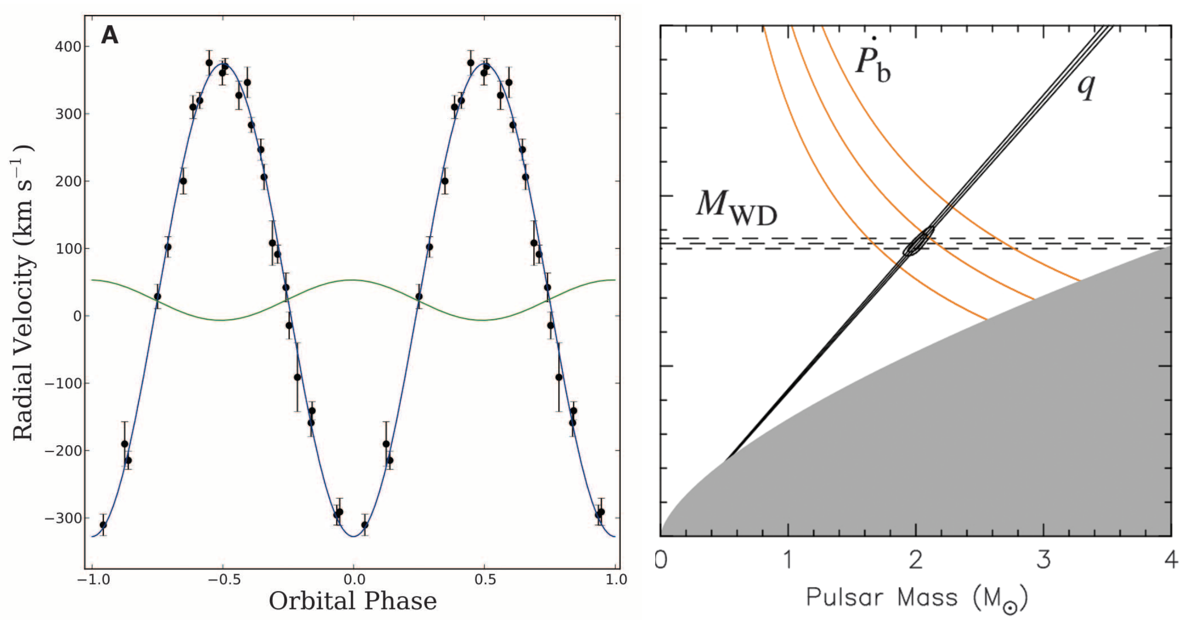




Hagen et al.,

# Constraints on the nuclear equation of state (EOS)

Science

**A Massive Pulsar in a Compact Relativistic Binary**

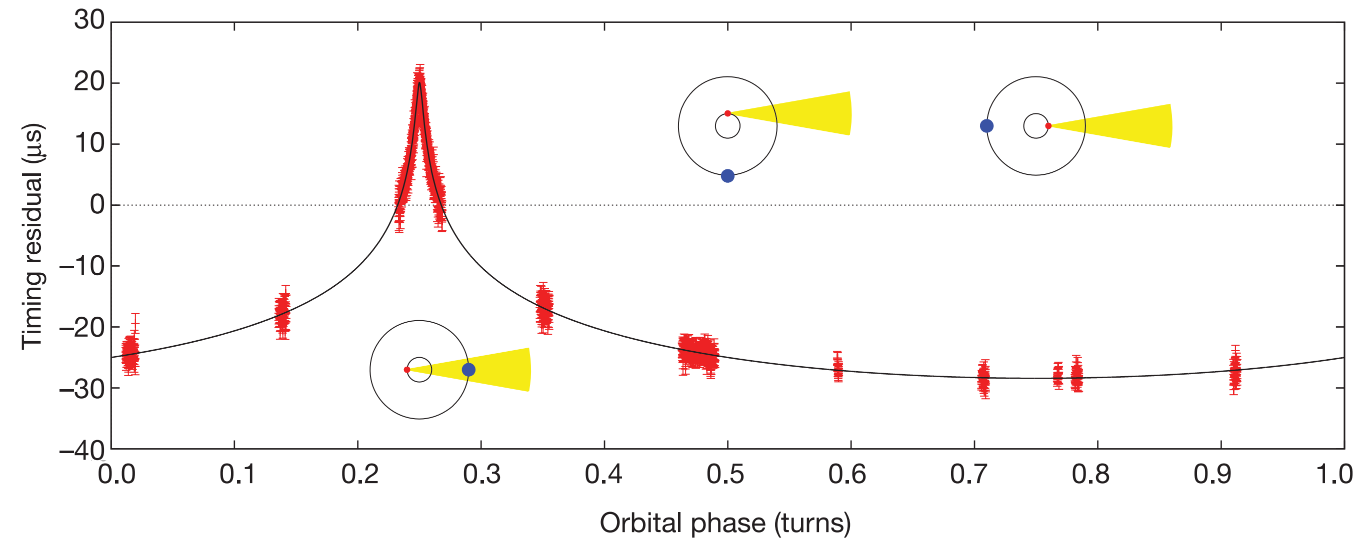



Antoniadis et al., Science 340, 448 (2013)

nature

**A two-solar-mass neutron star measured using Shapiro delay**

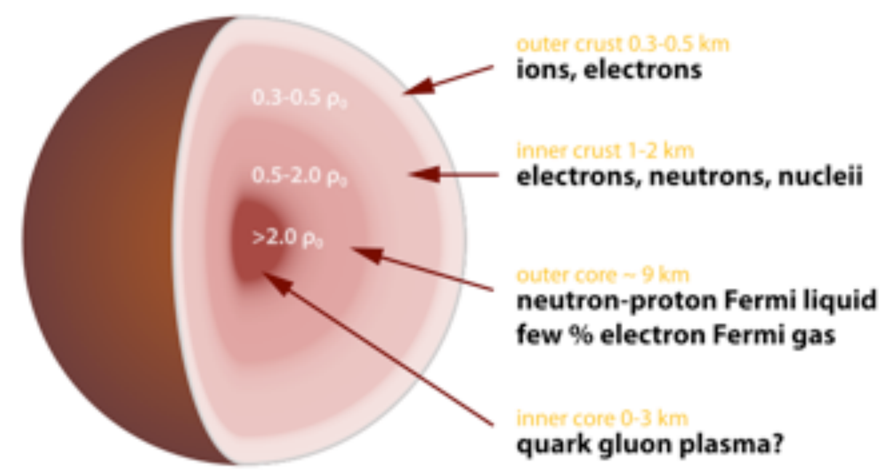
P. B. Demorest[1], T. Pennucci[2], S. M. Ransom[1], M. S. E. Roberts[3] & J. W. T. Hessels[4,5]




Demorest et al., Nature 467, 1081 (2010)

New constraints from recent observations:

$$M_{\rm max} = 1.65 M_\odot \rightarrow 1.97 \pm 0.04\, M_\odot$$
$$\rightarrow 2.01 \pm 0.04\, M_\odot$$




Calculation of neutron star properties require EOS up to high densities.

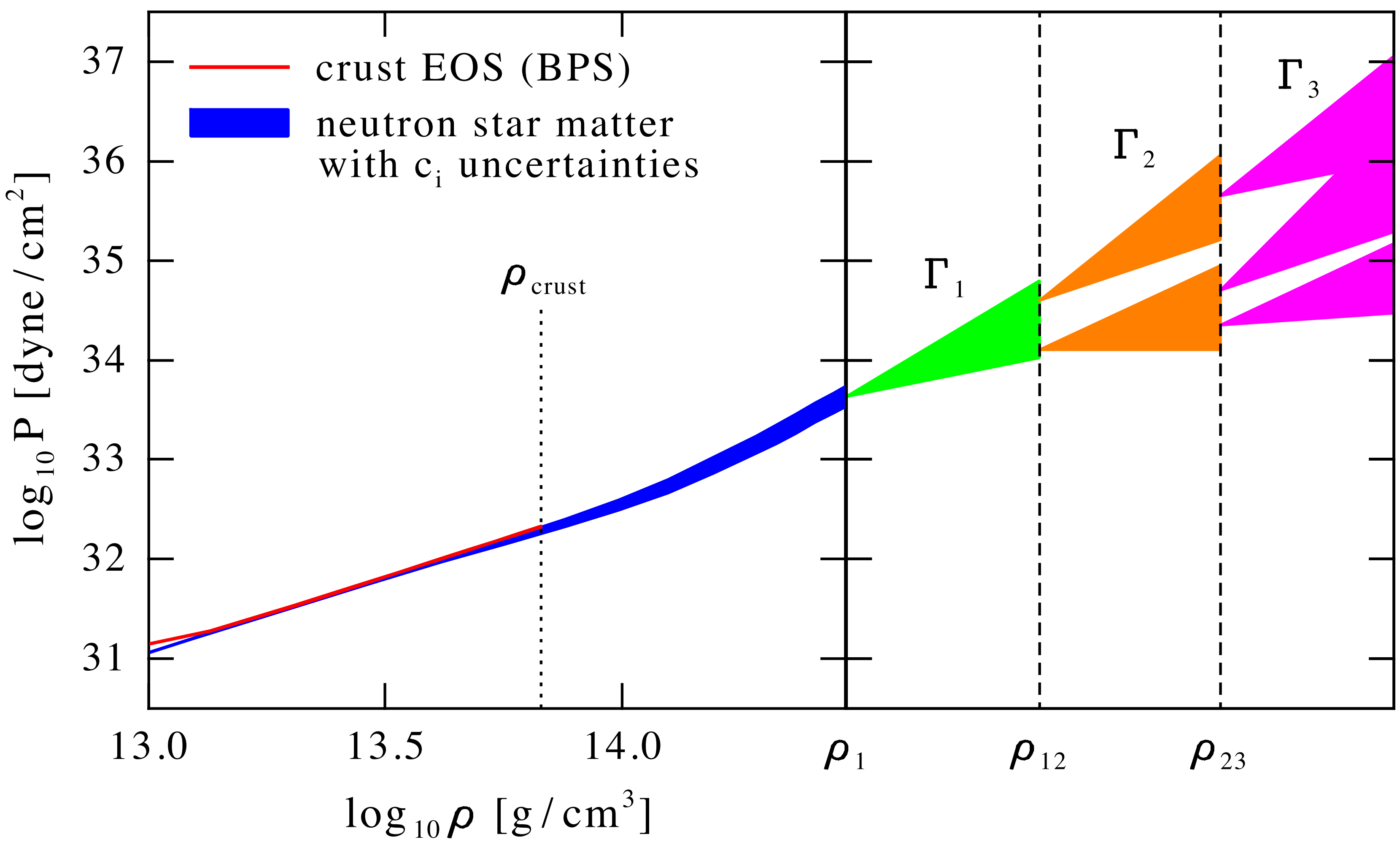
Strategy:
Use observations to constrain the high-density part of the nuclear EOS.

# Neutron star radius constraints

incorporation of beta-equilibrium: neutron matter $\longrightarrow$ neutron star matter

parametrize piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^{\Gamma}$
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics




KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)
KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

# Constraints on the nuclear equation of state

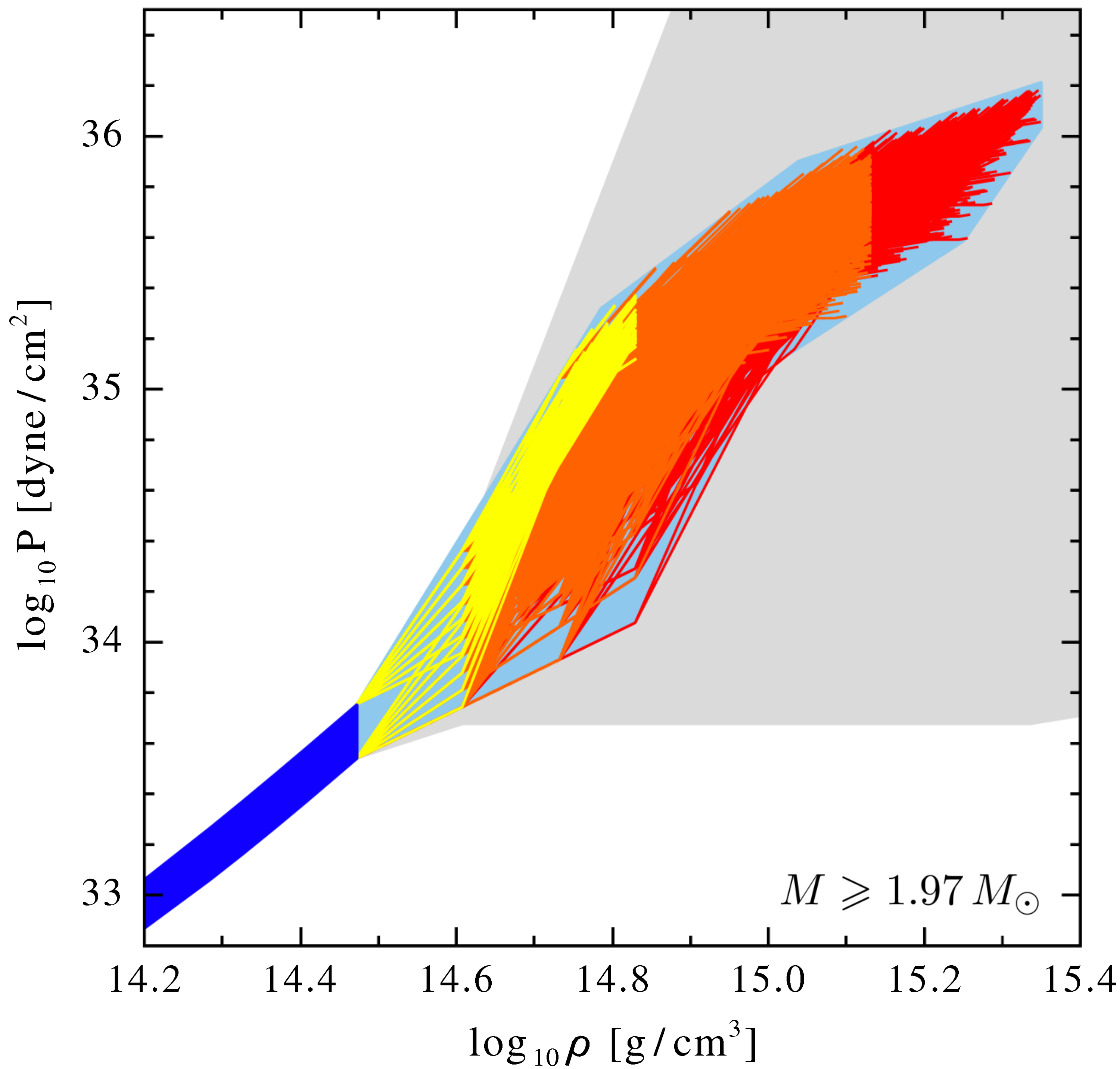
use the constraints:

recent NS observations

$$M_{\max} > 1.97\, M_\odot$$

causality

$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$




KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)

constraints lead to significant reduction of EOS uncertainty band

# Constraints on the nuclear equation of state

use the constraints:

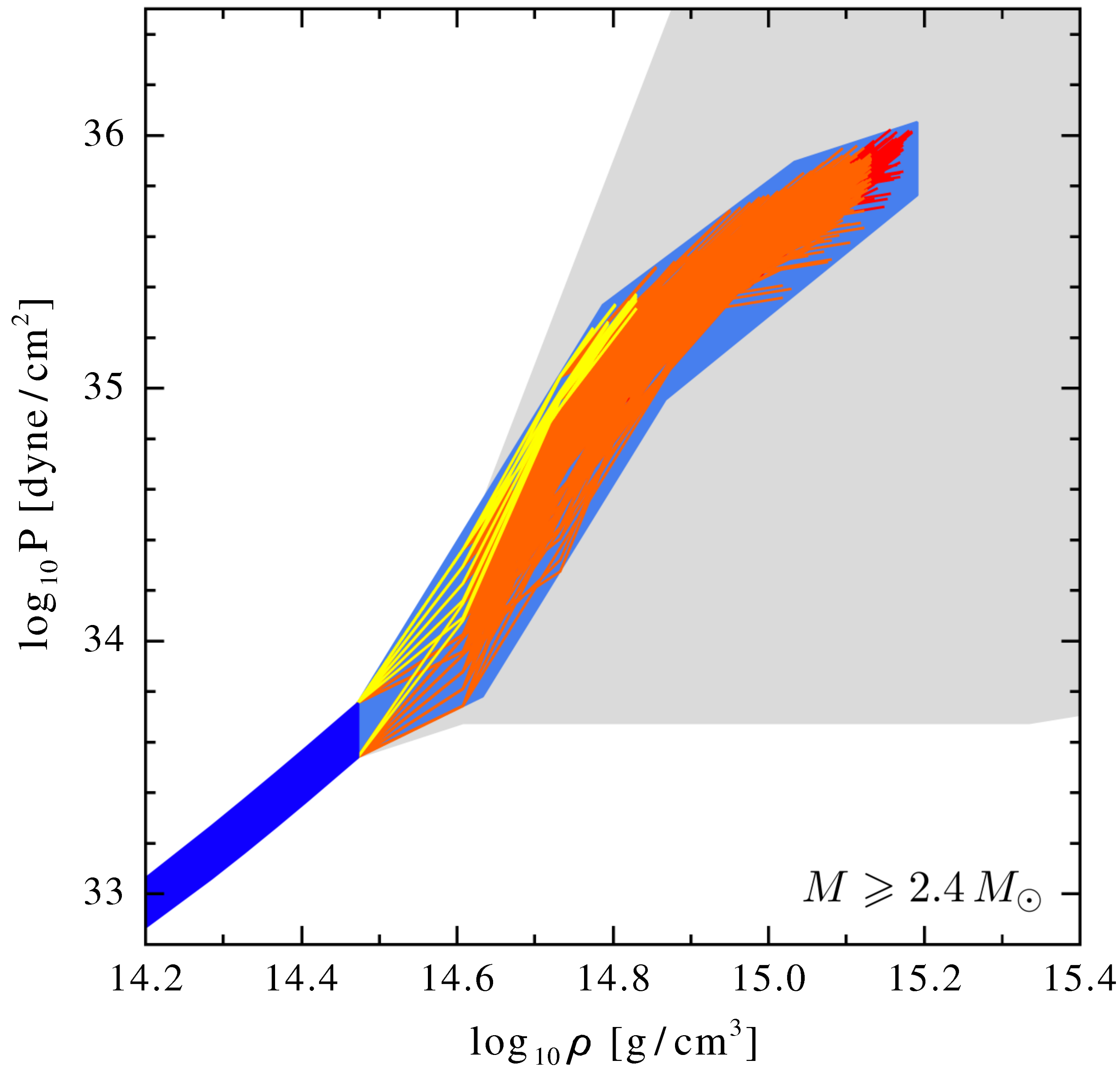
fictitious NS mass

$M_{\max} > 2.4\, M_\odot$

causality

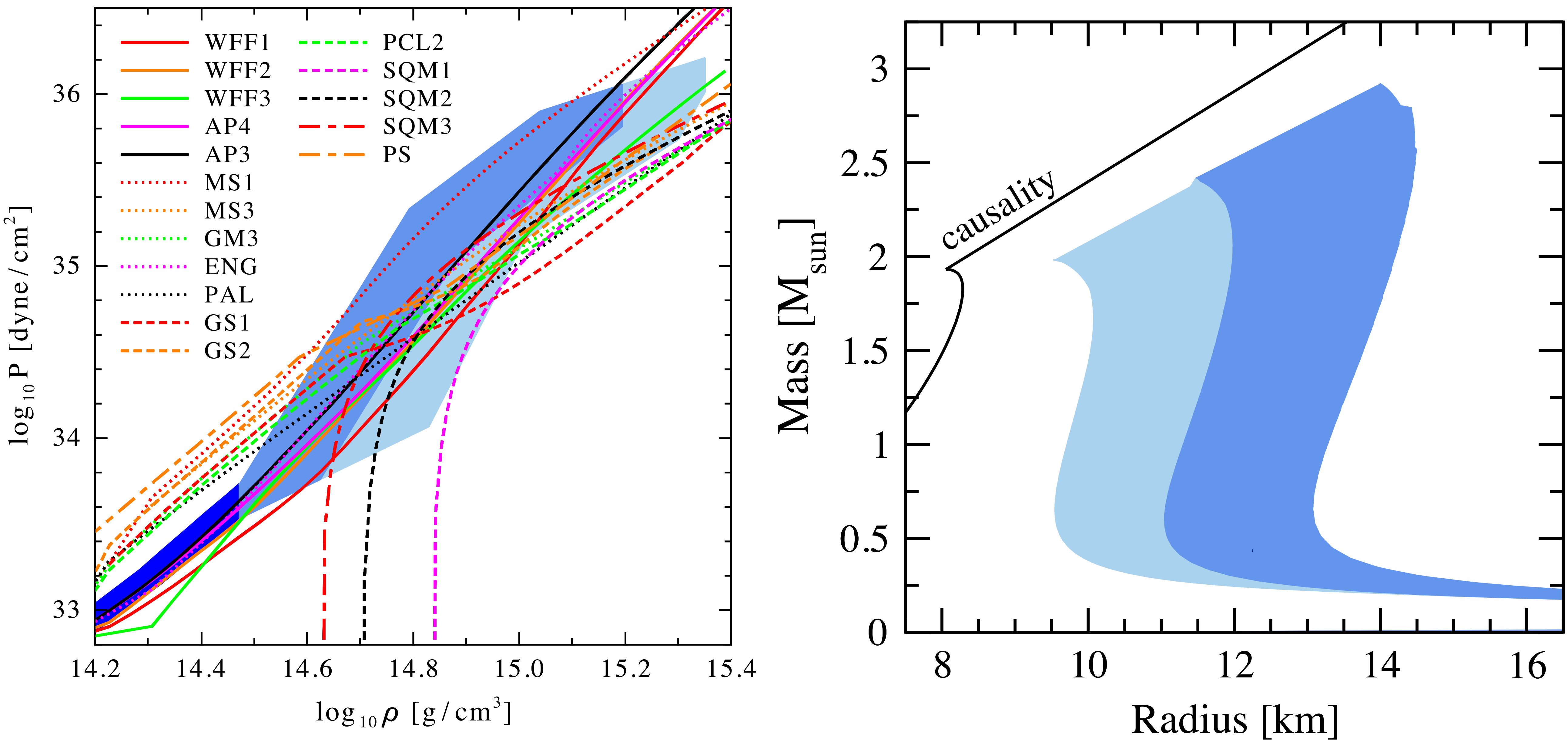
$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$

KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)




increased $M_{\max}$ systematically reduces width of band

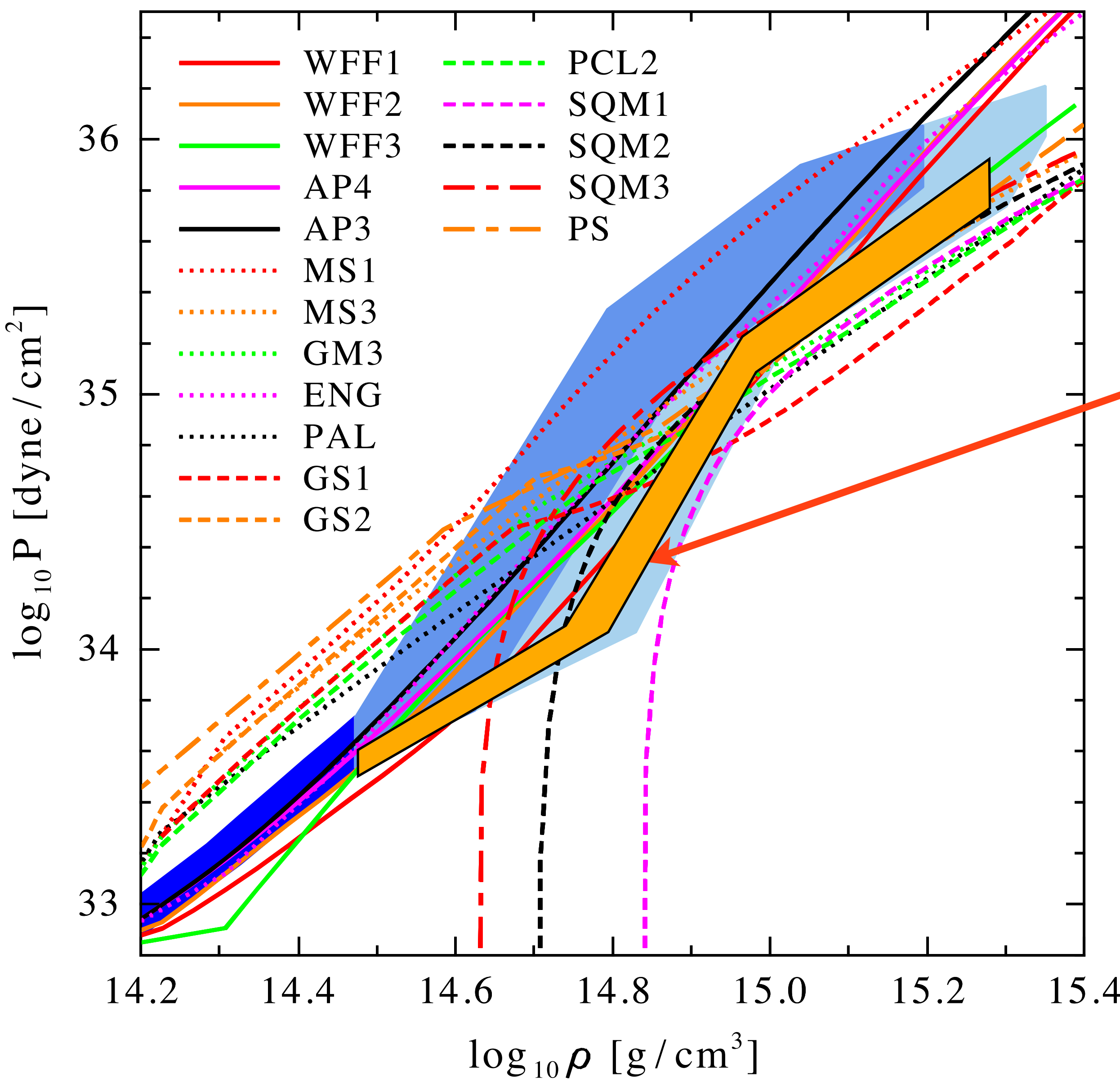
# Constraints on neutron star radii

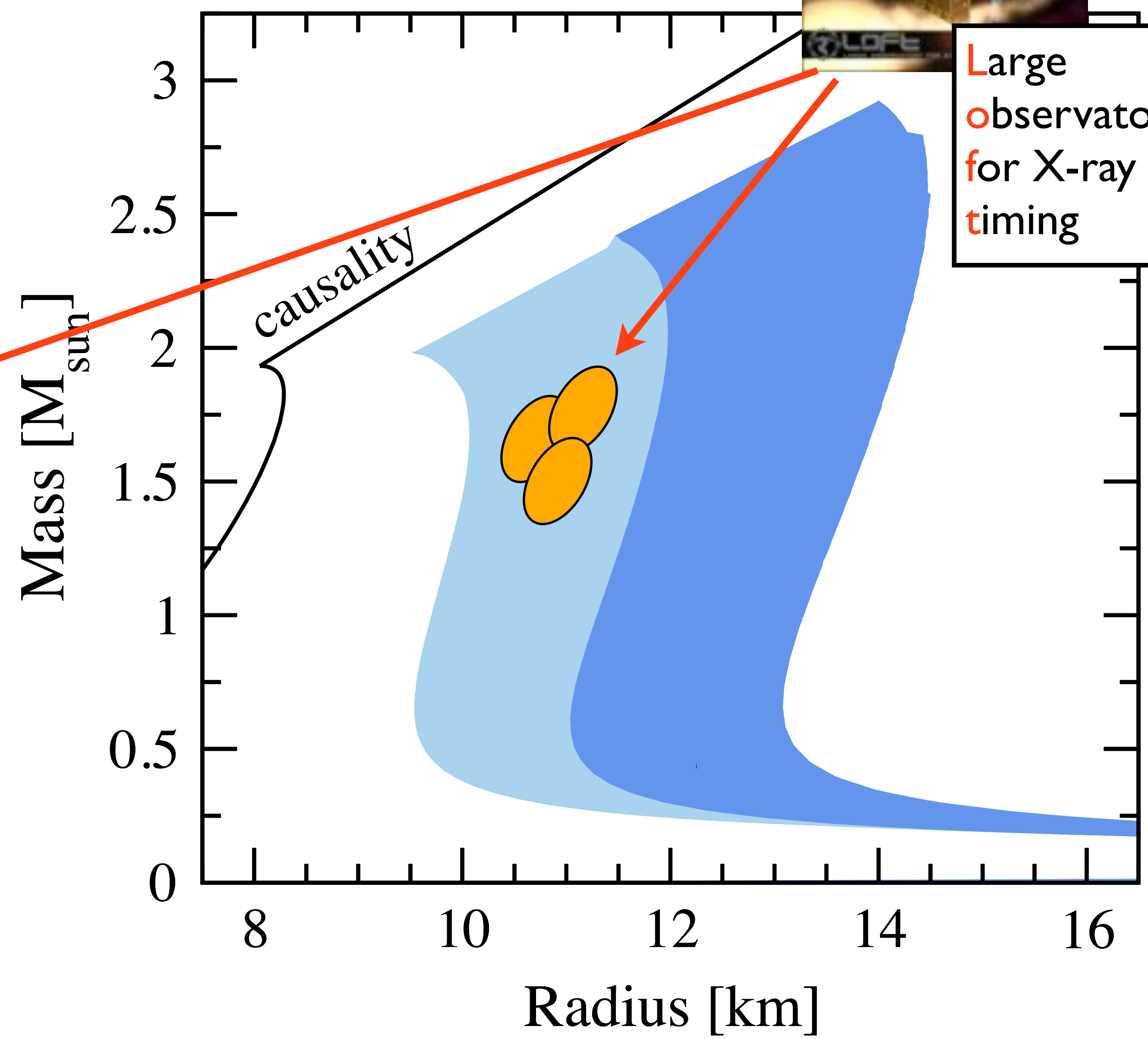



KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)
see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- current radius prediction for typical $1.4\,M_\odot$ neutron star: $9.7 - 13.9\,\mathrm{km}$

# Constraints on neutron star radii



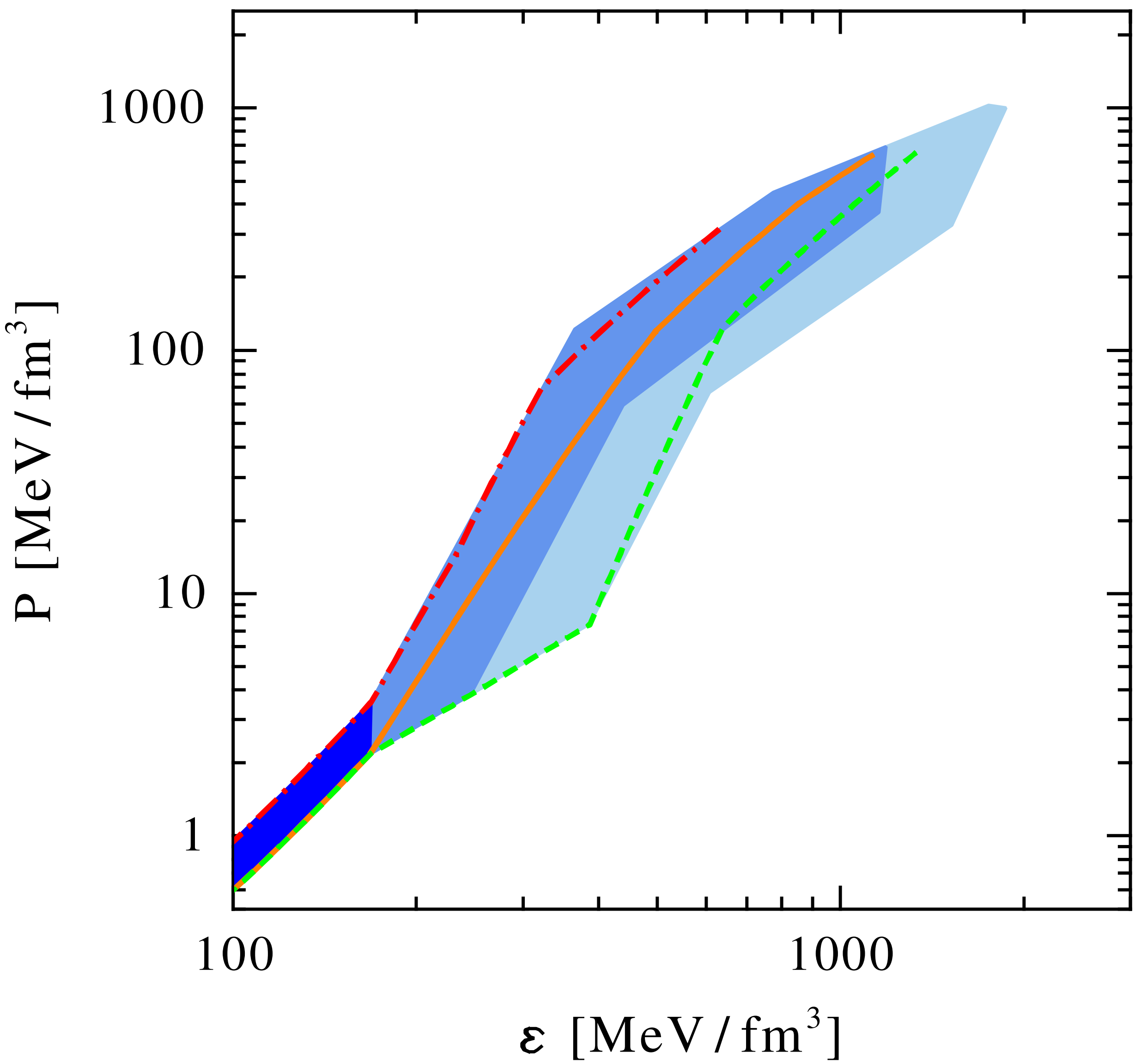



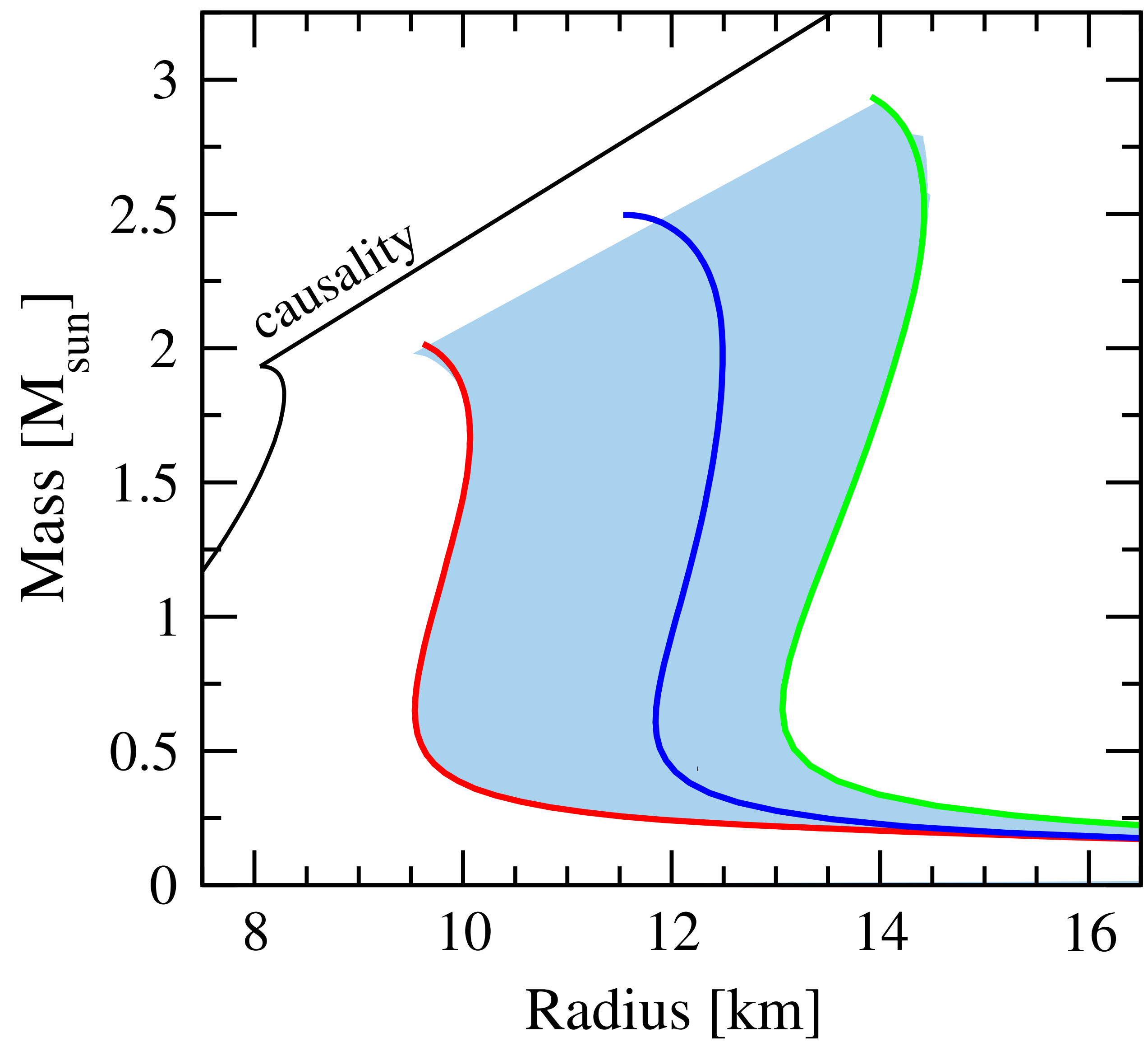


KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)
see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- current radius prediction for typical $1.4\,M_\odot$ neutron star: $9.7 - 13.9\,\mathrm{km}$
- new observatories could significantly improve constraints

# Representative set of EOS

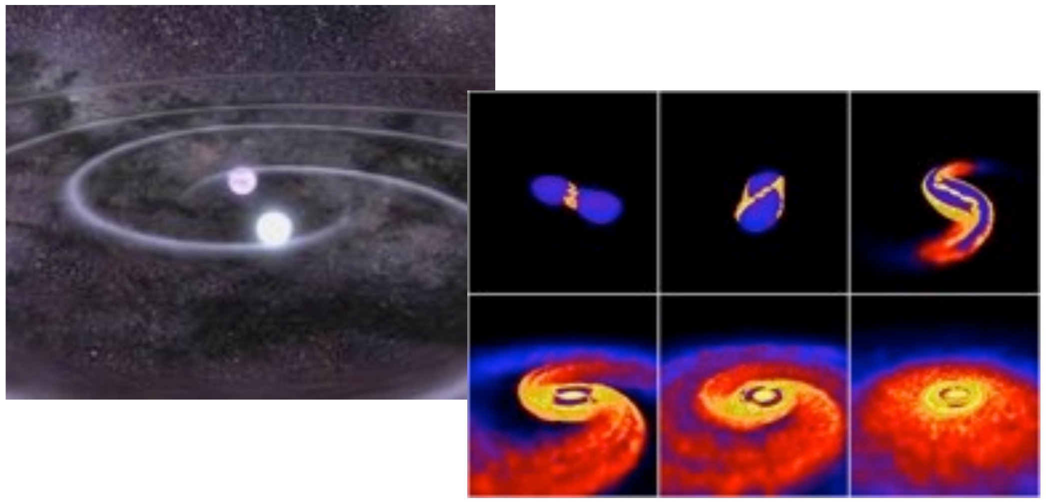


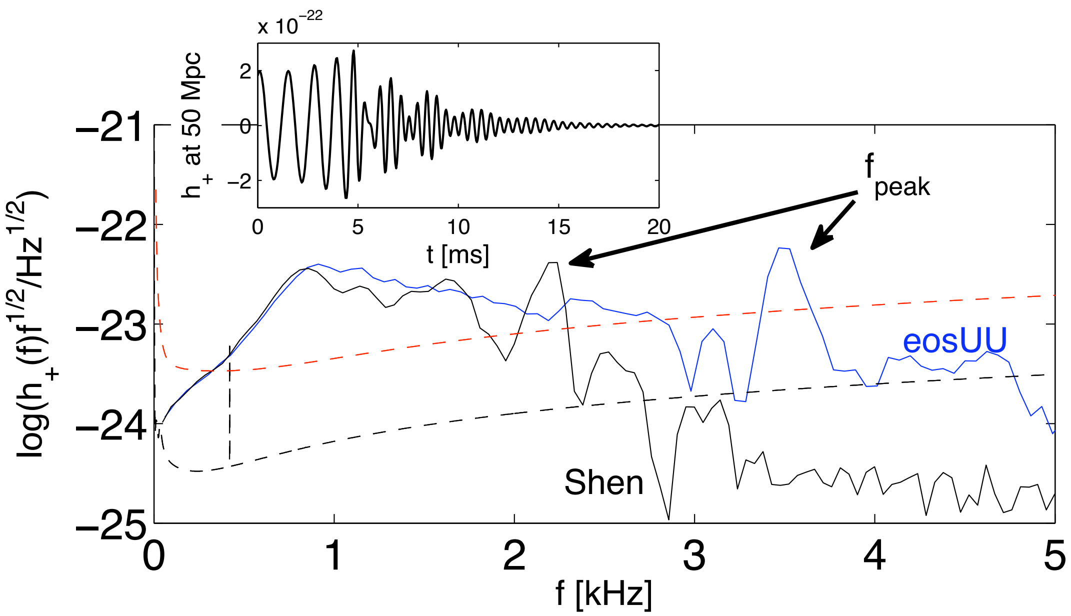



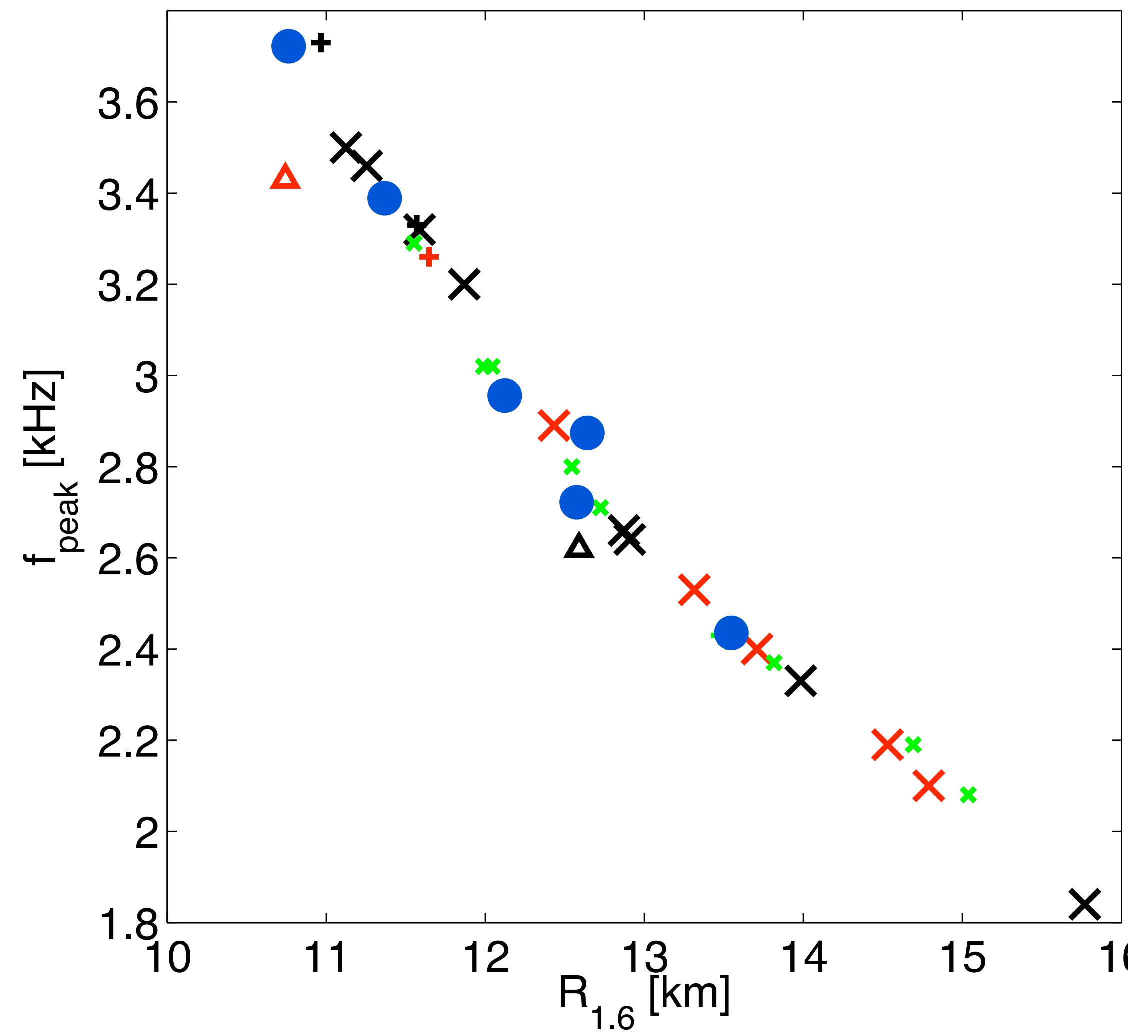

KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)

- constructed 3 representative EOS compatible with uncertainty bands for astrophysical applications: soft, intermediate and stiff
- allows to probe impact of current theoretical EOS uncertainties on astrophysical observables

# Gravitational wave signals from neutron star binary mergers



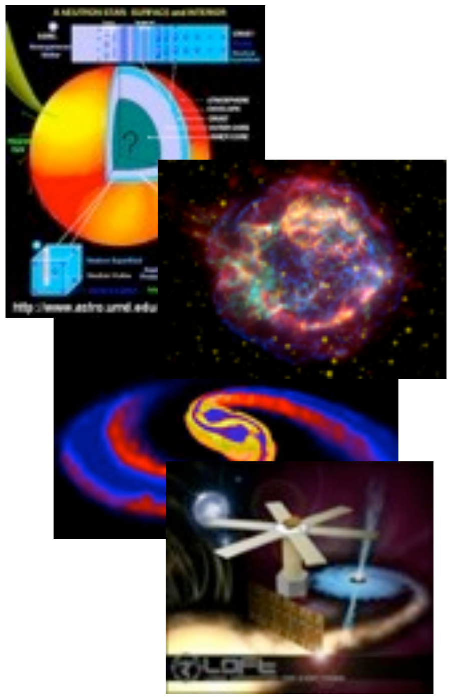



Bauswein and Janka, PRL 108, 011101 (2012),
Bauswein, Janka, KH, Schwenk, PRD 86, 063001

- simulations of NS binary mergers show strong correlation between between $f_{\rm peak}$ of the GW spectrum and the radius of a NS
- measuring $f_{\rm peak}$ is key step for constraining EOS systematically at large $\rho$

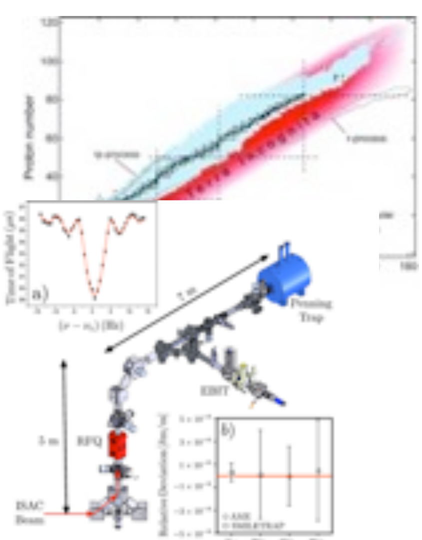
# Future directions, open problems



- develop the most advanced chiral Hamiltonians to enable controlled microscopic calculations of matter and light as well as medium-mass nuclei

- improve EOS constraints at high densities (LOFT, GW waves, ?), explore limits of chiral EFT interactions

- extend EOS calculations to finite temperature



- calculate response functions and neutrino interactions in matter

- benchmarks between different many-body frameworks based on a set of Hamiltonians

- **derivation of systematic uncertainty estimates** by performing order-by-order calculations in chiral expansion

# In collaboration with:

C. Drischler, T. Krüger, R. Roth, A. Schwenk, I. Tews

S. Bogner

A. Gezerlis,

J. Lattimer

J. Golak, R. Skibinski

R. Furnstahl, S. More

E. Epelbaum, H. Krebs

A. Nogga

C. Pethick

international collaborator in NUCLEI Nuclear Computational Low-Energy Initiative

---

## computing support:

HRZ Hochschulrechenzentrum

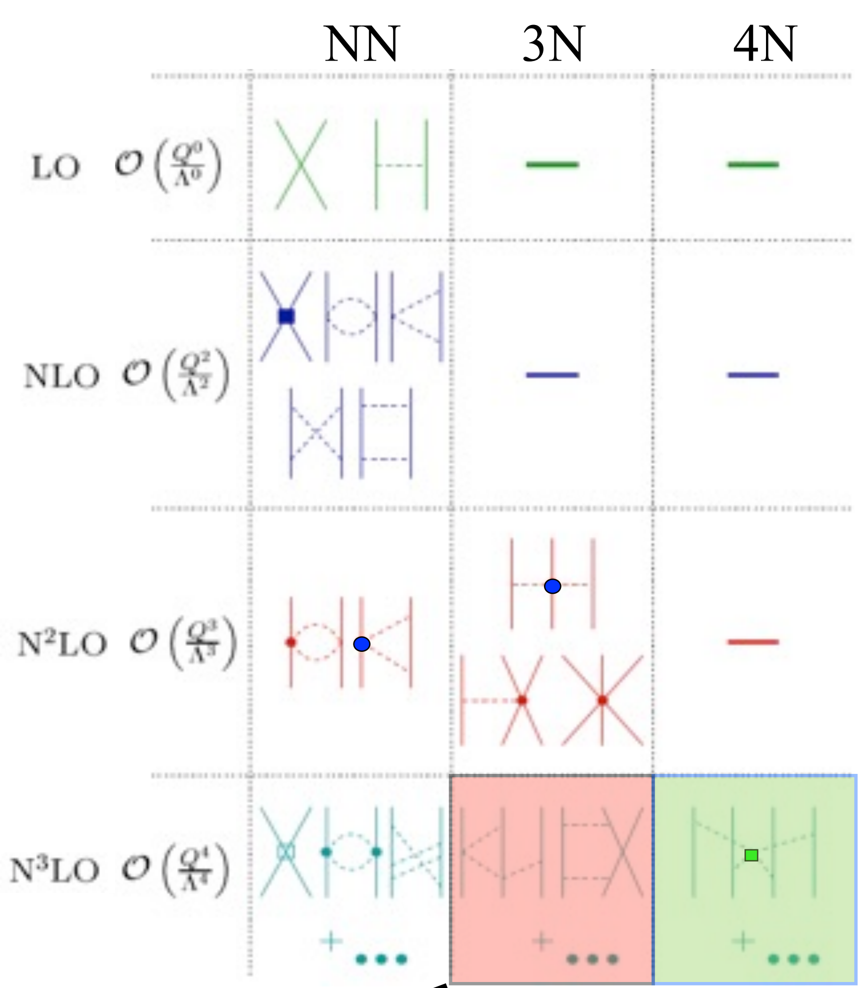
Ohio Supercomputer Center

JÜLICH FORSCHUNGSZENTRUM JUROPA

Thank you!

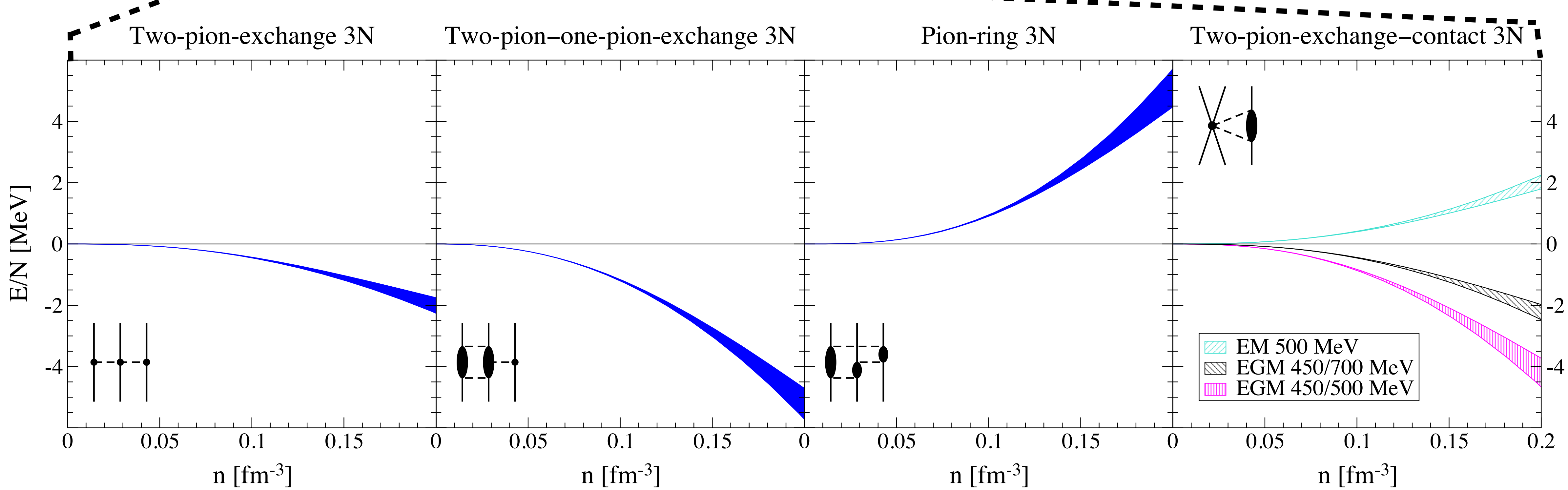
backup slides

# Contributions of many-body forces at $N^3LO$ in neutron matter

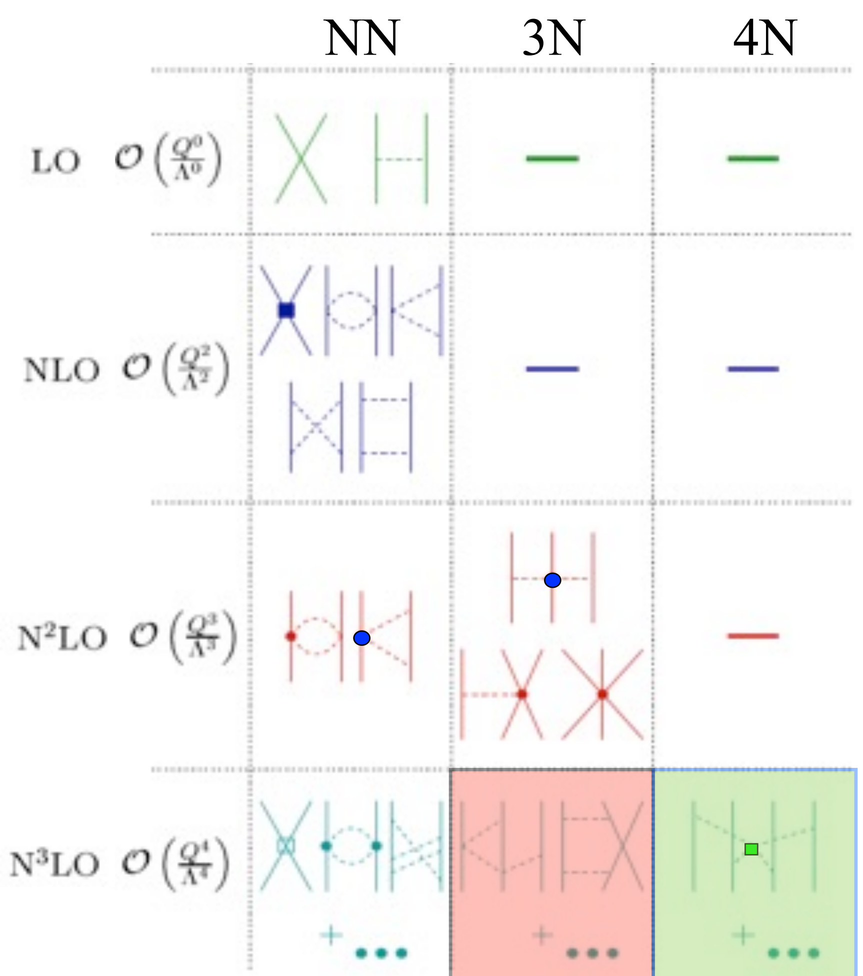



- first calculations of $N^3LO$ 3NF and 4NF contributions to EOS of neutron matter
- found large contributions in Hartree Fock appr., comparable to size of $N^2LO$ contributions

Tews, Krüger, KH, Schwenk
PRL 110, 032504 (2013)

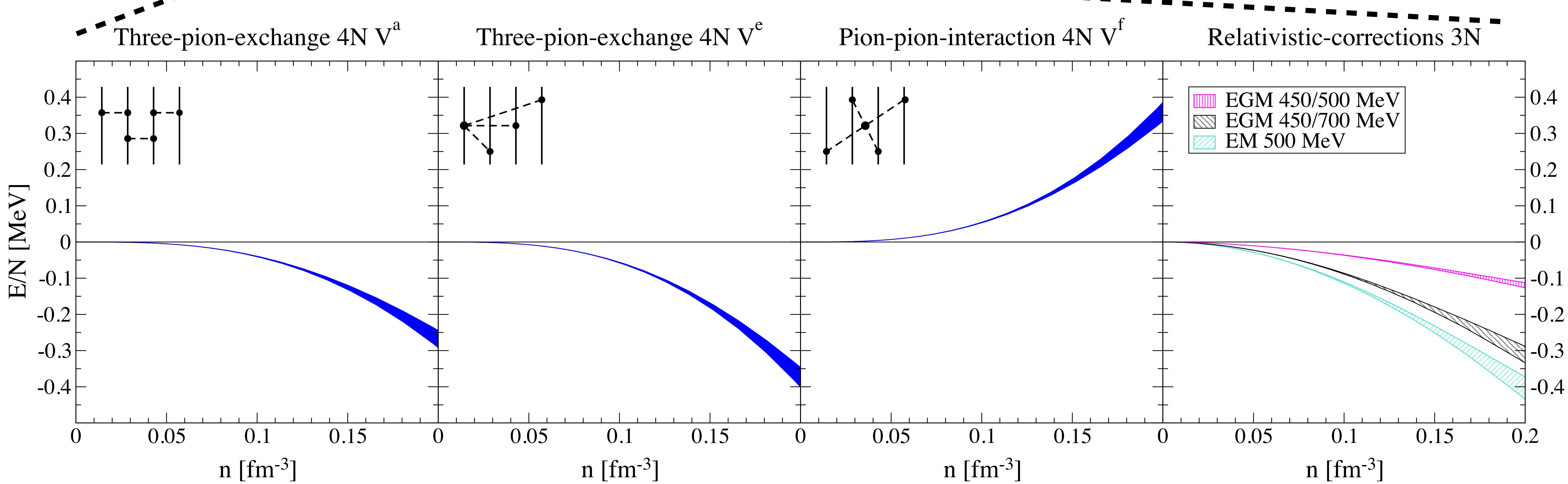


# Contributions of many-body forces at N$^3$LO in neutron matter

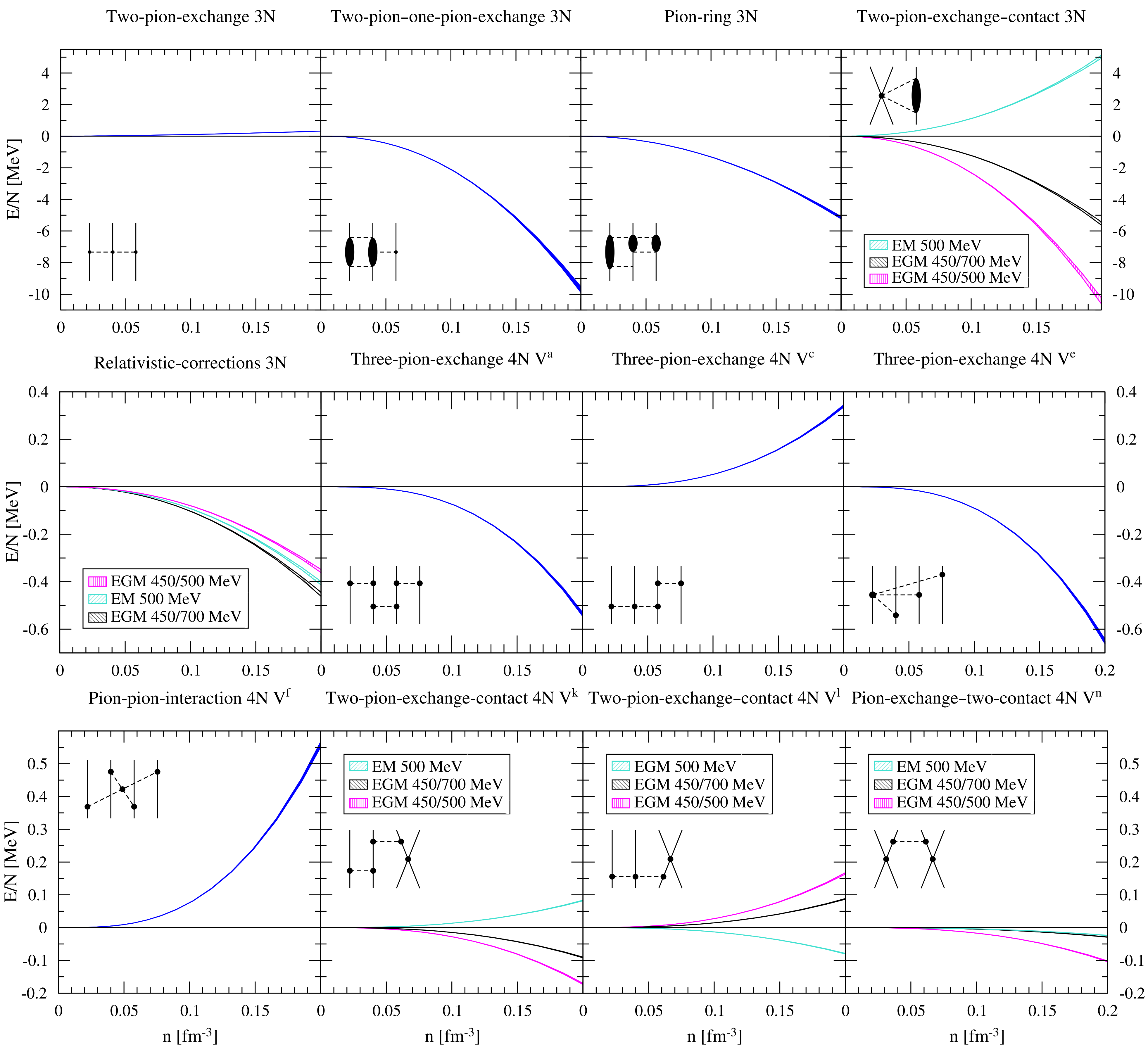



- first calculations of N$^3$LO 3NF and 4NF contributions to EOS of neutron matter
- found large contributions in Hartree Fock appr., comparable to size of N$^2$LO contributions
- 4NF contributions small

Tews, Krüger, KH, Schwenk
PRL 110, 032504 (2013)



# $N^3LO$ contributions in nuclear matter (Hartree Fock)




Krüger, Tews, KH, Schwenk
PRC88, 025802 (2013)

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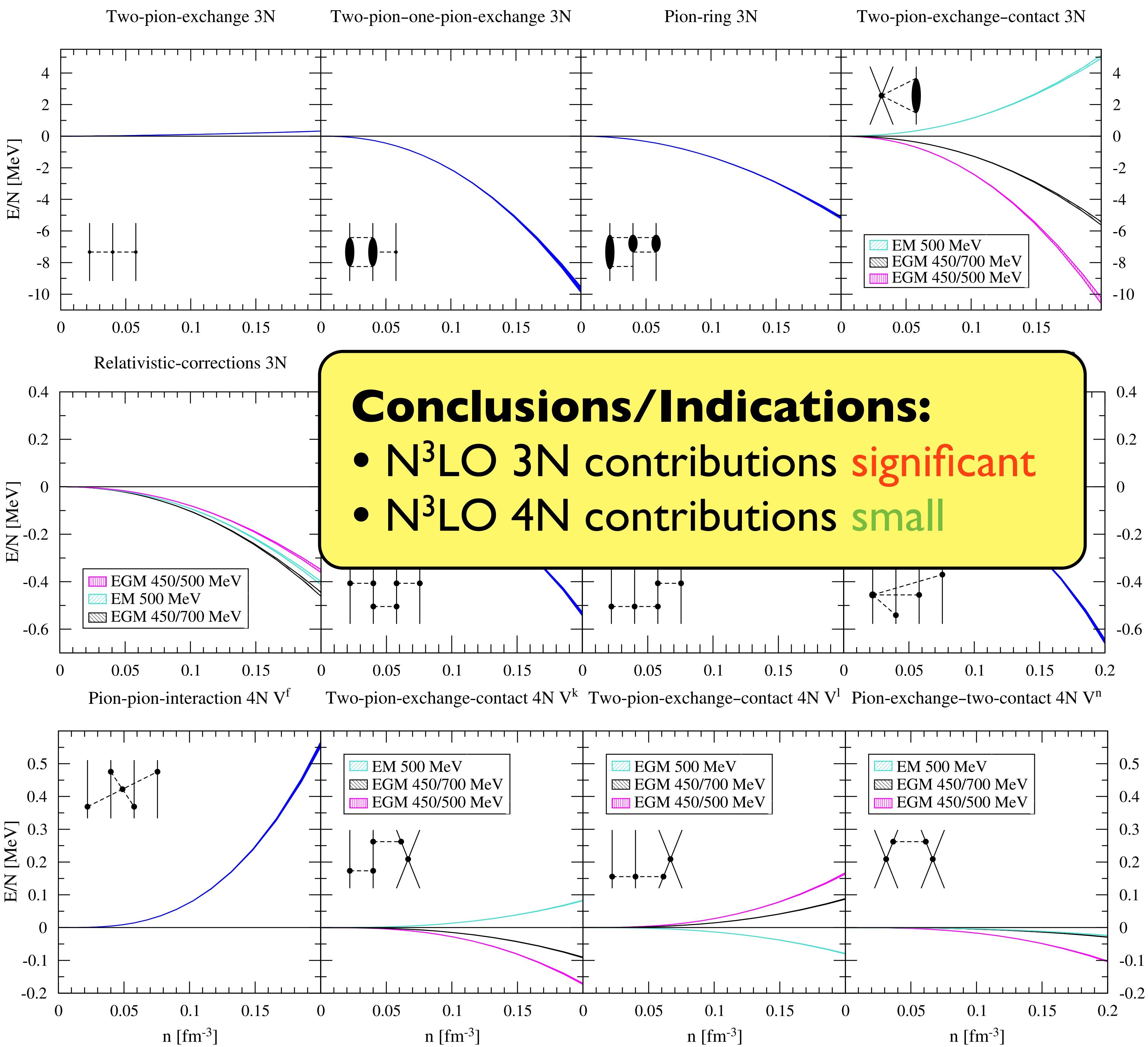



**Conclusions/Indications:**

- $N^3LO$ 3N contributions significant
- $N^3LO$ 4N contributions small

Krüger, Tews, KH, Schwenk
PRC88, 025802 (2013)

# Chiral 3N forces at subleading order ($N^3LO$)

**Goal**

Calculate matrix elements of 3NF in a momentum partial-wave decomposed form, which is suitable for all these few- and many-body frameworks.

# Chiral 3N forces at subleading order ($N^3LO$)

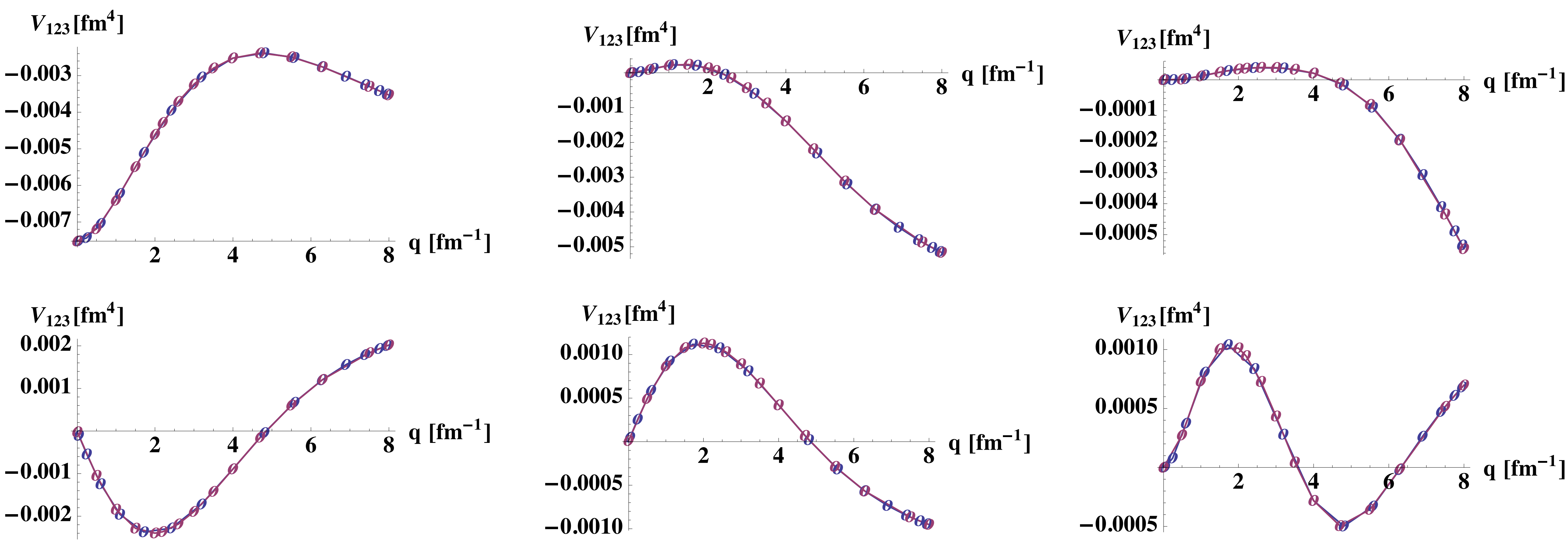
**Goal**

Calculate matrix elements of 3NF in a momentum partial-wave decomposed form, which is suitable for all these few- and many-body frameworks.

**Challenge**

Due to the large number of matrix elements, the traditional way of computing matrix elements requires extreme amounts of computer resources.

# Chiral 3N forces at subleading order ($N^3LO$)




**Strategy**

Development of a general framework, which allows to decompose efficiently arbitrary local 3N interactions.

- **perfect agreement** with results based on traditional approach
- **speedup** factors of >1000
- **very general**, can also be applied to pion-full EFT, $N^4LO$ terms, currents...

# Incorporation in different many-body frameworks

## Hyperspherical harmonics

Bacca (TRIUMF), Barnea (Hebrew U.)


## Faddeev, Faddeev-Yakubovski

Nogga (Juelich), Witala (Kracow)

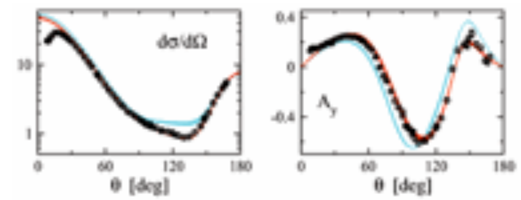



FIG. 4: Nd elastic observables at 65 MeV.

## no-core shell model

Roth (TU Darmstadt),
Navratil (TRIUMF), Vary (Iowa)

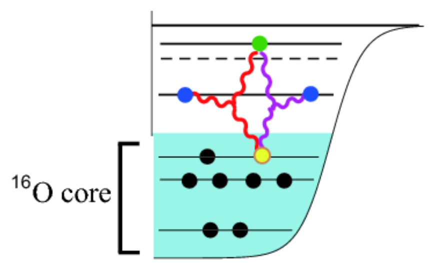
## coupled cluster method

Binder, Hagen, Papenbrock
(Oak Ridge)

$$|\Psi\rangle = e^{\hat{T}}|\Phi_0\rangle = \left(1 + \hat{T} + \frac{1}{2}\hat{T}^2 + \frac{1}{3!}\hat{T}^3 + \cdots\right)|\Phi_0\rangle,$$

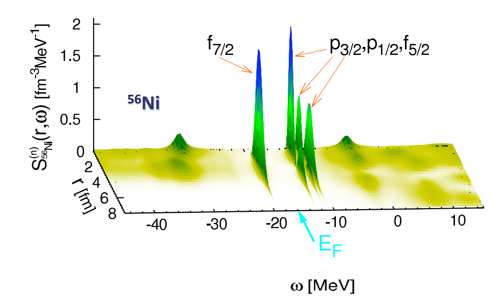
## valence shell model

Holt (TRIUMF), Menendez, Simonis,
Schwenk (TU Darmstadt)

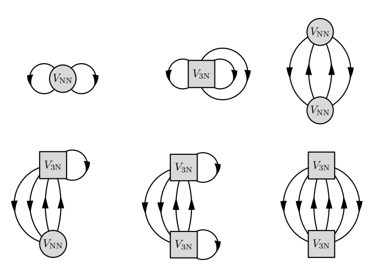



## Self-consistent Greens function

Barbieri (Surrey), Duguet, Soma (CEA)

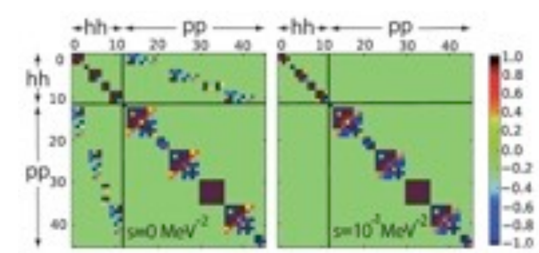



## Many-body perturbation theory



## In-medium SRG

Bogner (MSU), Hergert (OSU),
Holt (TRIUMF)




**Required inputs:**

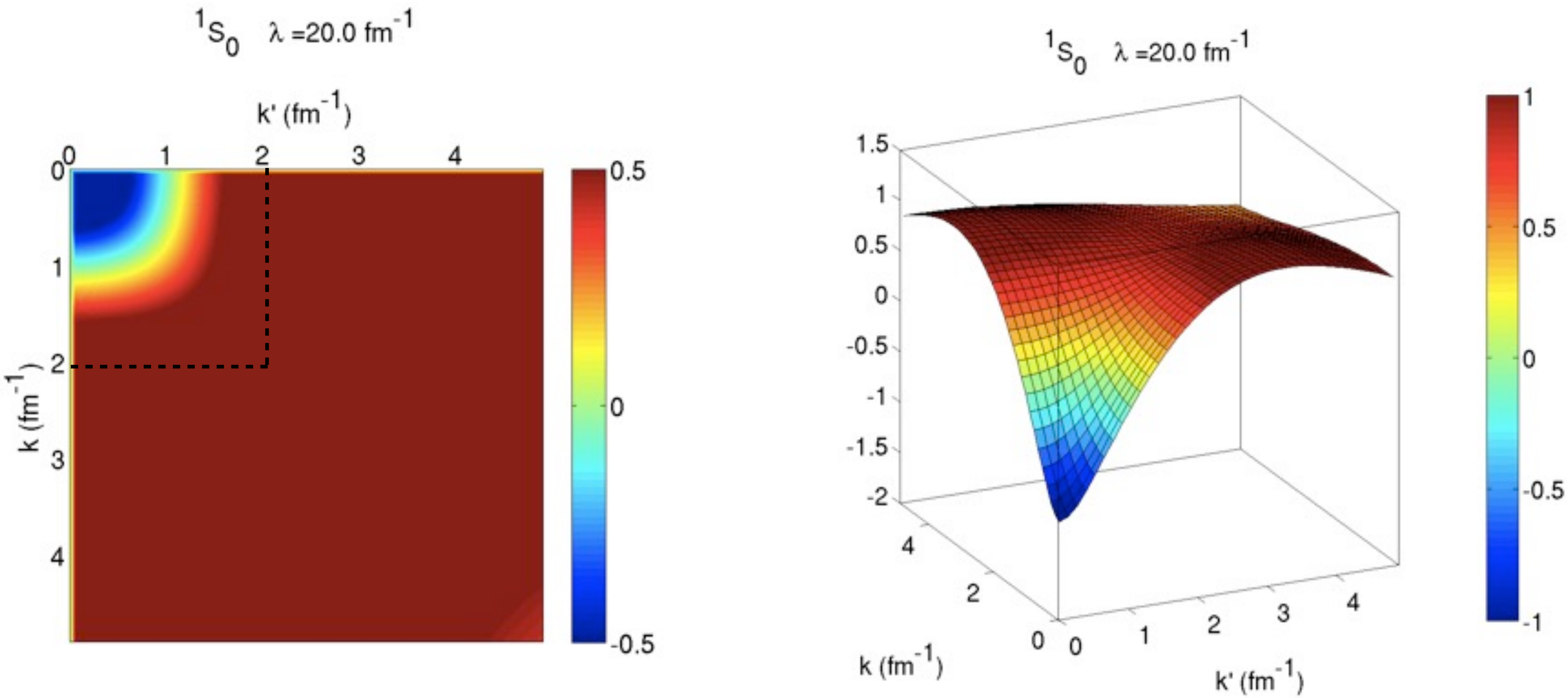
1. consistent NN and 3N forces at $N^3LO$ in partial-wave-decomposed form
2. softened forces for judging approximations and pushing to heavier nuclei

# Systematic decoupling of high-momentum physics: The Similarity Renormalization Group

- generate unitary transformation which decouples low- and high momenta

$$H_\lambda = U_\lambda H U_\lambda^\dagger \quad \text{with the resolution parameter } \lambda$$

- basic idea: change resolution successively in small steps: $\frac{dH_\lambda}{d\lambda} = [\eta_\lambda, H_\lambda]$
- generator $\eta_\lambda$ can be chosen and tailored to different applications
- observables are preserved due to unitarity of transformation



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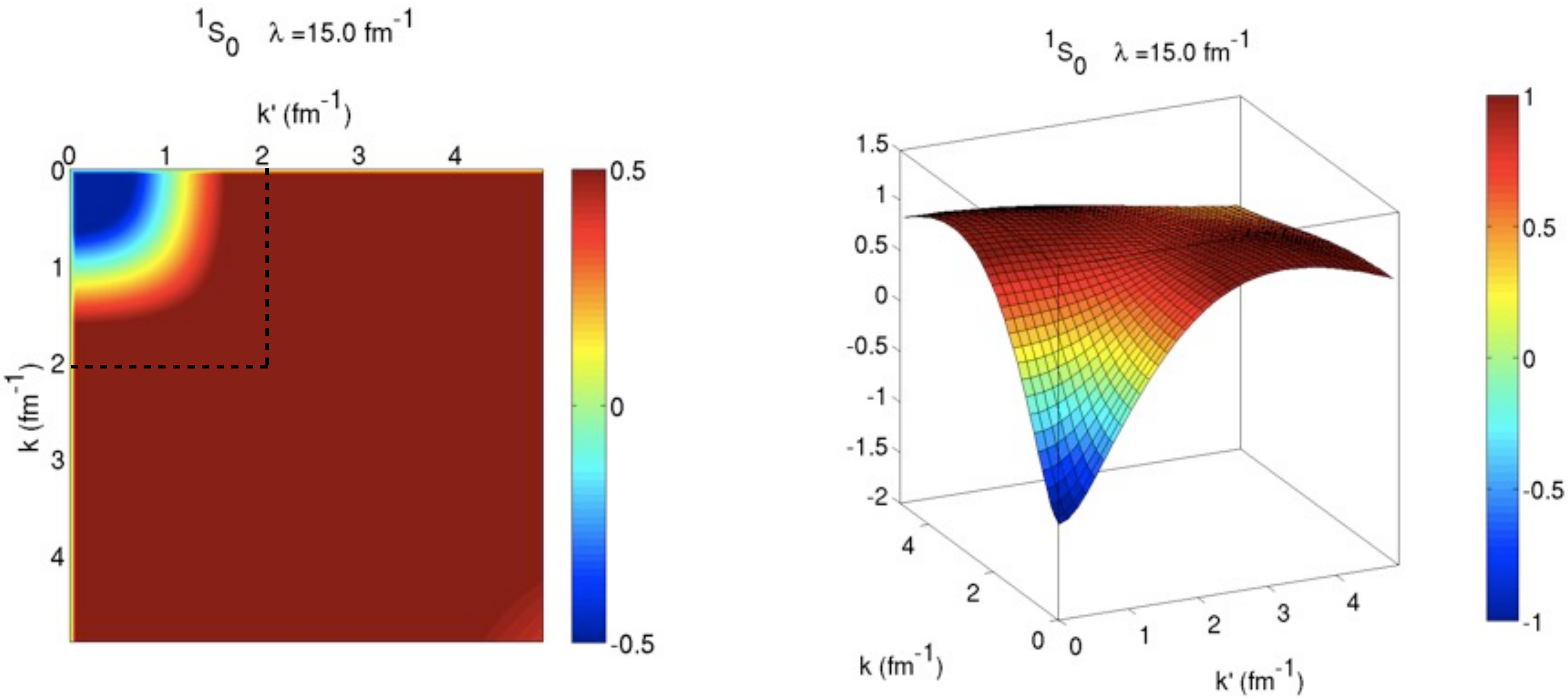

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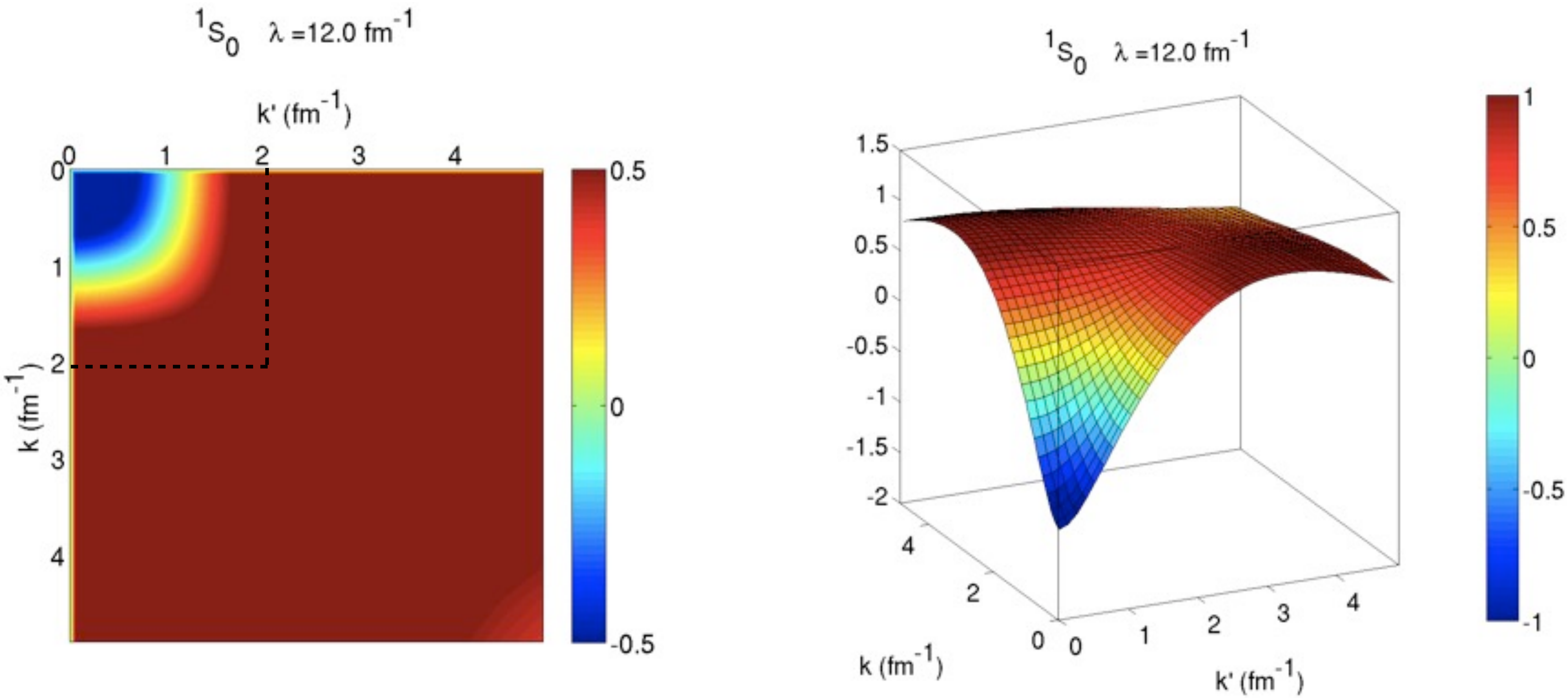

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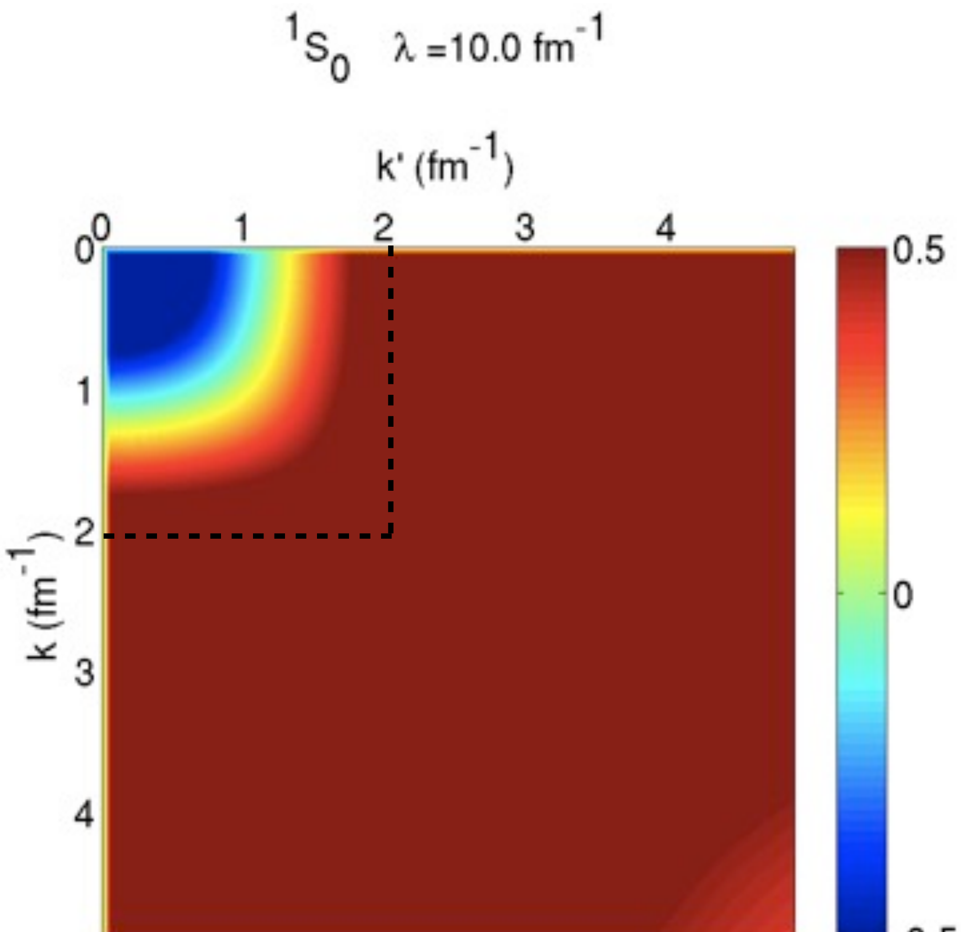


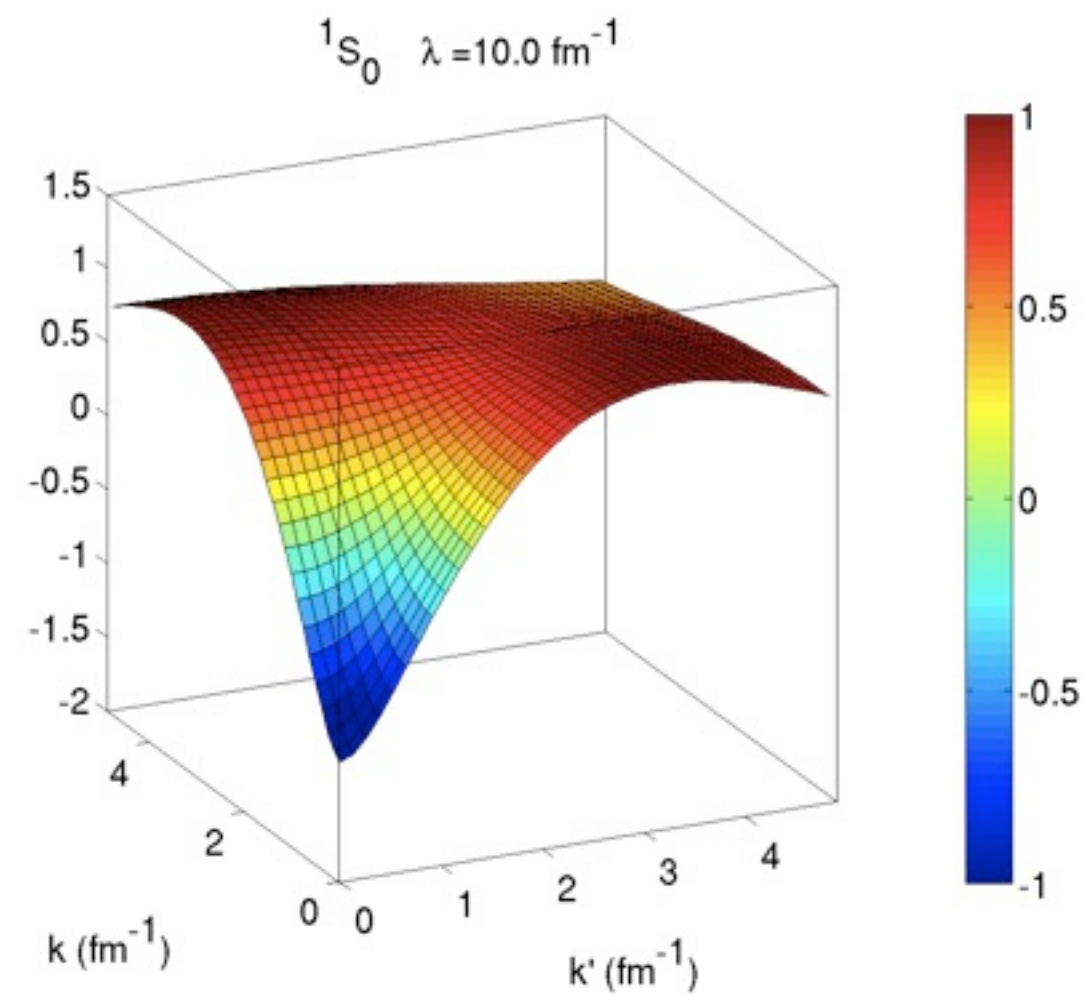

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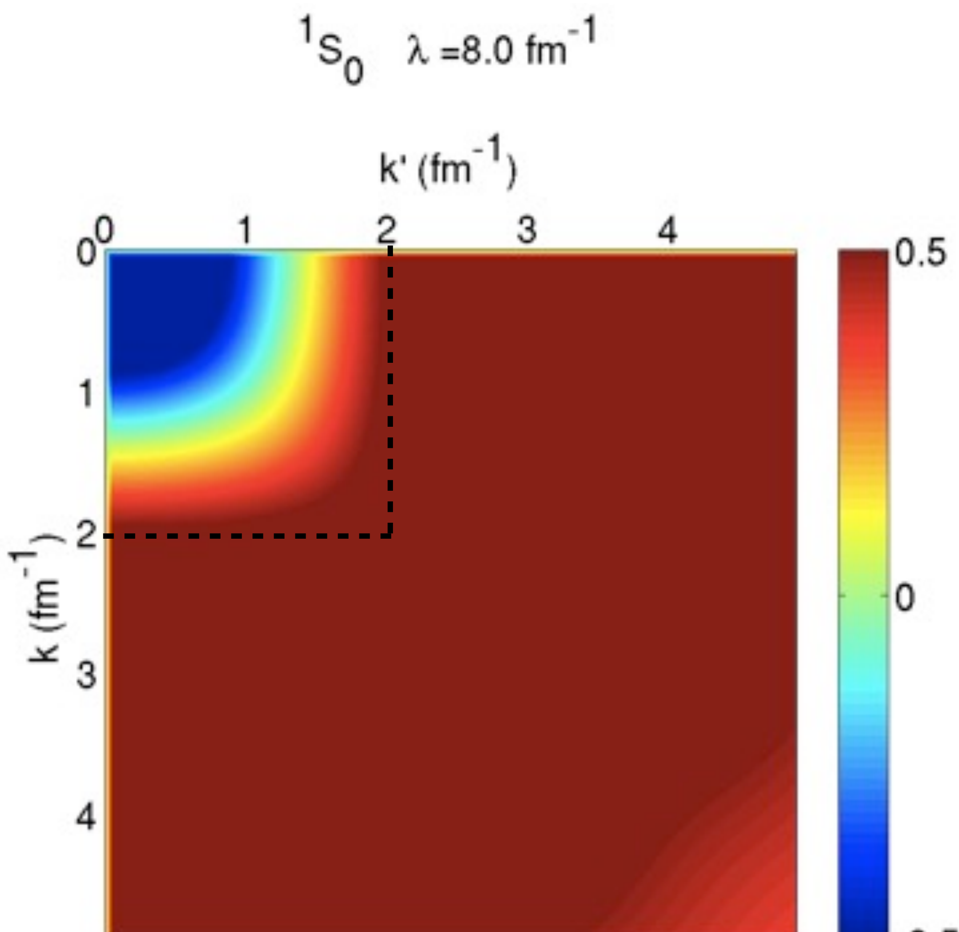


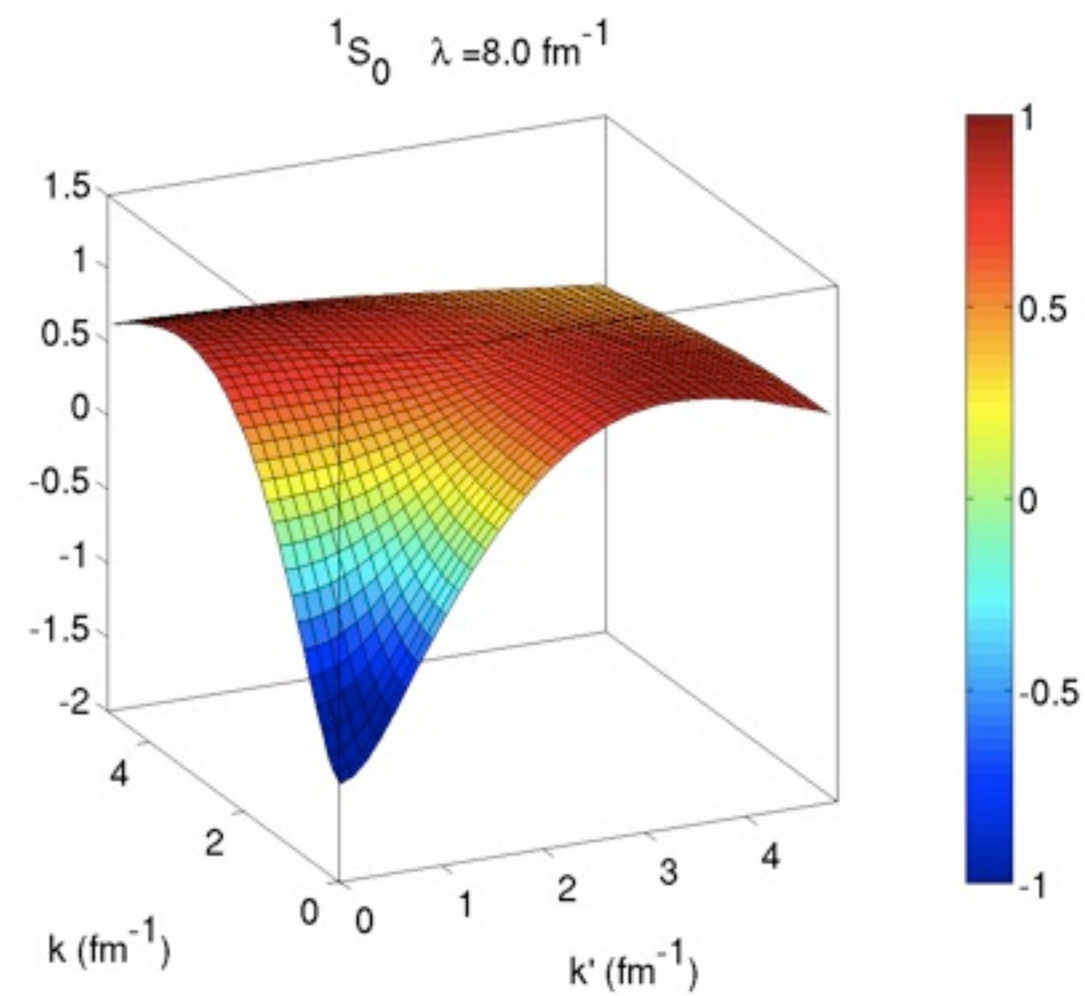

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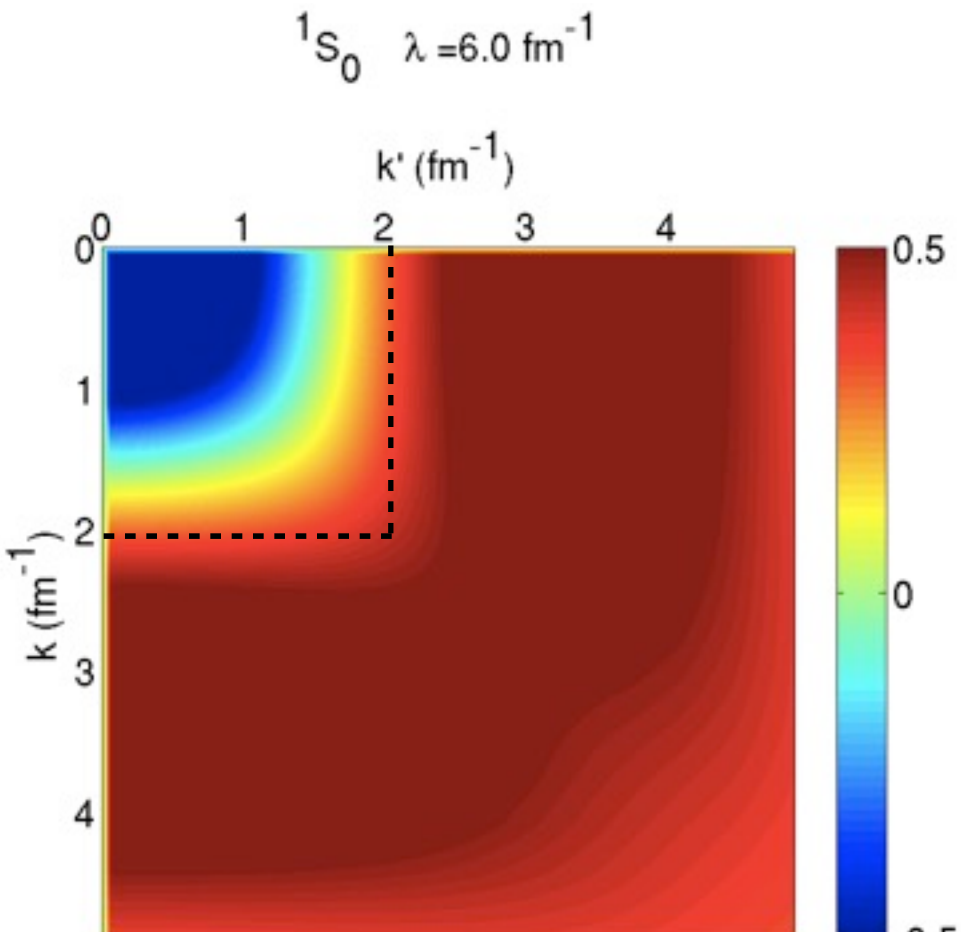


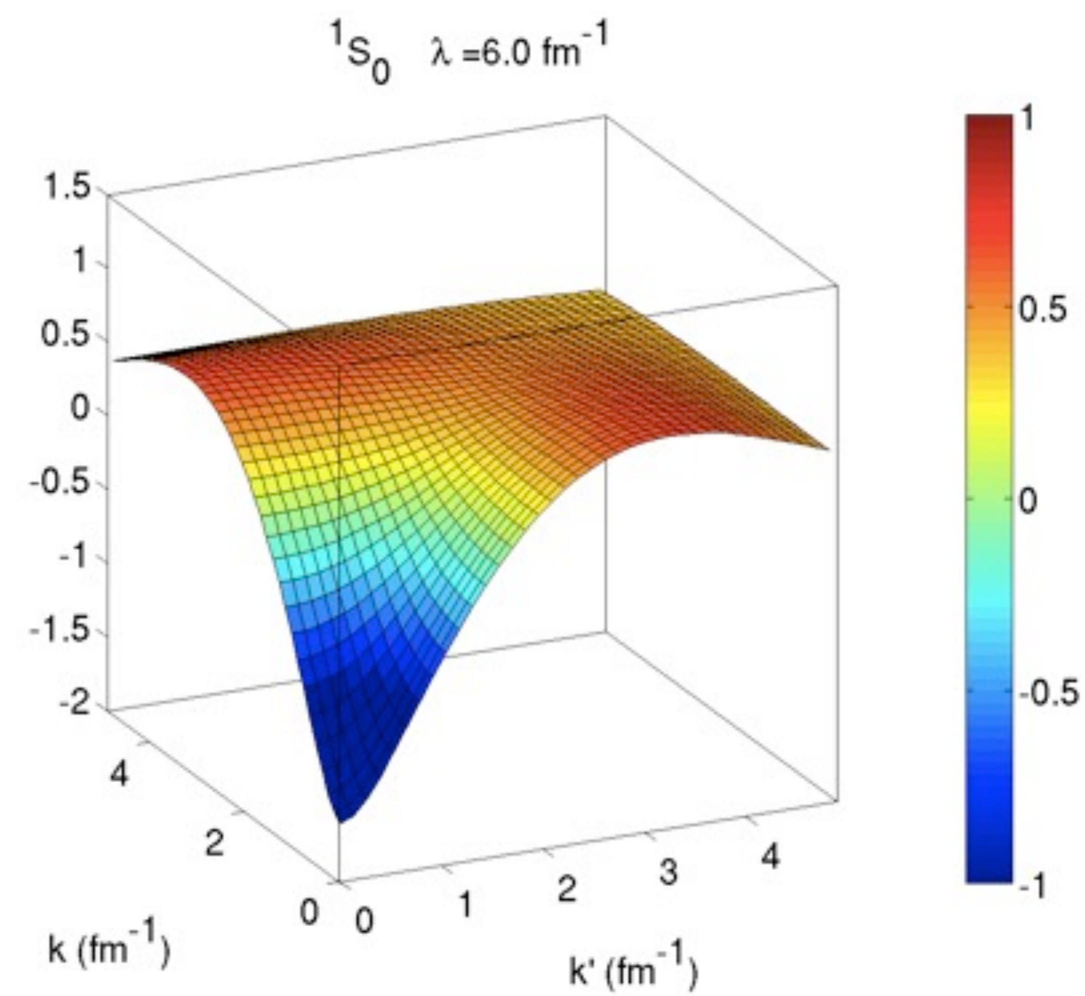

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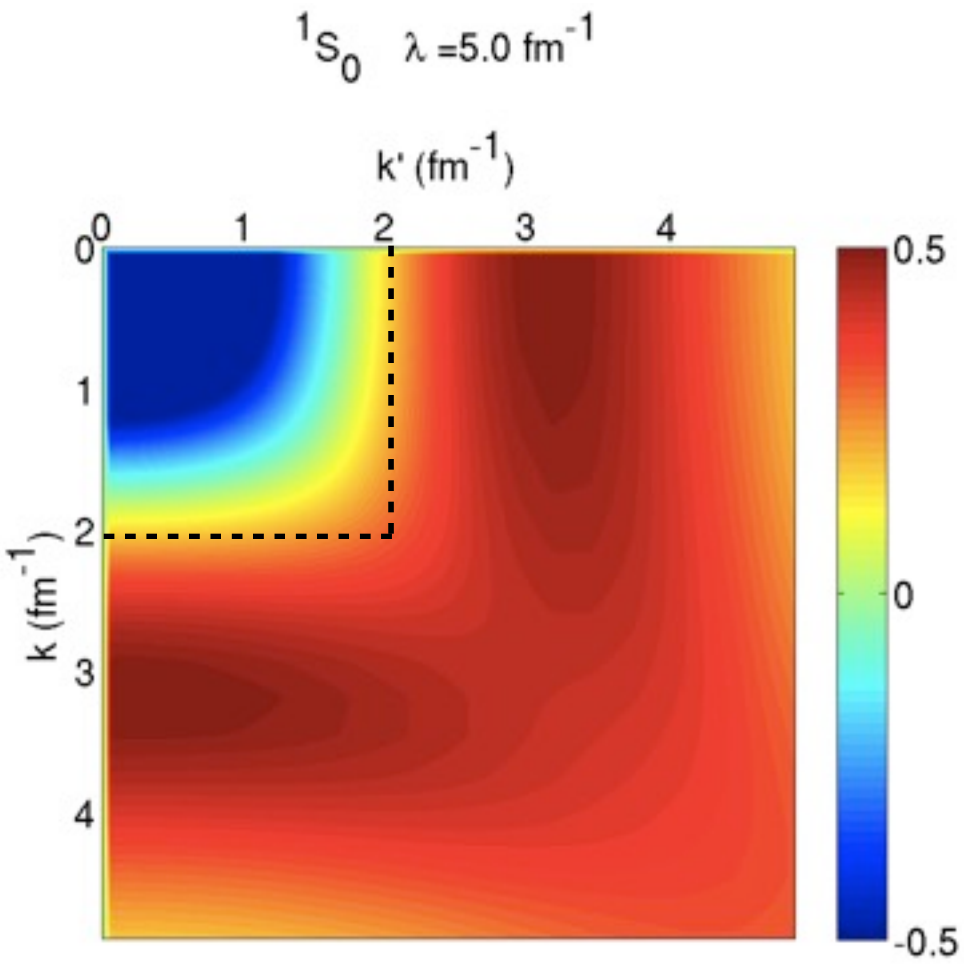


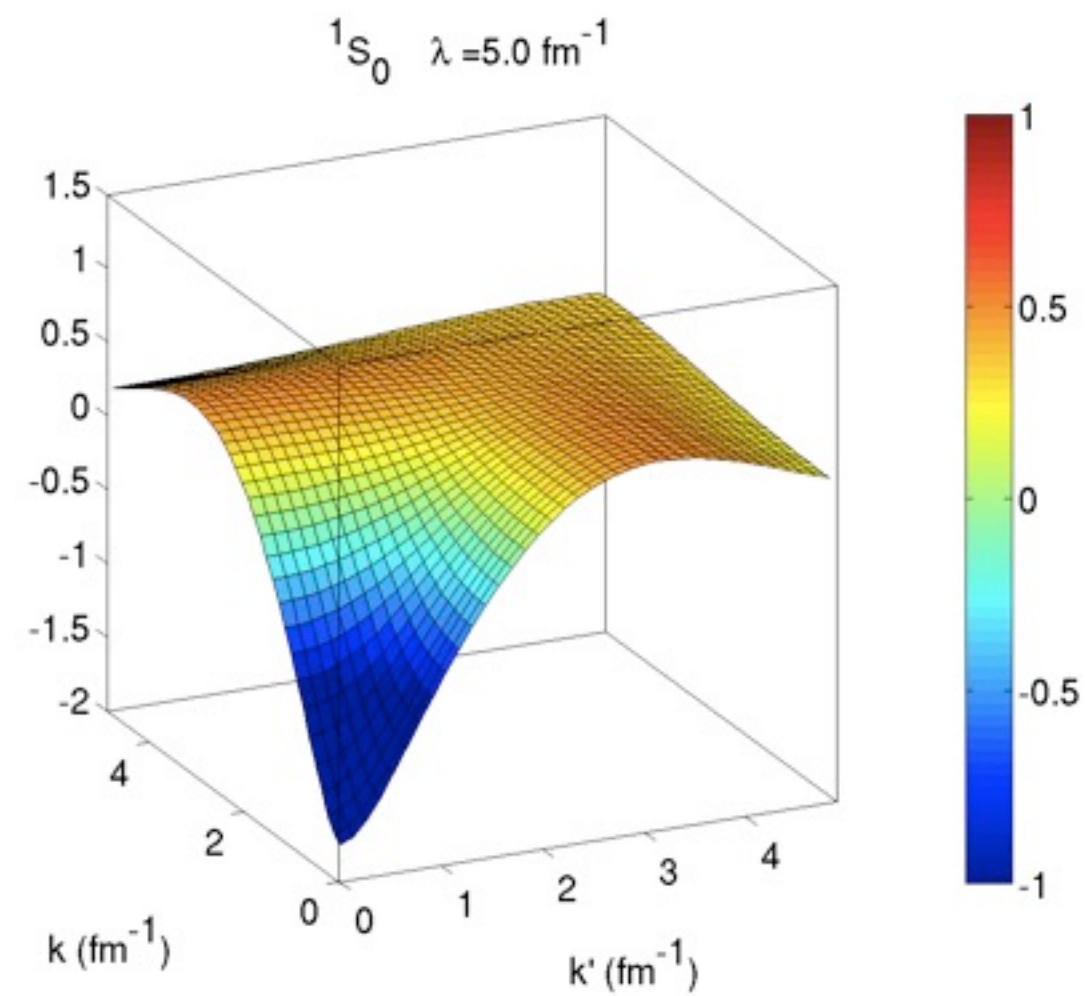

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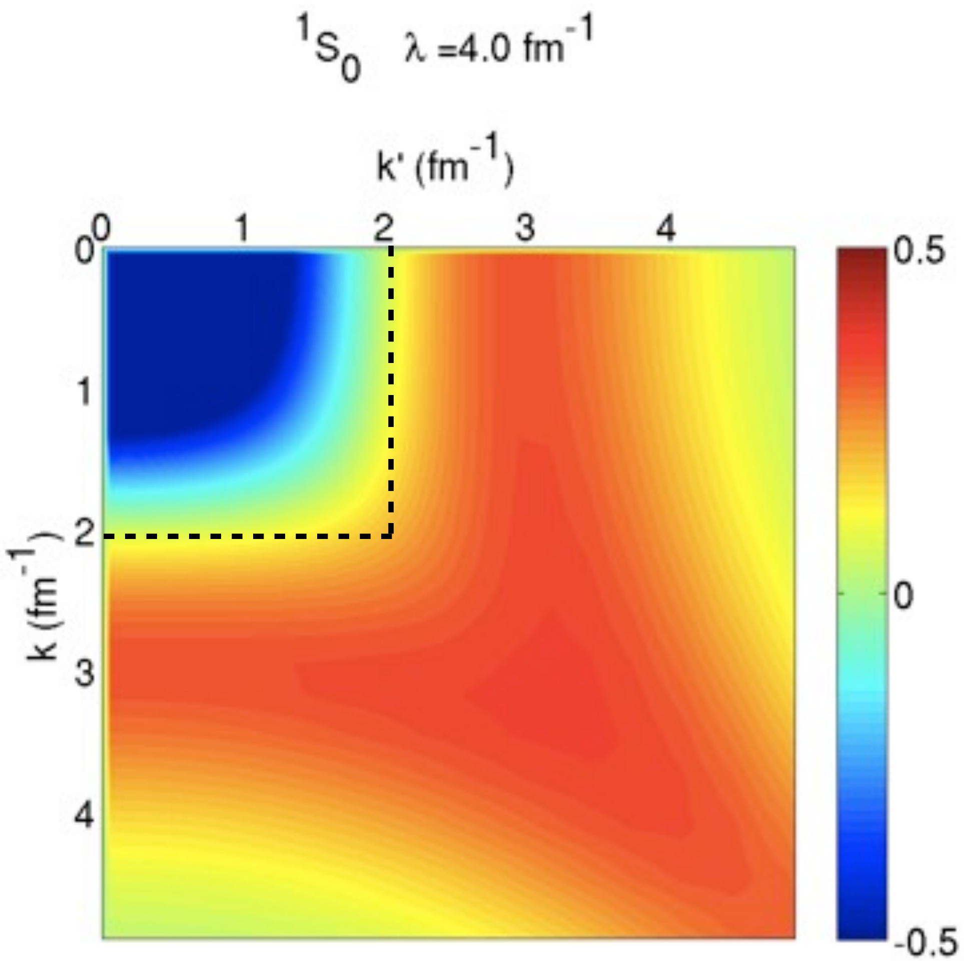


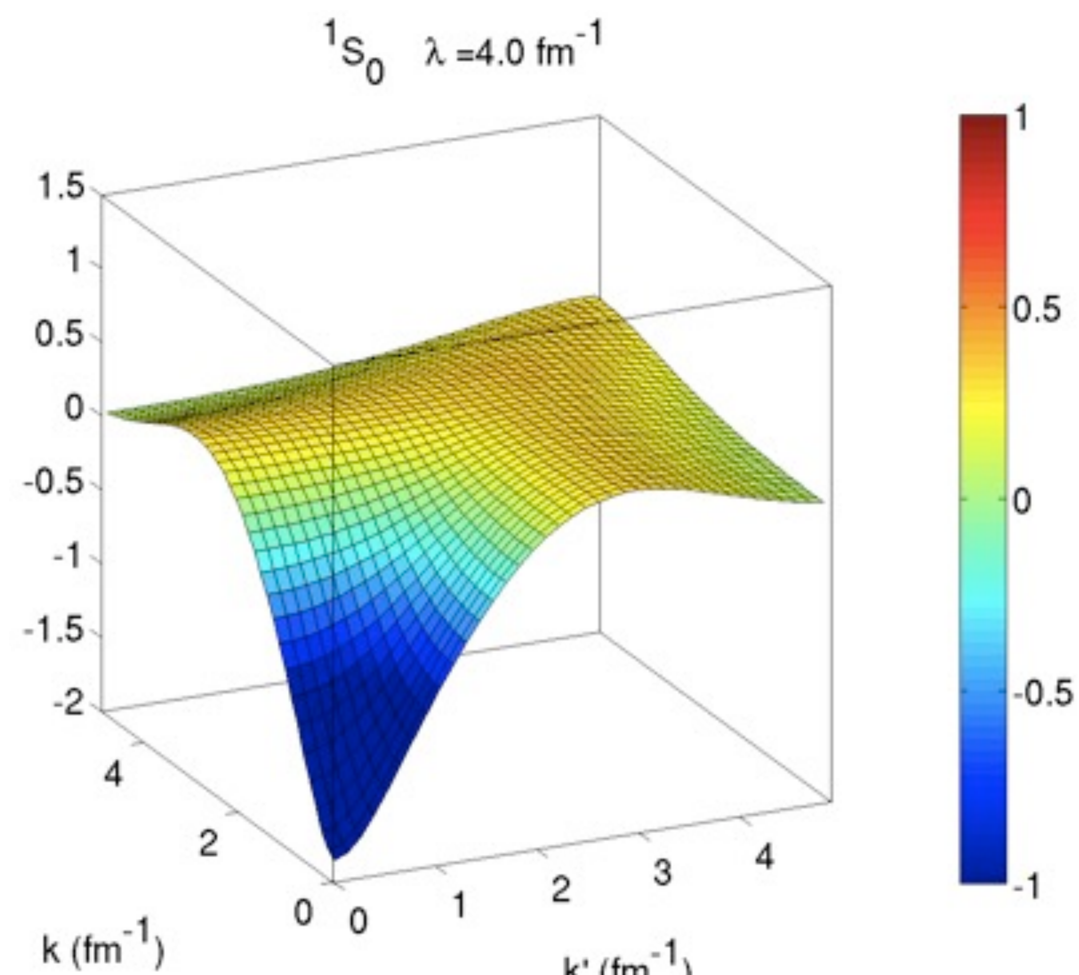

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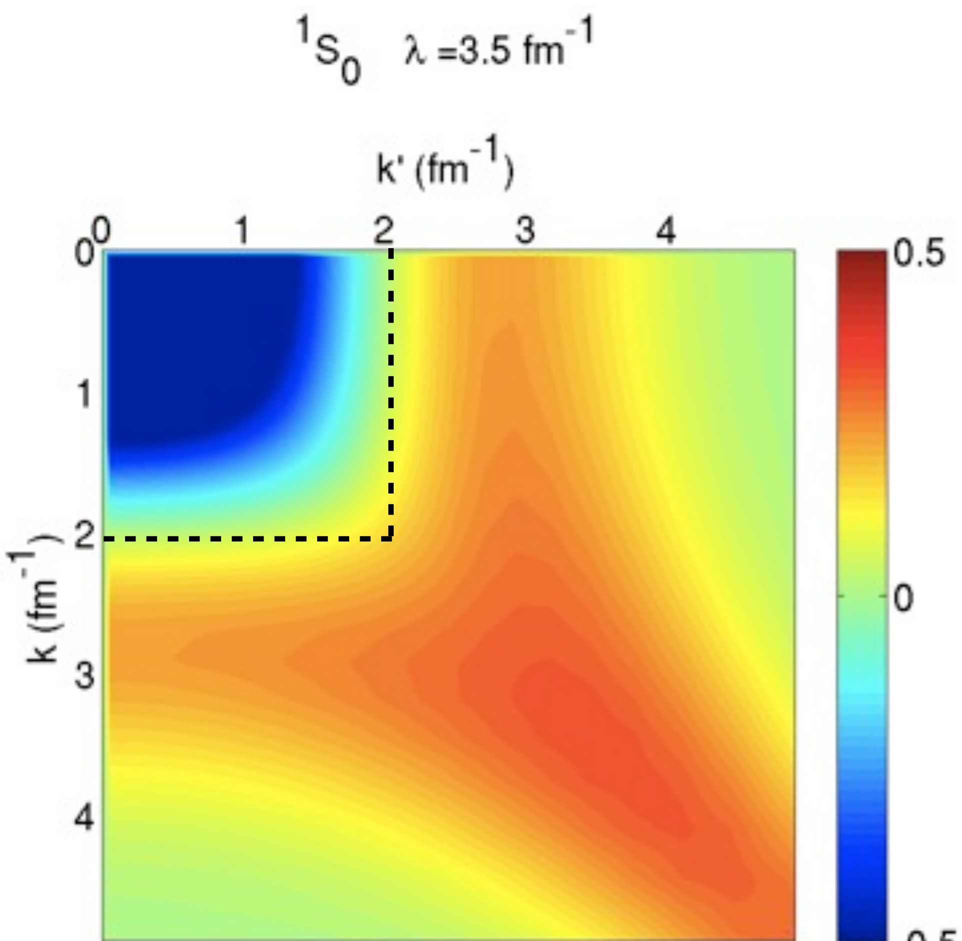


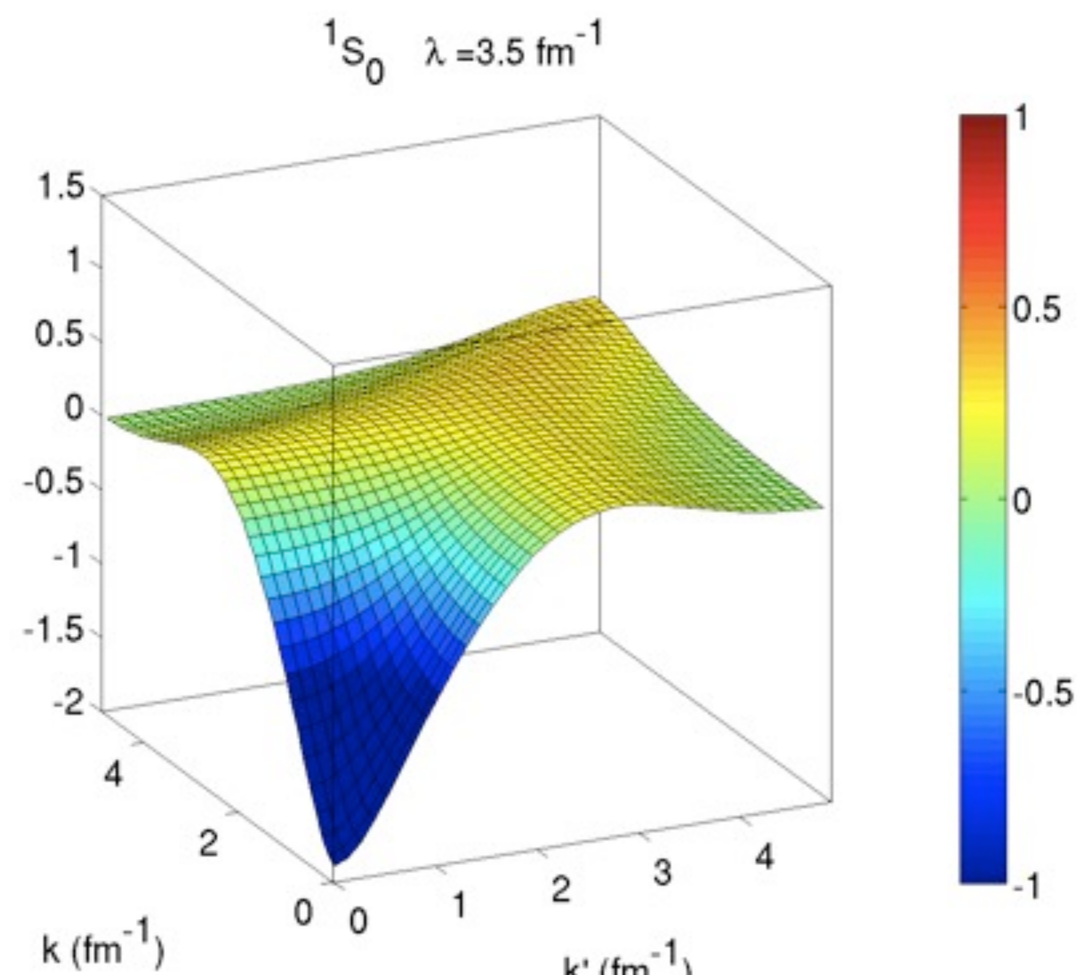

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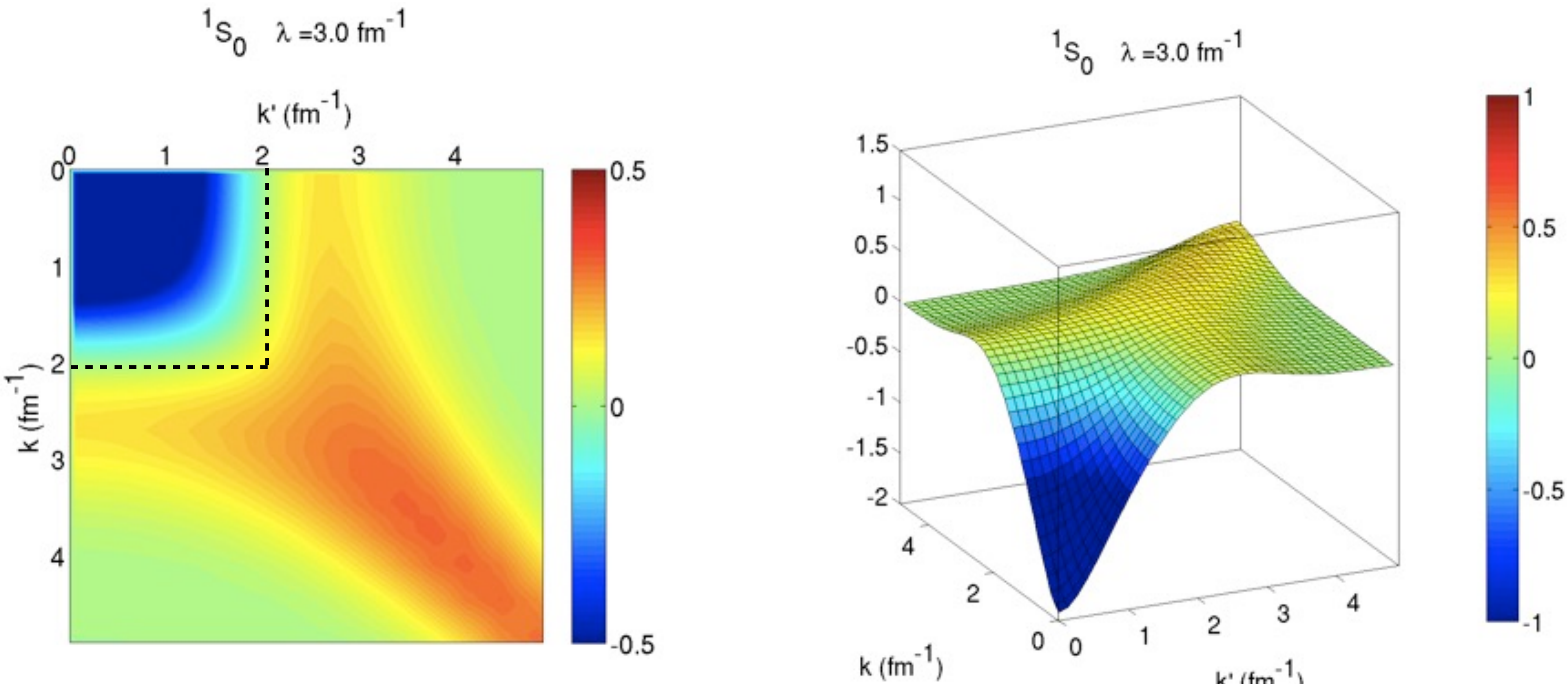

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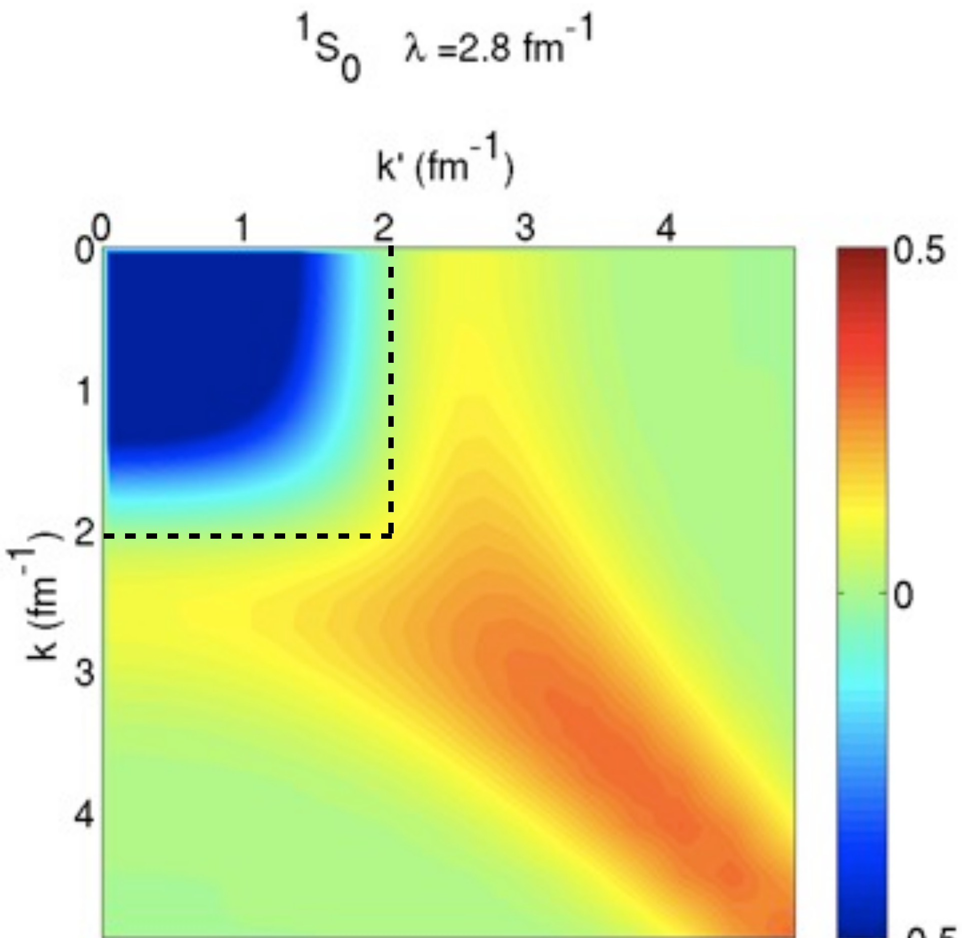


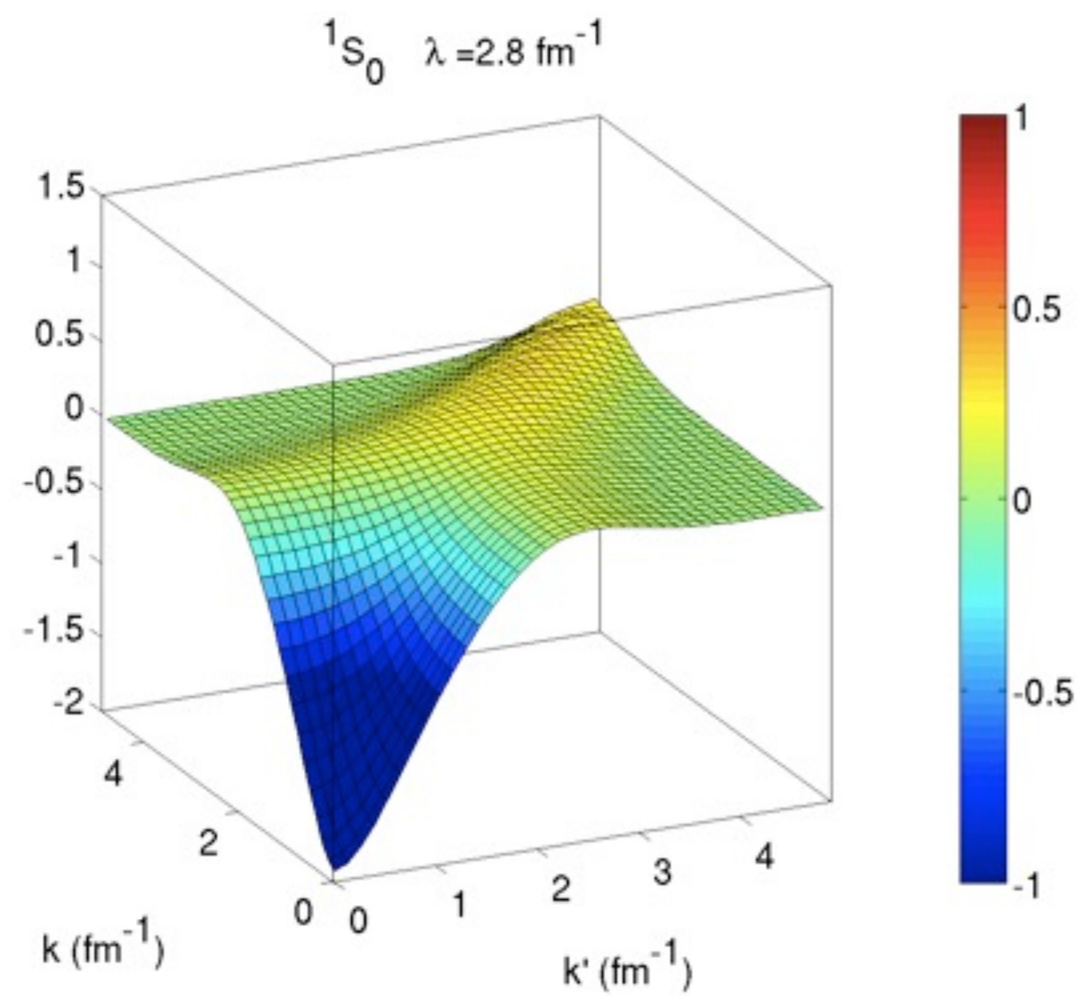

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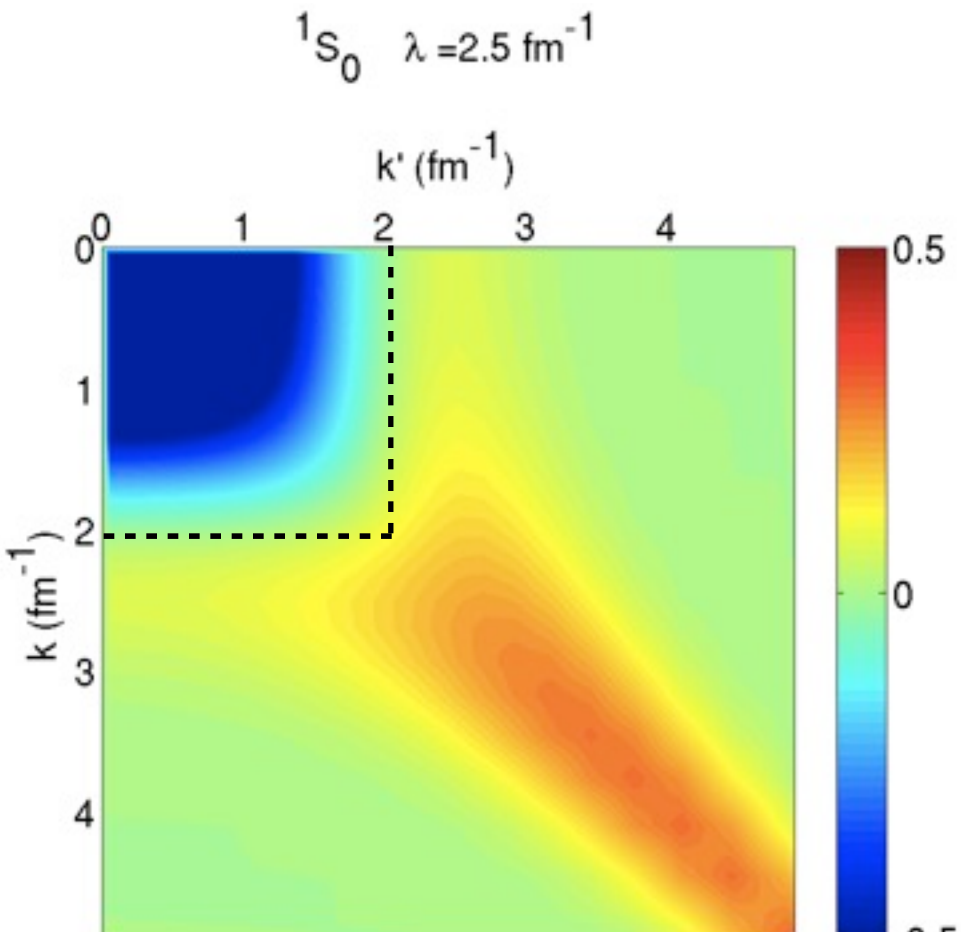


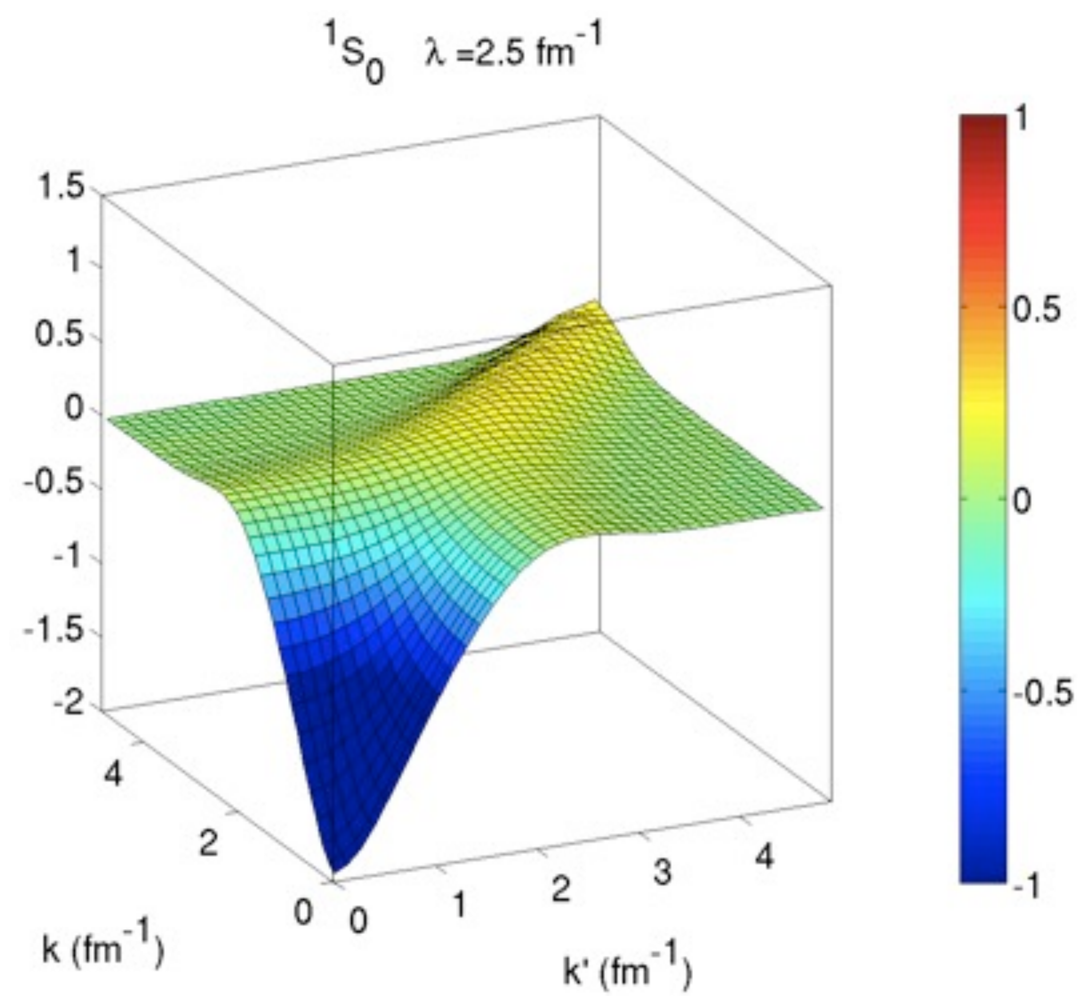

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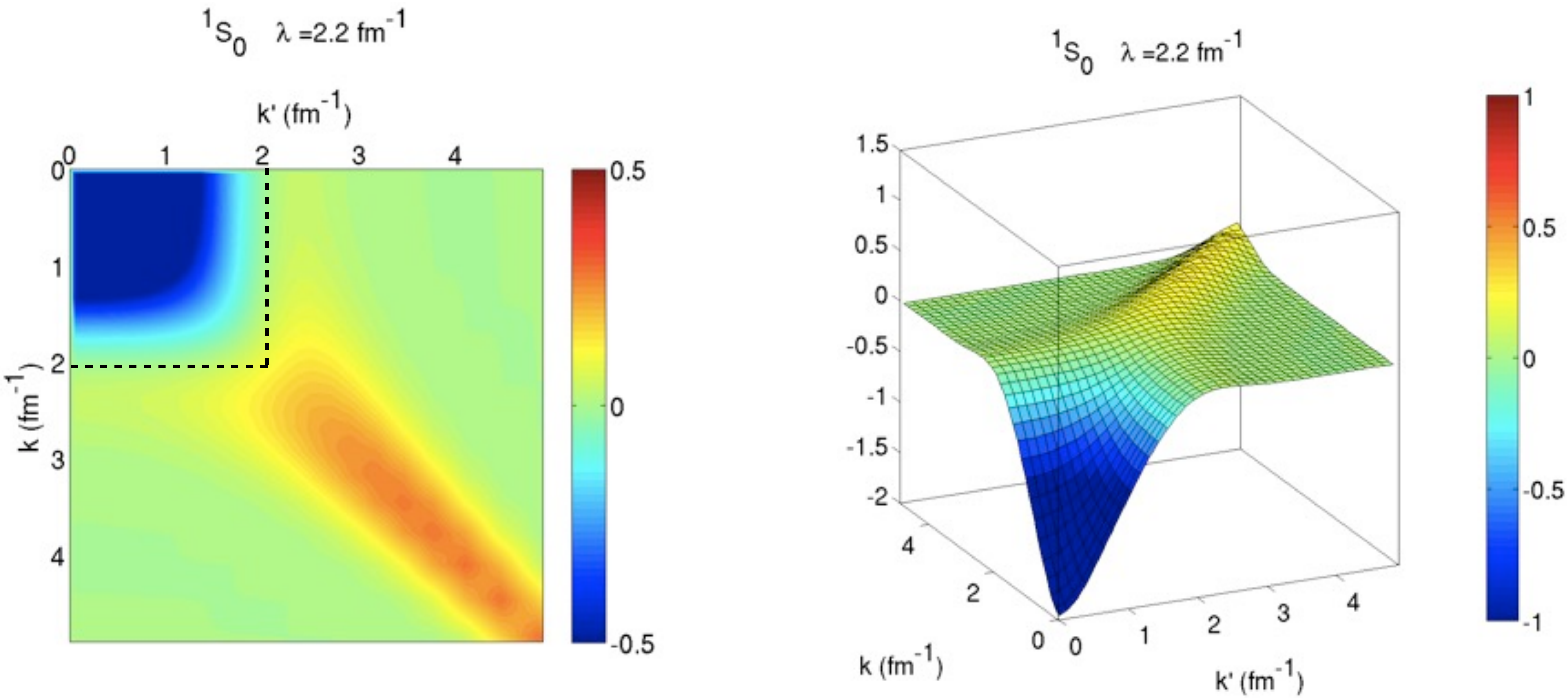

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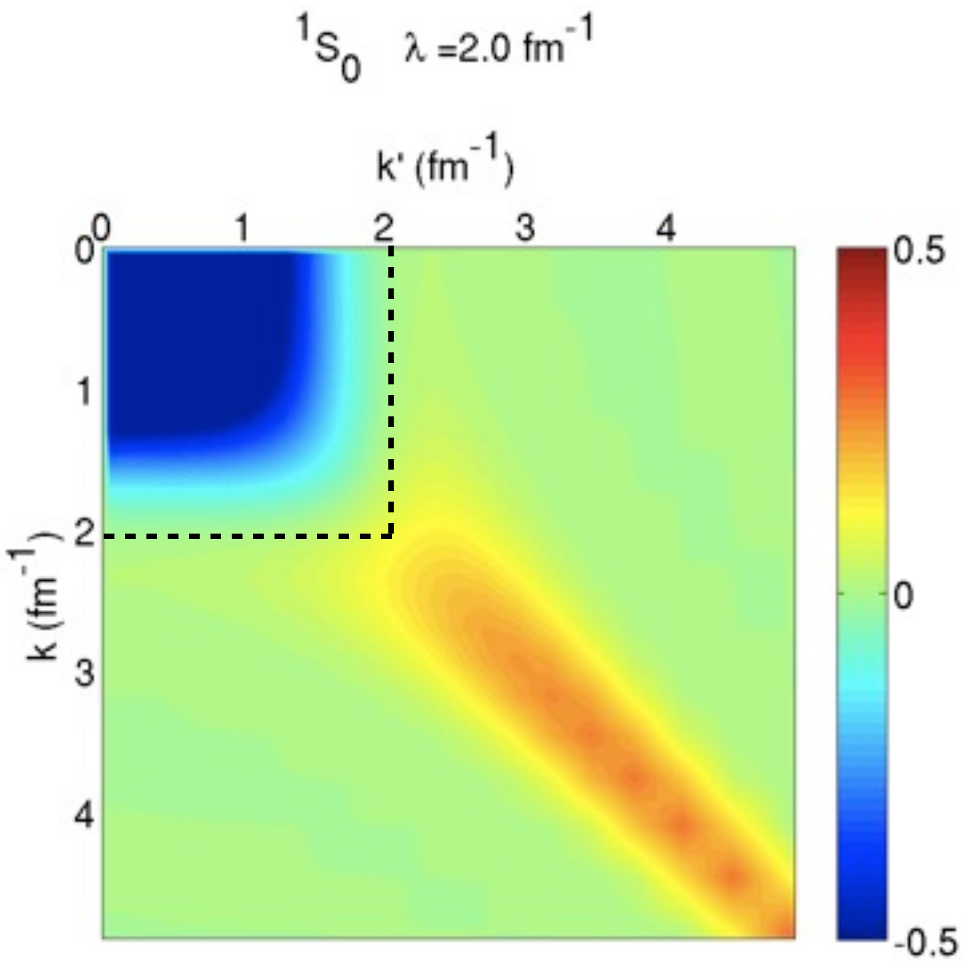
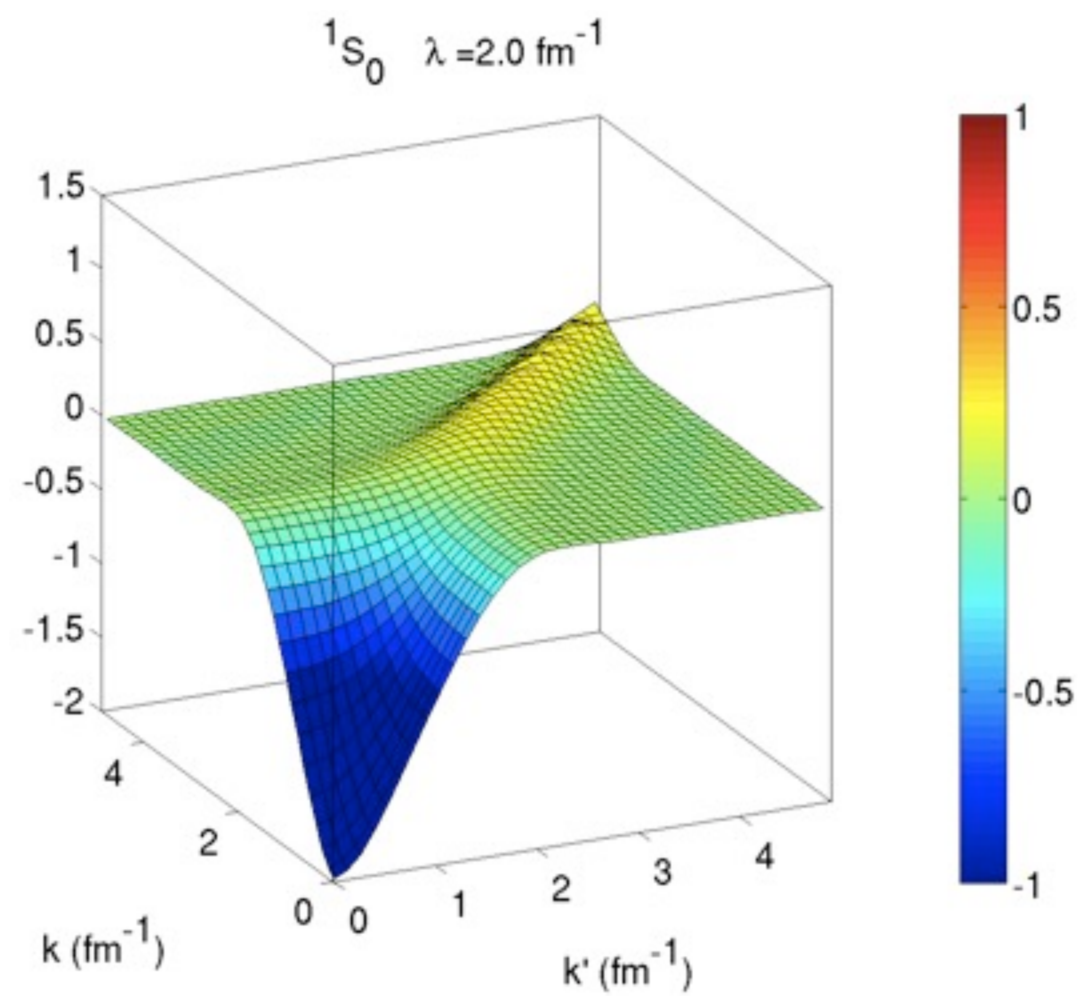
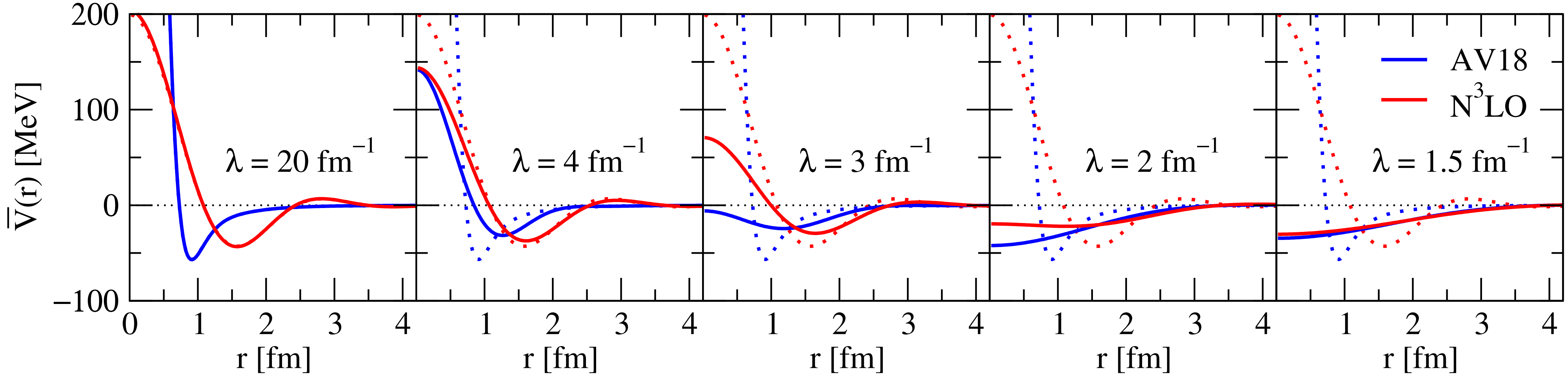
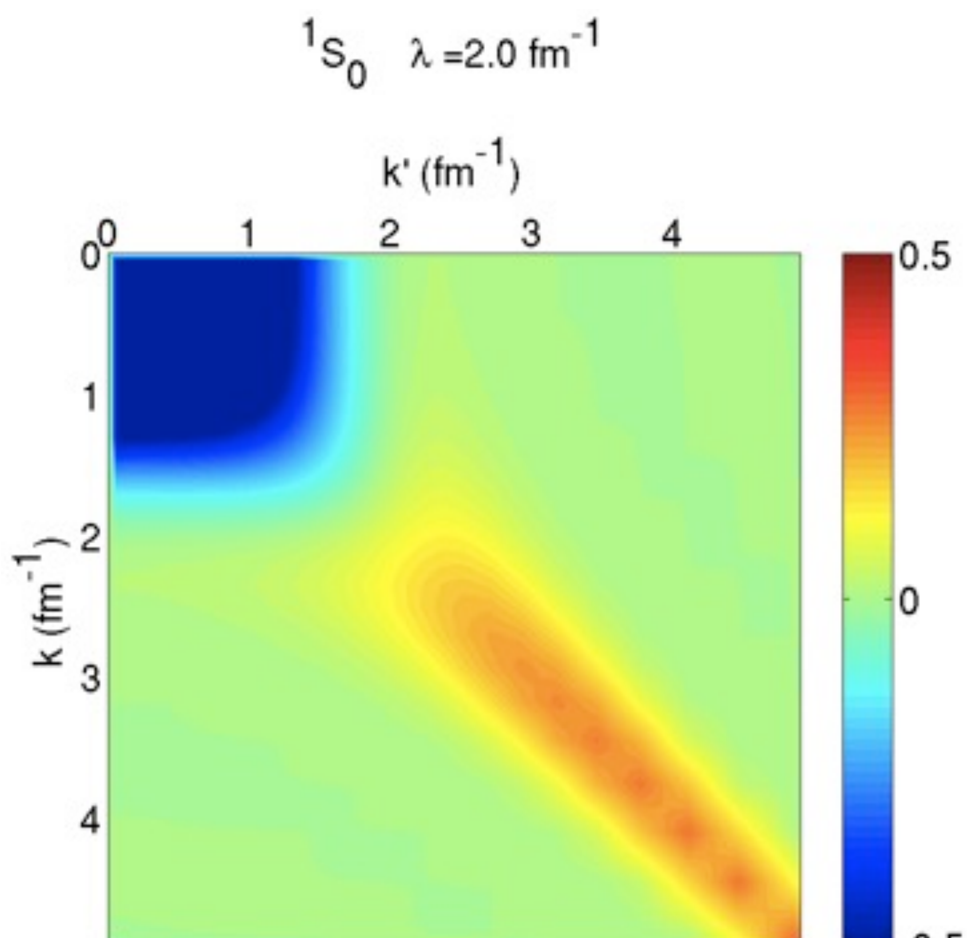


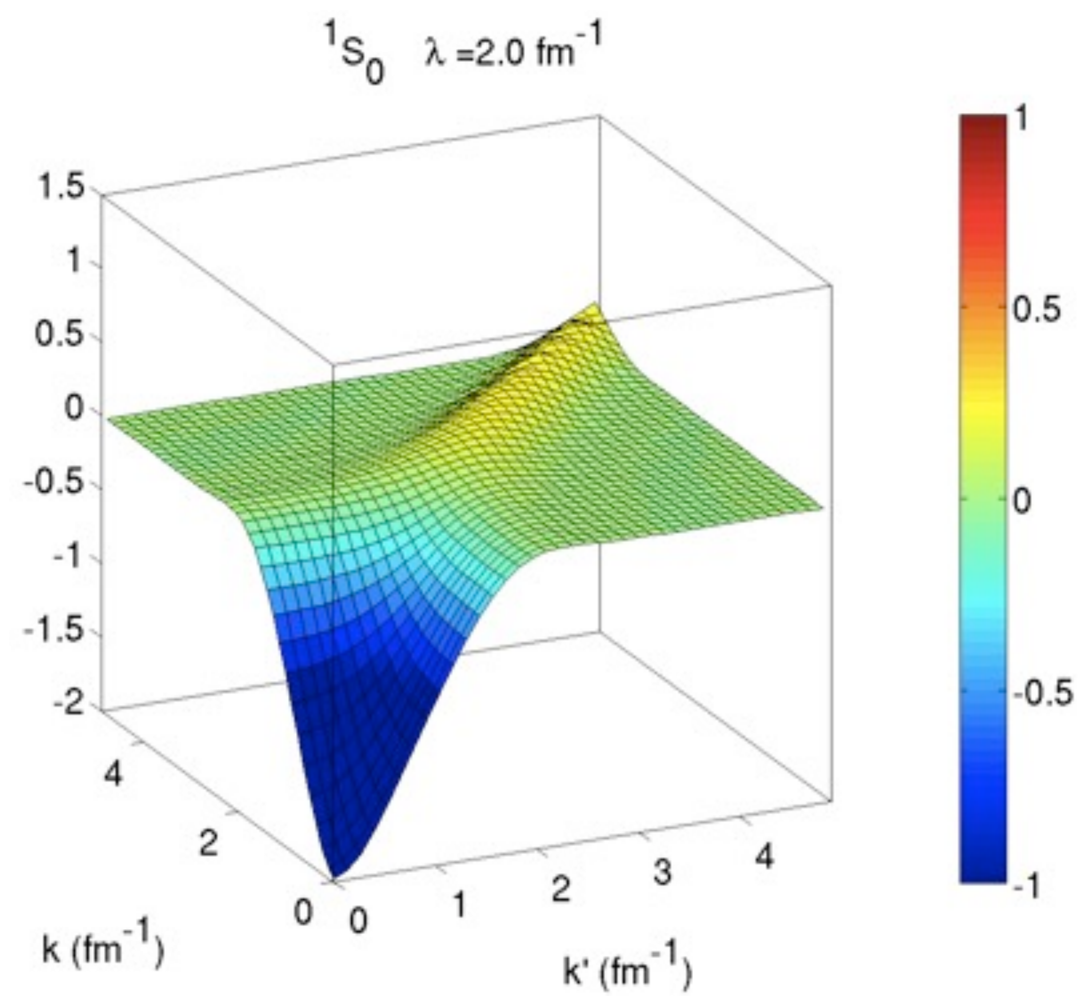



# Systematic decoupling of high-momentum physics: The Similarity Renormalization Group

$$\overline{V}_\lambda(r) = \int dr' r'^2 V_\lambda(r,r')$$



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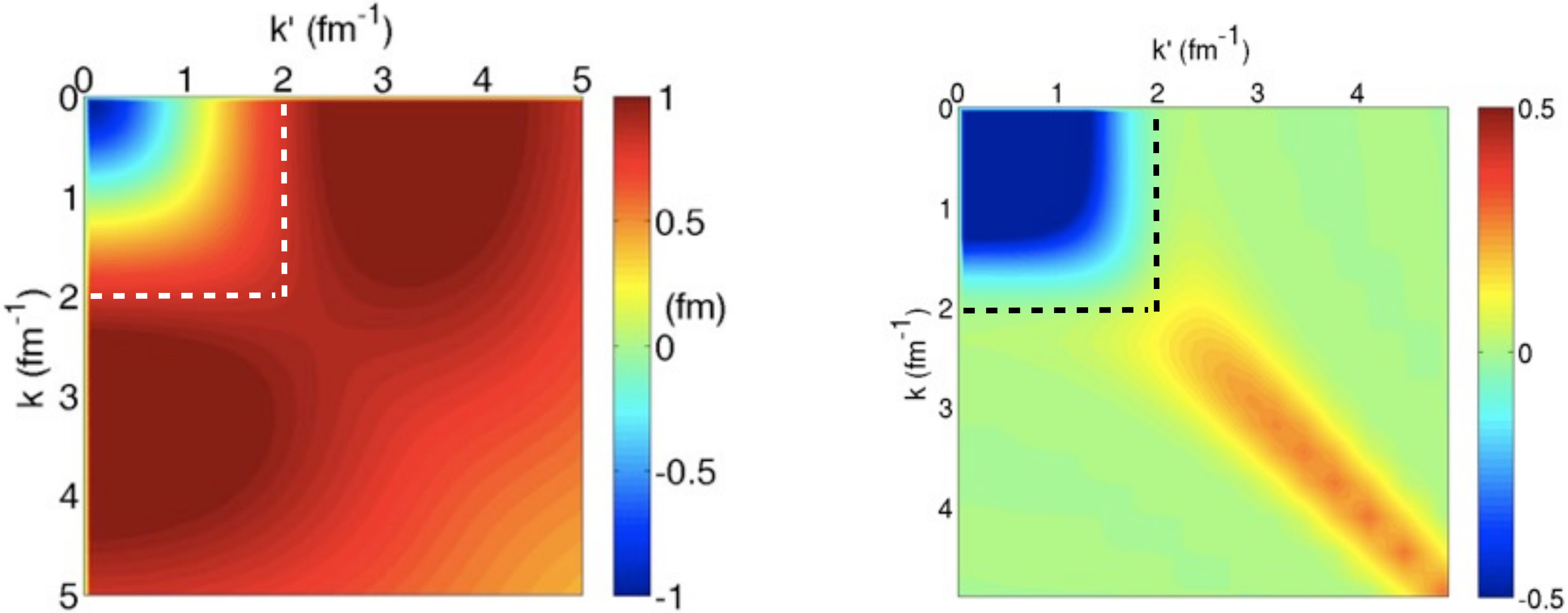



- elimination of coupling between low- and high momentum components, ⟶ simplified many-body calculations
- observables unaffected by resolution change (for exact calculations)
- residual resolution dependences can be used as tool to test calculations

Not the full story:
RG transformation also changes three-body (and higher-body) interactions.

# Applications of chiral 3N forces at $N^3LO$

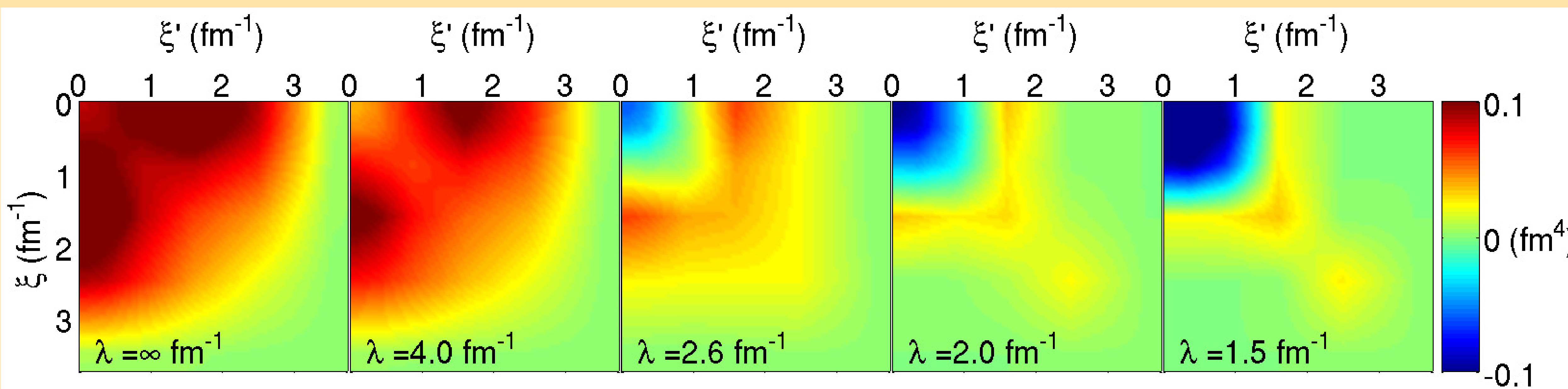
**Problem:**

Basis size for converged results of ab initio calculations including 3N forces grows rapidly with the number of particles.
Calculations limited to light nuclei.

**Strategy:**

Use SRG transformations to **decouple** low- and high momentum states.
Required basis size decreases drastically.

First implementation of consistent
SRG evolution of 3NF in a momentum basis:

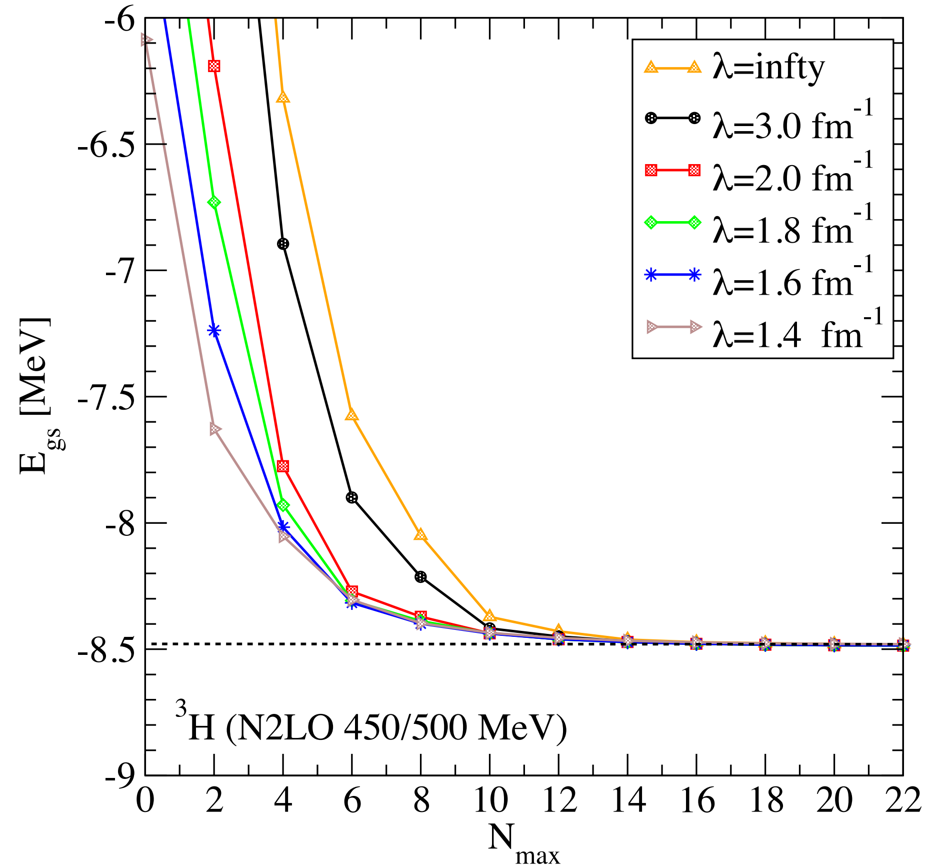



Hebeler PRC(R) 85, 021002 (2012)

# Applications of chiral 3N forces at $N^3LO$

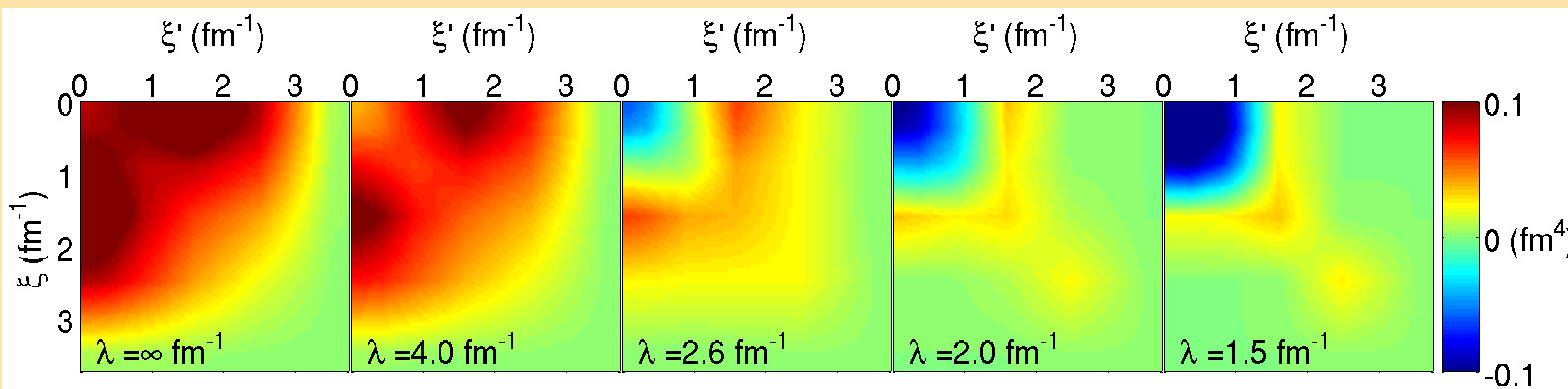
Basis size for conve… …tions including
3N forces gro… …particles.
Calc…

Transformation to HO basis:



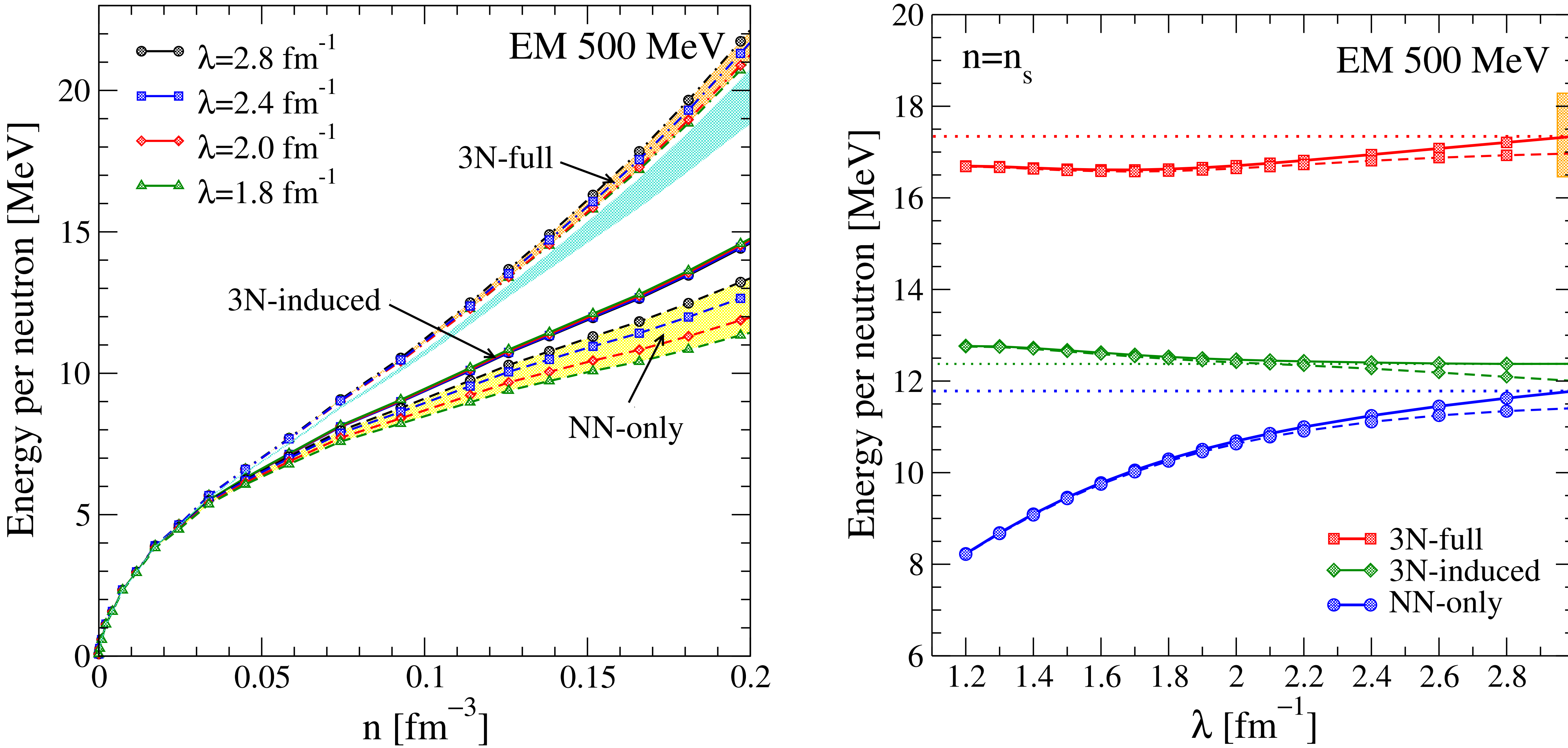

Use SRG transformati… …gh momentum states.
Requ… …cally.

Fir… …nt
SRG ev… …n basis:




Hebeler PRC(R) 85, 021002 (2012)

# First results for neutron matter equation of state based on consistently evolved 3N ($N^2LO$) forces

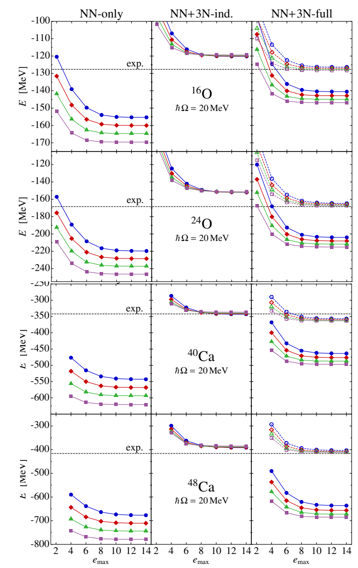



KH and Furnstahl, PRC 87, 031302(R) (2013)

- 3NF contributions treated in Hartree-Fock approximation
- no indications for unnaturally large 4N force contributions

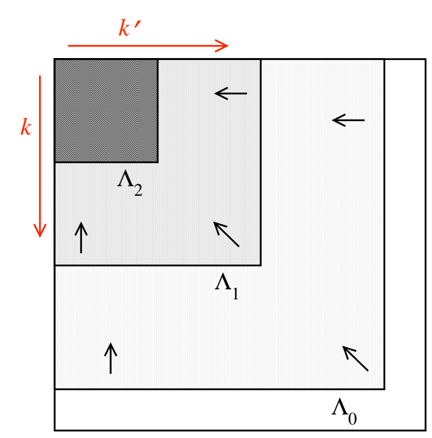
# 3NF evolution in momentum basis: Current developments and applications

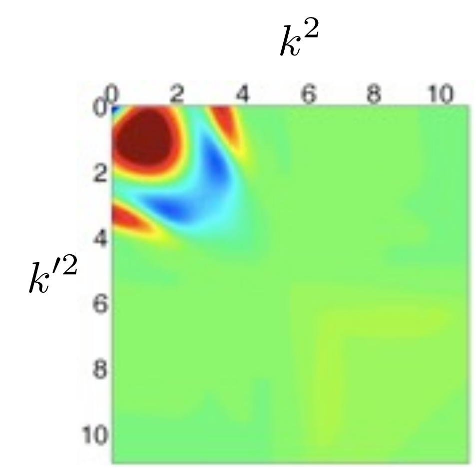
- application to infinite systems
  - equation of state (first applications to neutron matter)
  - systematic study of induced many-body contributions




- transformation of evolved interactions to oscillator basis
  - application to nuclei, complimentary to HO evolution (already implemented and tested)

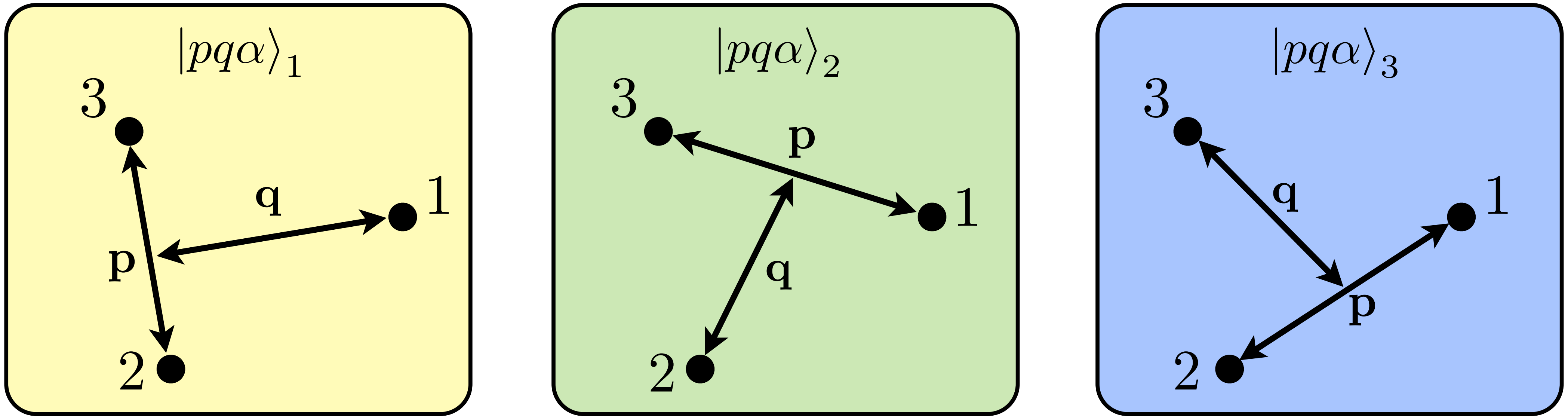
- study of various generators
  - different decoupling patterns (e.g. $V_{\text{low k}}$)
  - improved efficiency of evolution
  - suppression of many-body forces




- evolution of arbitrary operators
  - needed for all observables
  - study of correlations in nuclear systems ⟶ factorization

# RG evolution of 3N interactions in momentum space

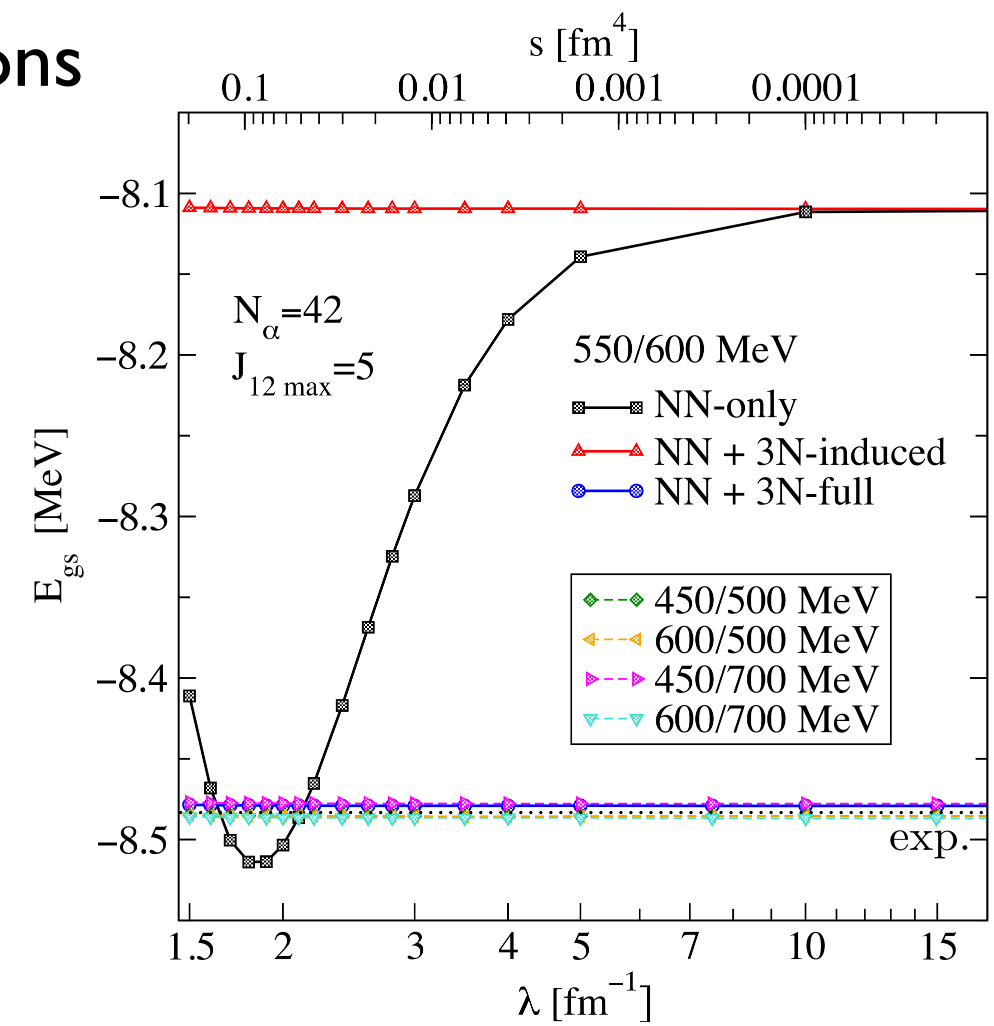



- represent interaction in basis $|pq\alpha\rangle_i \equiv |p_i q_i; [(LS)J(ls_i)j]\,\mathcal{J}\mathcal{J}_z(Tt_i)\mathcal{T}\mathcal{T}_z\rangle$
- explicit equations for NN and 3N flow equations

$$\frac{dV_{ij}}{ds} = [[T_{ij}, V_{ij}], T_{ij} + V_{ij}],$$

$$\begin{aligned}\frac{dV_{123}}{ds} &= [[T_{12}, V_{12}], V_{13} + V_{23} + V_{123}] \\ &+ [[T_{13}, V_{13}], V_{12} + V_{23} + V_{123}] \\ &+ [[T_{23}, V_{23}], V_{12} + V_{13} + V_{123}] \\ &+ [[T_{\rm rel}, V_{123}], H_s]\end{aligned}$$

Bogner, Furnstahl, Perry PRC 75, 061001(R) (2007)




Hebeler PRC(R) 85, 021002 (2012)

# Strategy: Use a lower-resolution version

low-pass filter →

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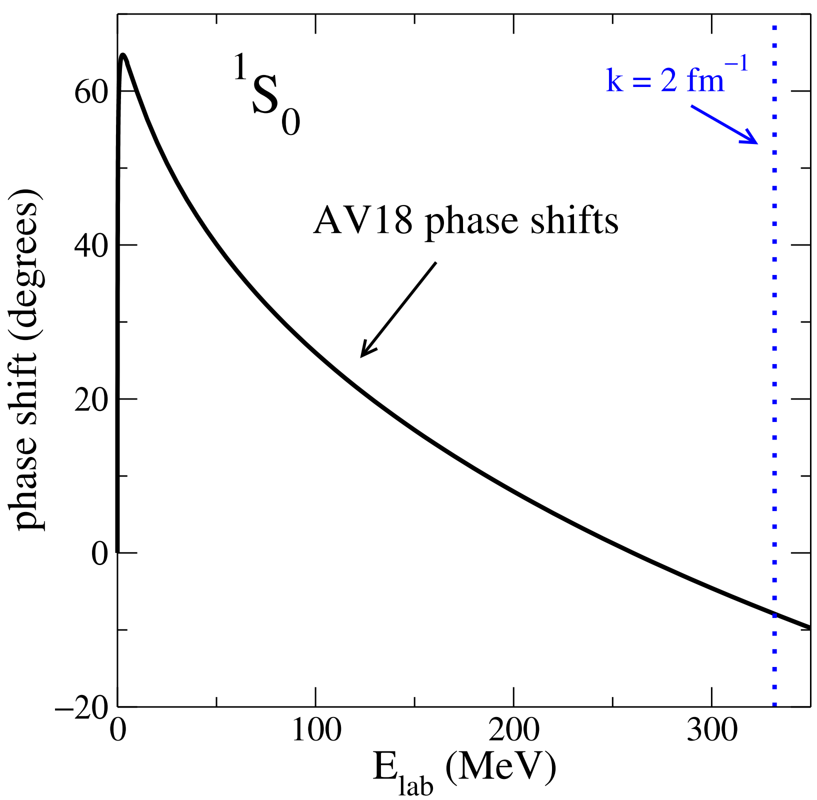


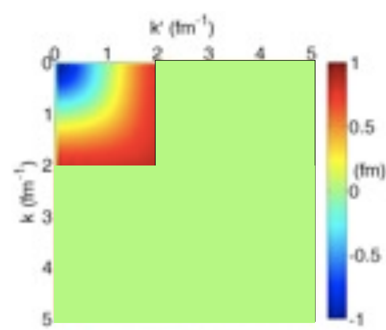


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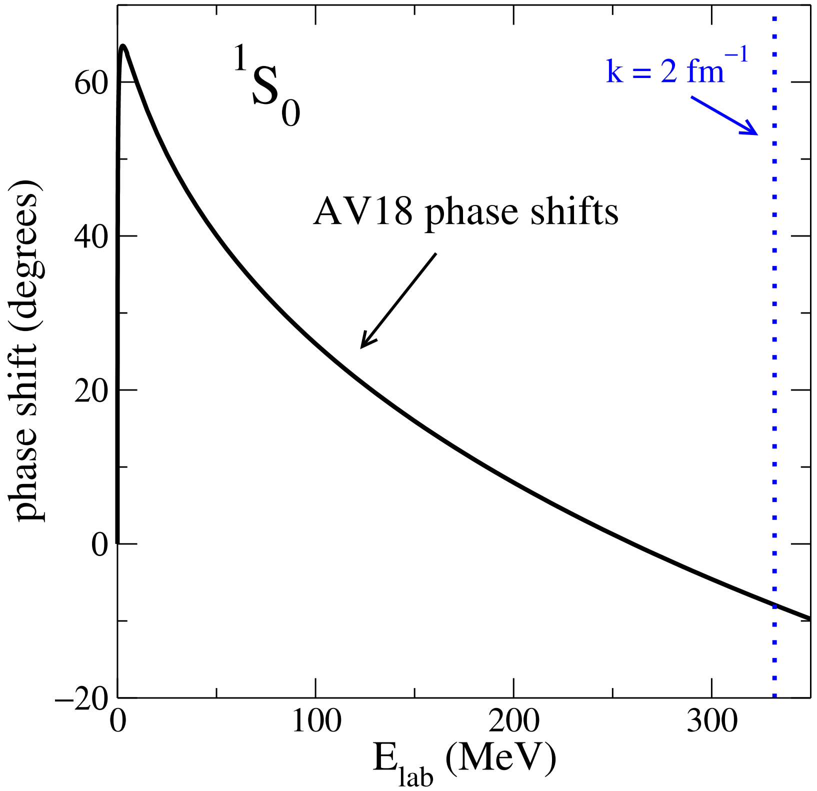
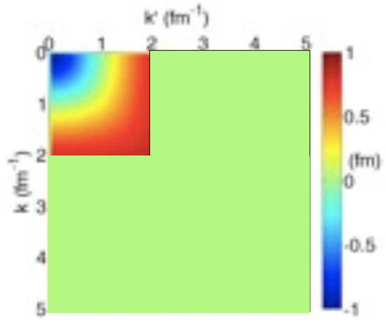
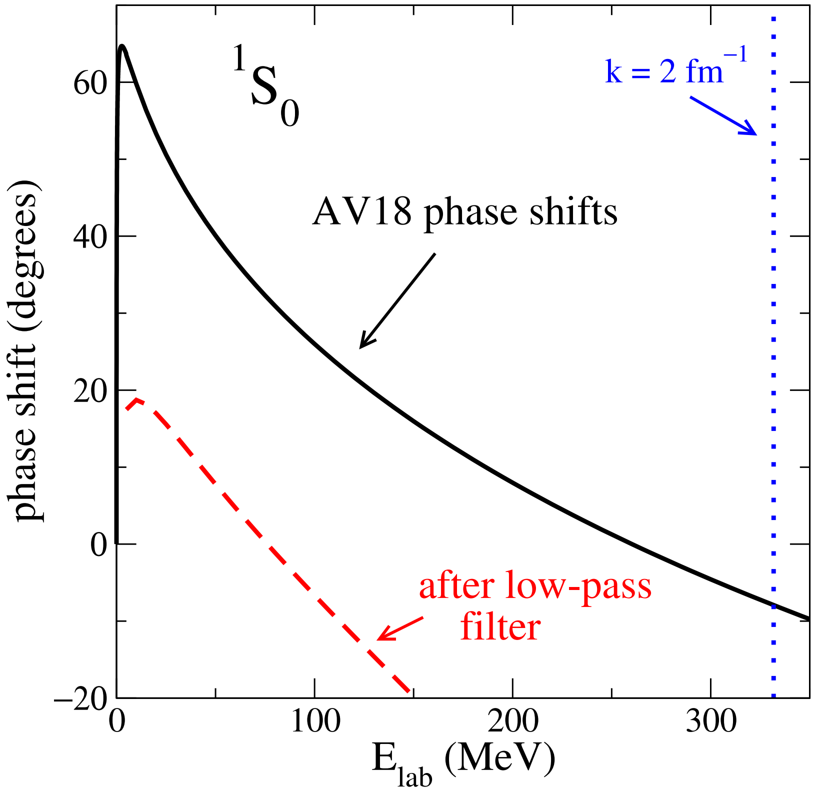



low-pass filter




- truncated interaction fails completely to reproduce original phase shifts
- problem: low- and high momentum states are coupled by interaction!

# First Quantum Monte Carlo based on chiral EFT interactions

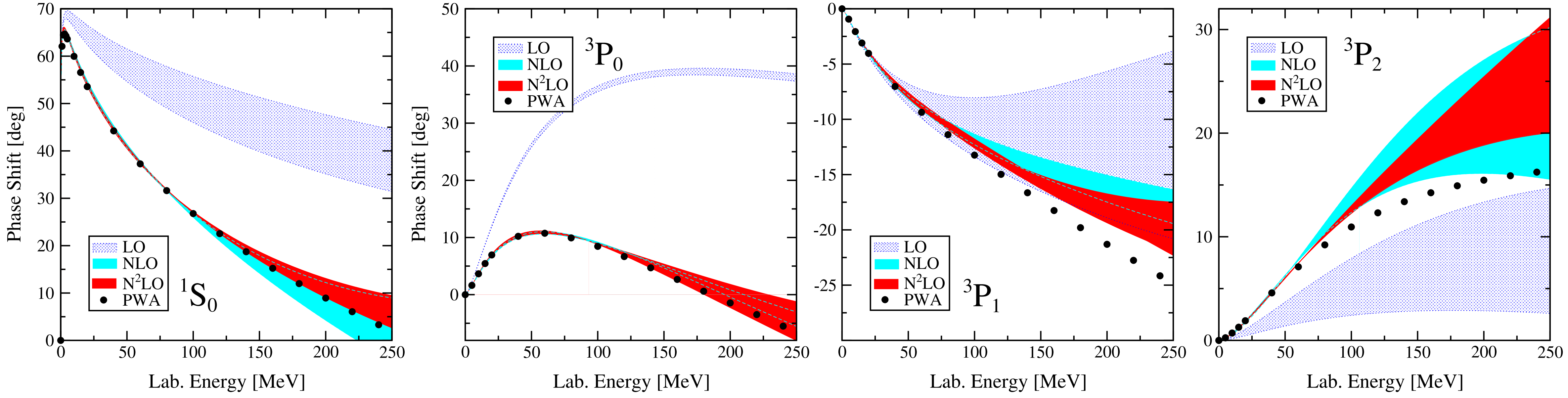
**Problem:**

Current QMC frameworks can only applied to **local** Hamiltonians.
Conventional interactions derived within chiral EFT are **nonlocal**.

**Strategy:**

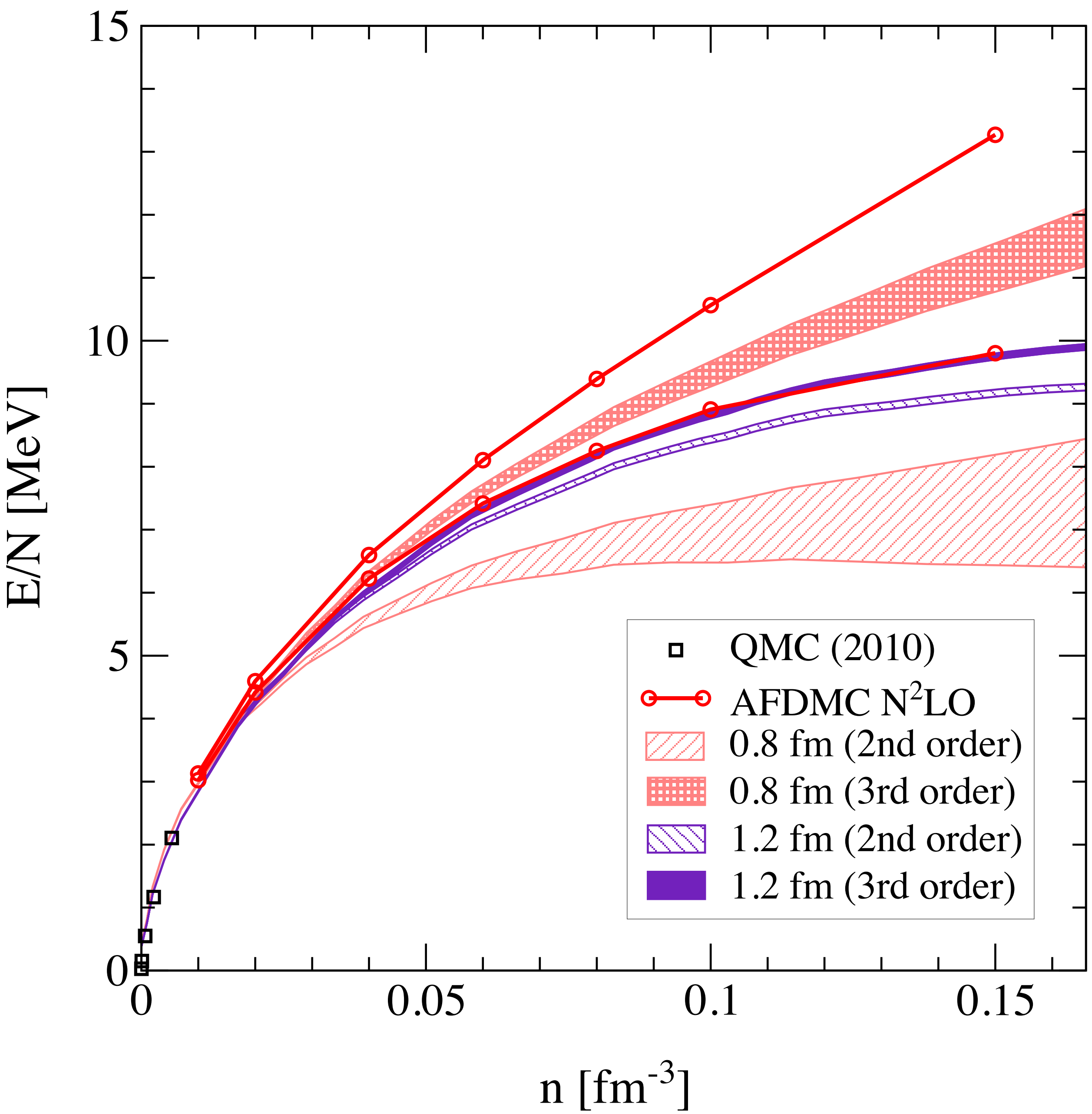
Use freedom in the choice of operators and the type of regulator to construct local Hamiltonians up to $N^2LO$:

- regulate in coordinate space in relative distance: $f(r) = 1 - e^{-(r/R_0)^4}$
- use isospin dependent terms instead of non-local operators at NLO




Gezerlis, Tews, Epelbaum, Gandolfi, KH, Nogga, Schwenk, PRL 111, 032501 (2013)

# First Quantum Monte Carlo based on chiral EFT interactions




perfect agreement for soft interactions, first direct validation of perturbative calculations

Gezerlis, Tews, Epelbaum, Gandolfi, KH, Nogga, Schwenk
PRL 111, 032501 (2013)

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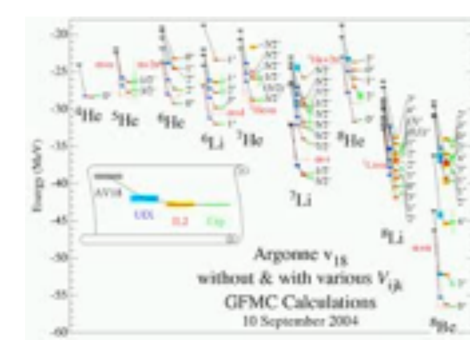
Greens Function Monte Carlo calculations for light nuclei based on chiral interactions currently in progress



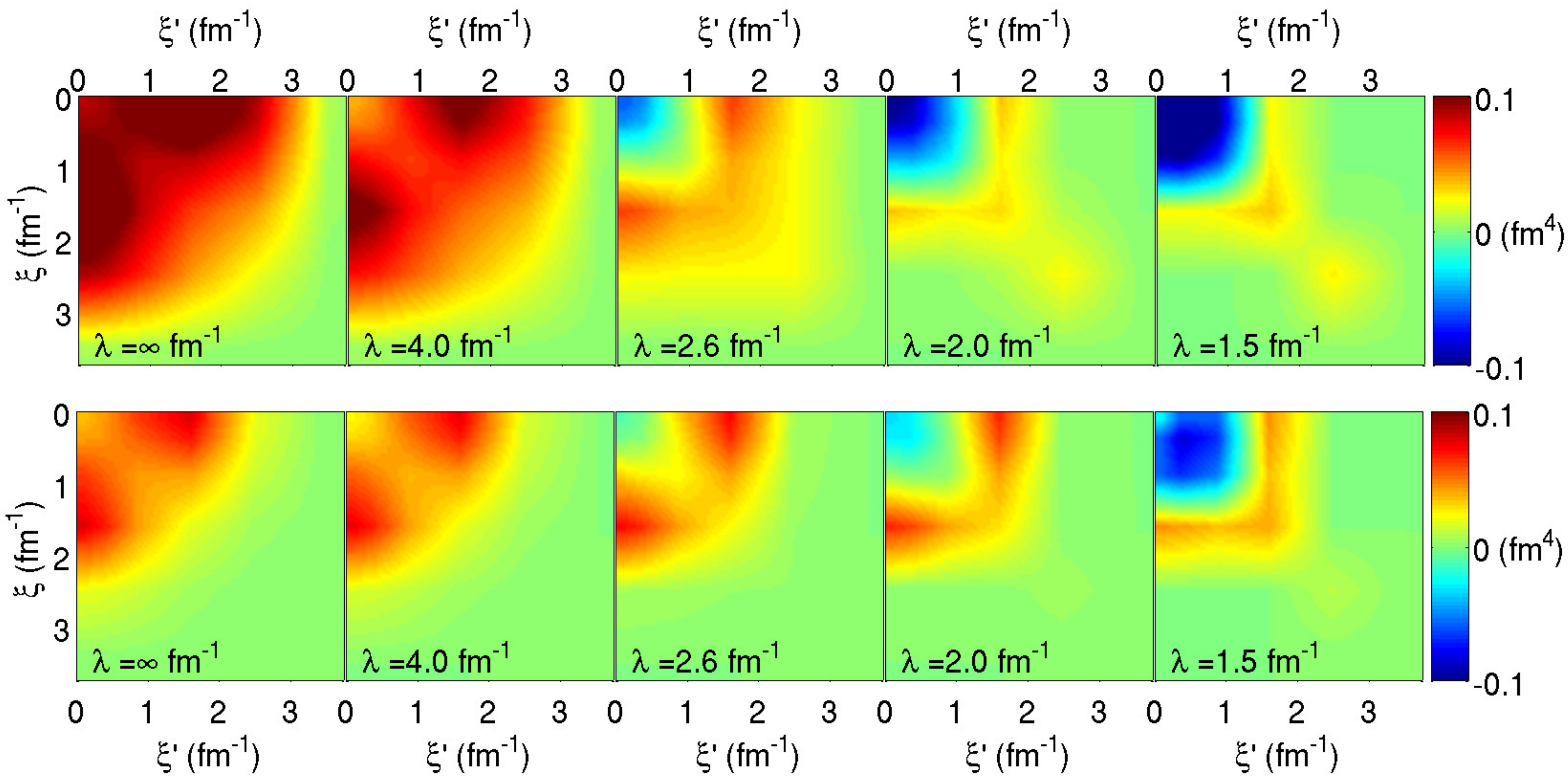
# Decoupling in 3NF matrix elements

$$\theta = \frac{\pi}{12} \qquad \mathcal{T} = \mathcal{J} = \frac{1}{2}$$

$\Lambda/\tilde{\Lambda}$

$550/600\,\mathrm{MeV}$

$450/500\,\mathrm{MeV}$




KH, PRC(R) 85, 021002 (2012)
see also KH, Furnstahl, PRC(R) 87, 031302 (2013)

hyperradius: $\xi^2 = p^2 + \frac{3}{4}q^2$ hyperangle: $\tan\theta = \frac{2\,p}{\sqrt{3}\,q}$

same decoupling patterns like in NN interactions

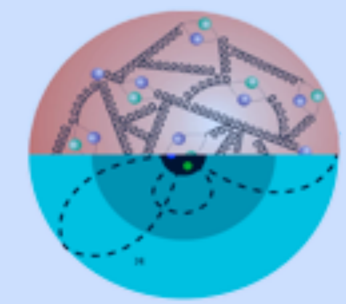
# Resolution dependence of nuclear forces

QCD $\rightarrow$ Effective theory for NN, 3N, many-N interactions:

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \ldots$$

$\Lambda \gg \Lambda_{\rm chiral}$

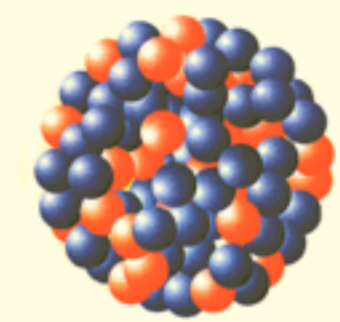
quarks+gluons/partons: $Q \gg m_\pi$



$\Lambda_{\rm chiral}$

typical momenta in nuclei: $Q \sim m_\pi$

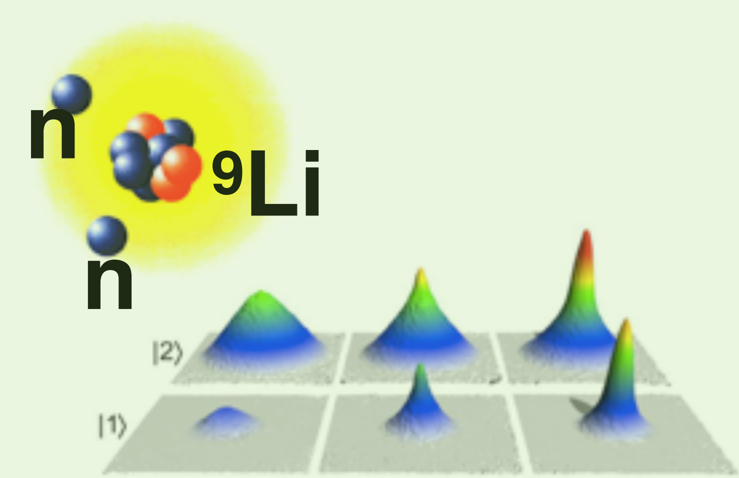
chiral EFT: nucleons interacting via pion exchanges and short-range contact interactions



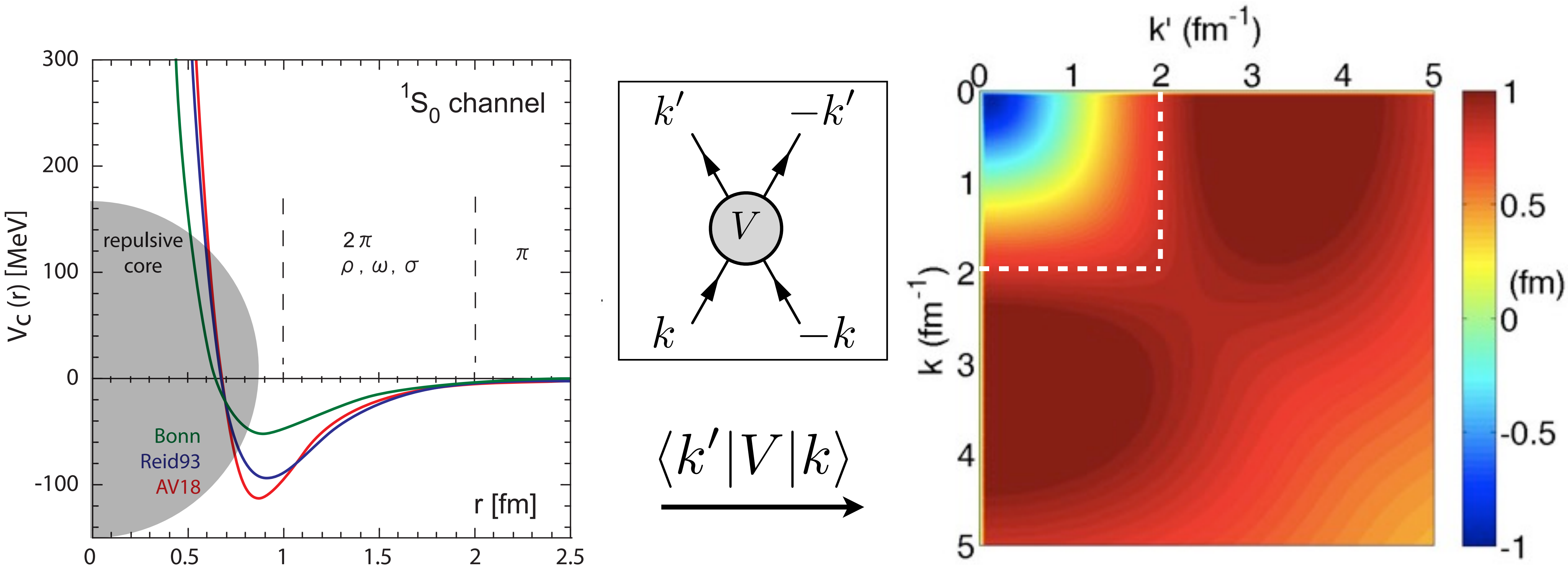
$\Lambda_{\rm pionless}$

large scattering length physics: $Q \ll m_\pi$

pionless EFT: unitary regime, non-universal corrections



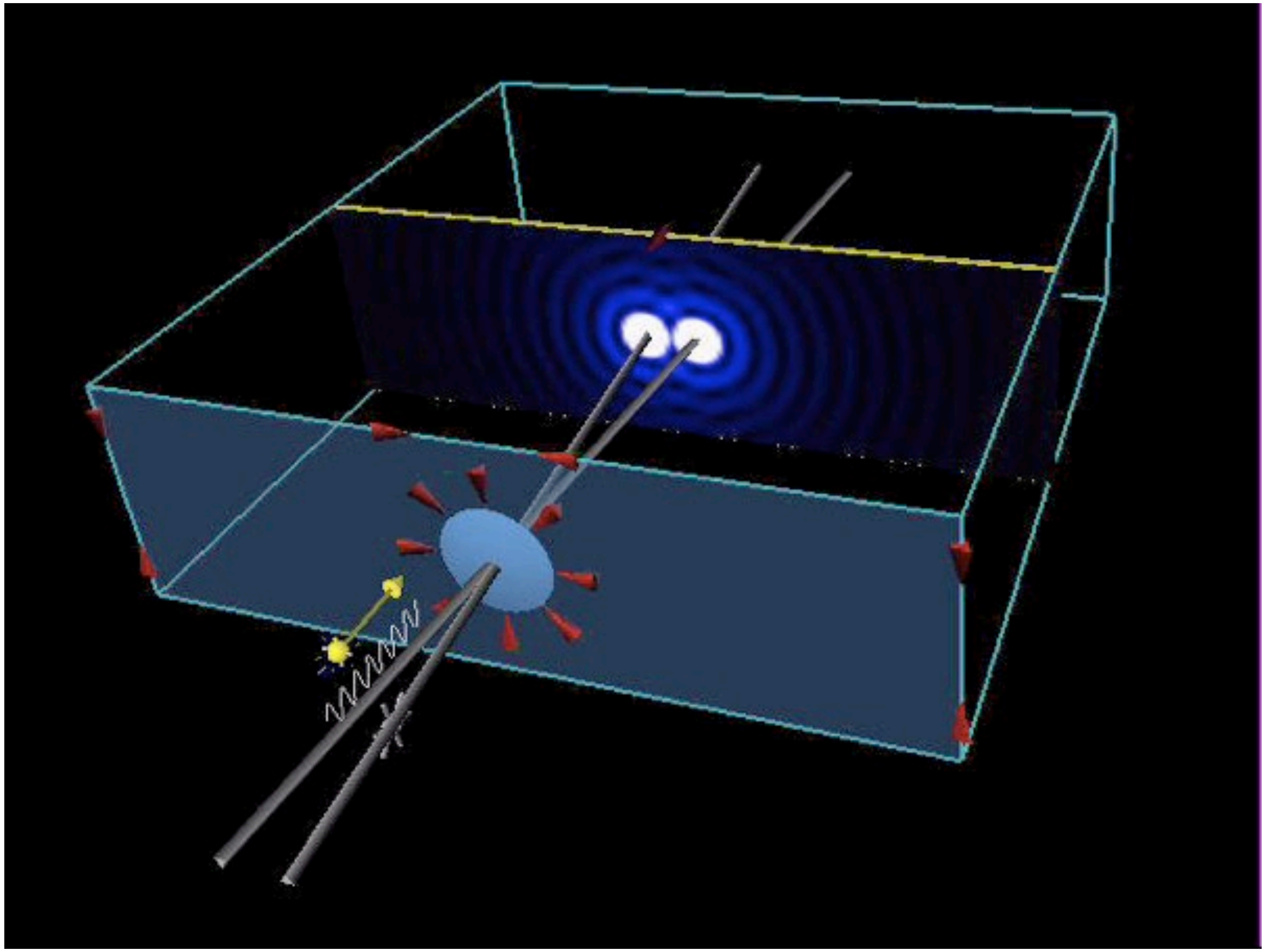
# Problem: Traditional “hard” NN interactions




- constructed to fit scattering data (long-wavelength information!)
- “hard” NN interactions contain repulsive core at small relative distance
- strong coupling between low and high-momentum components, hard to solve!

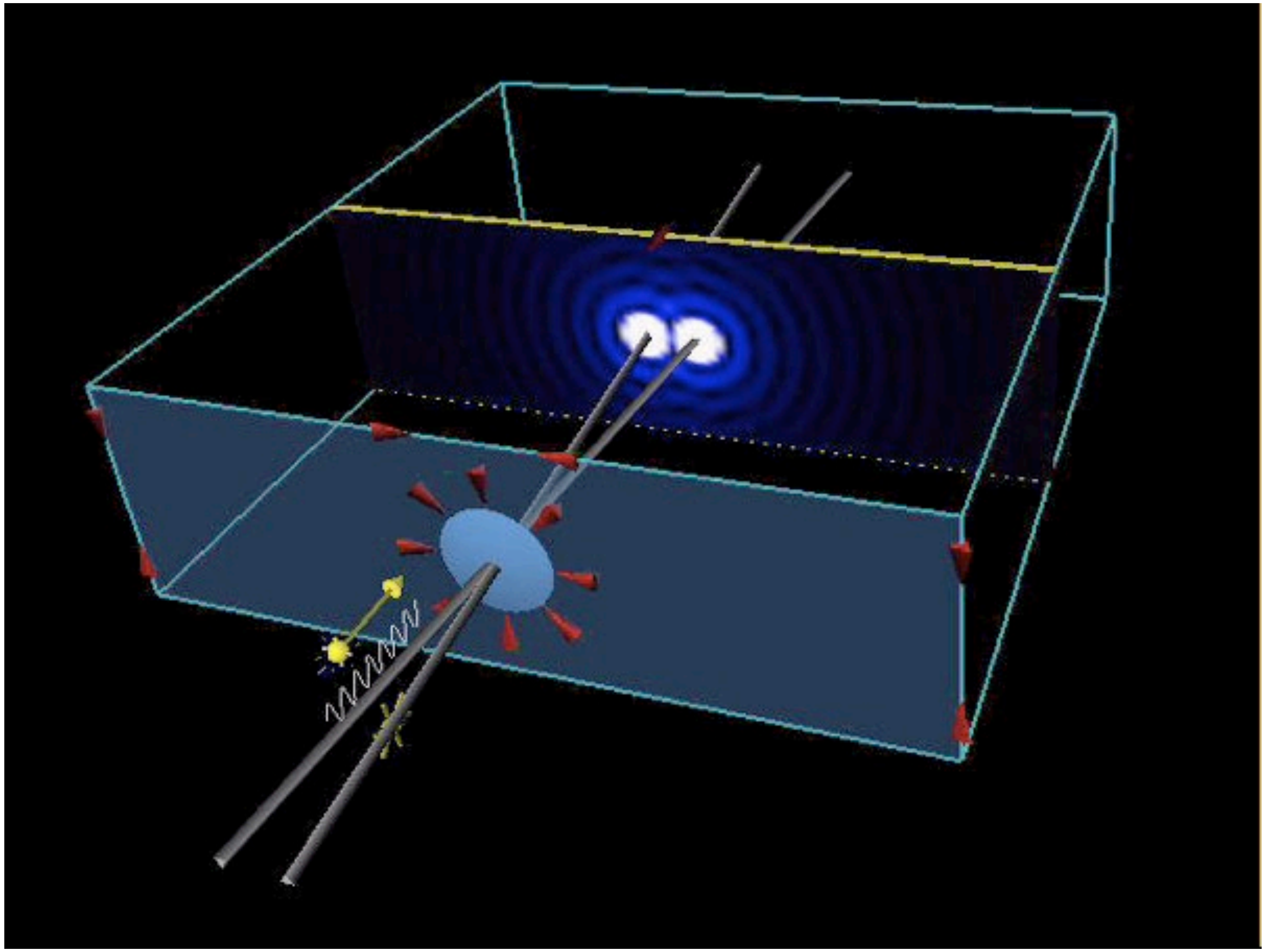
Claim:
Problems due to high resolution from interaction.

# Wavelength and resolution



size of resolvable structures depends on the wavelength

# Wavelength and resolution



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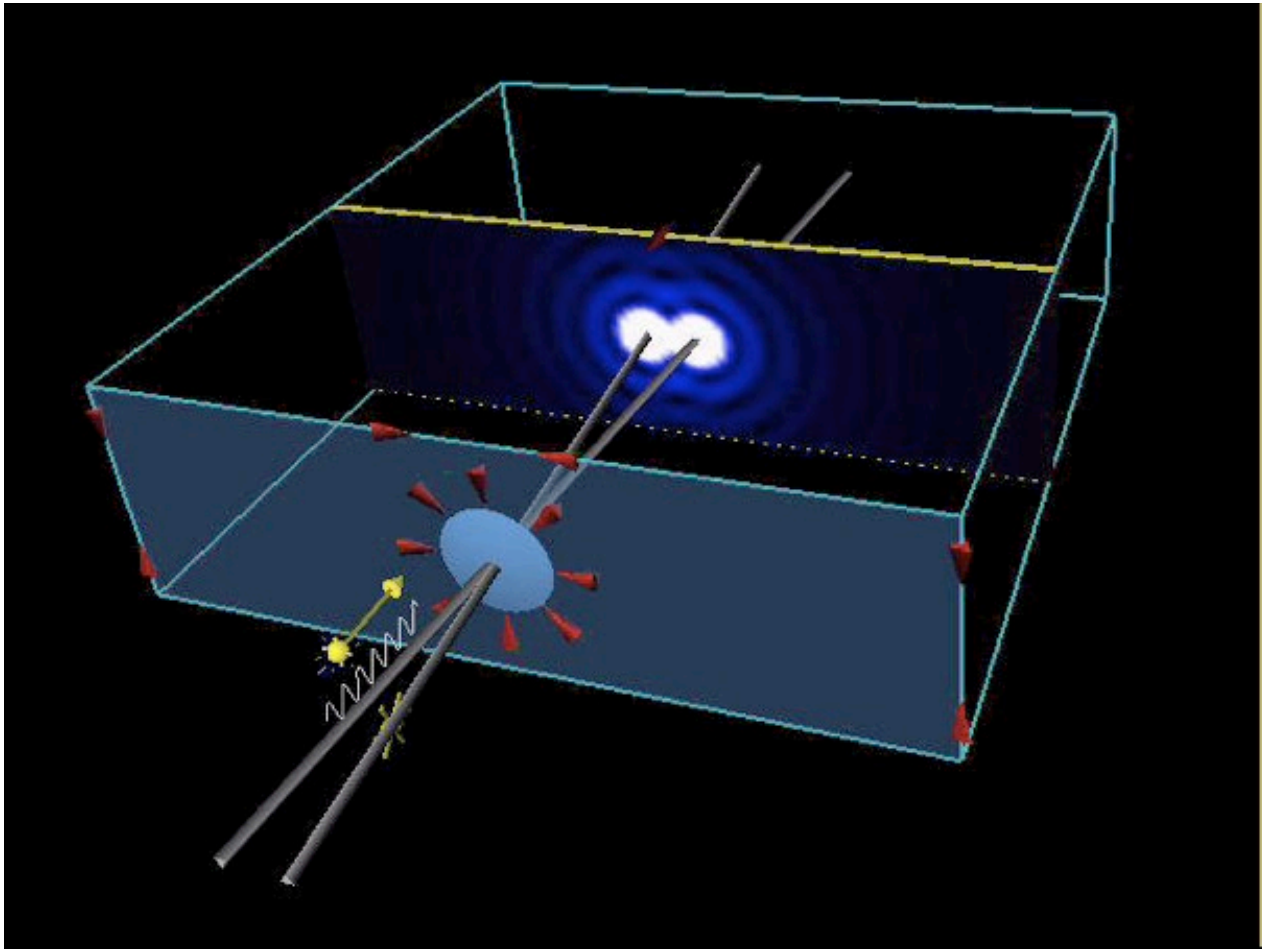

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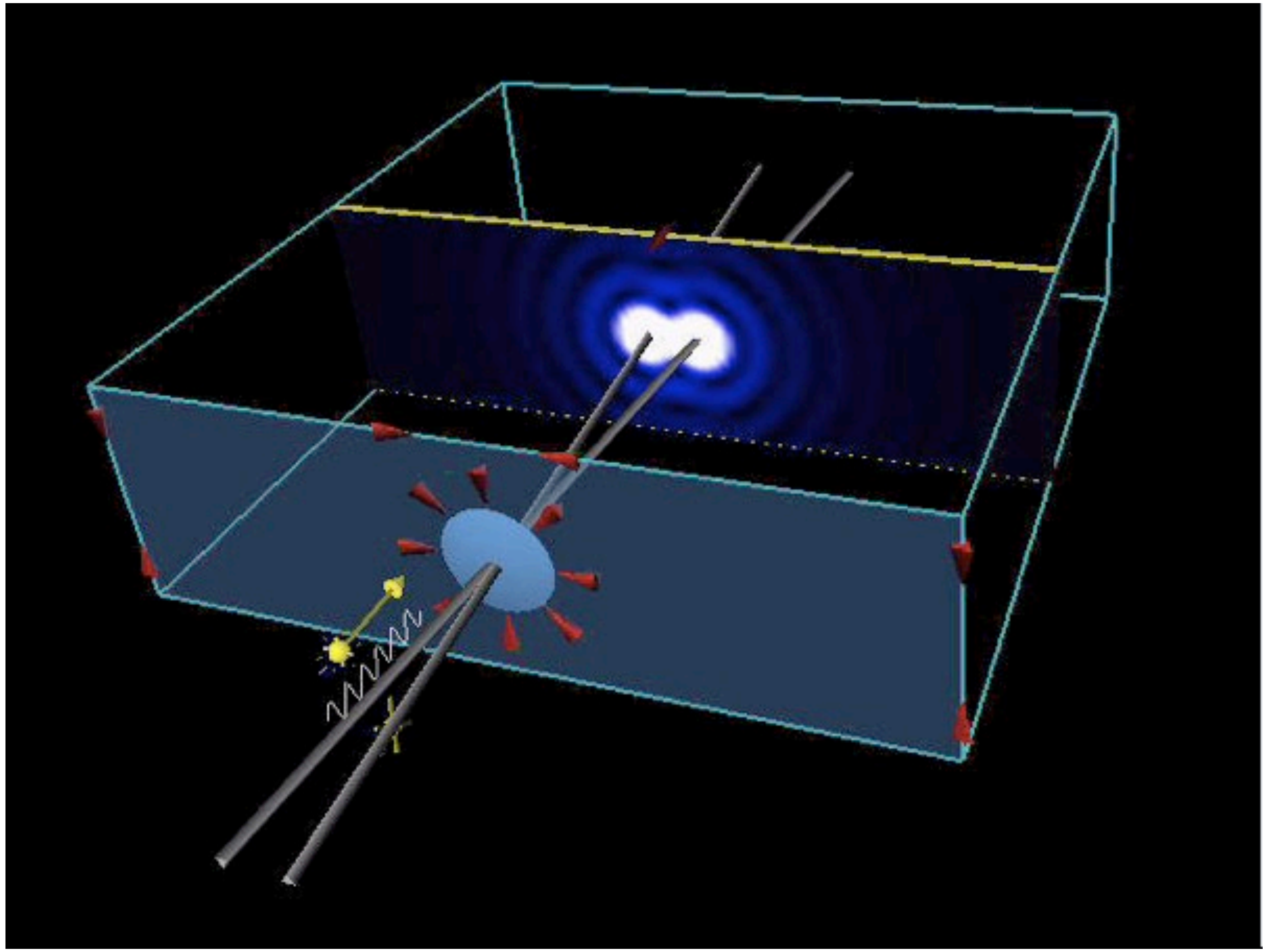

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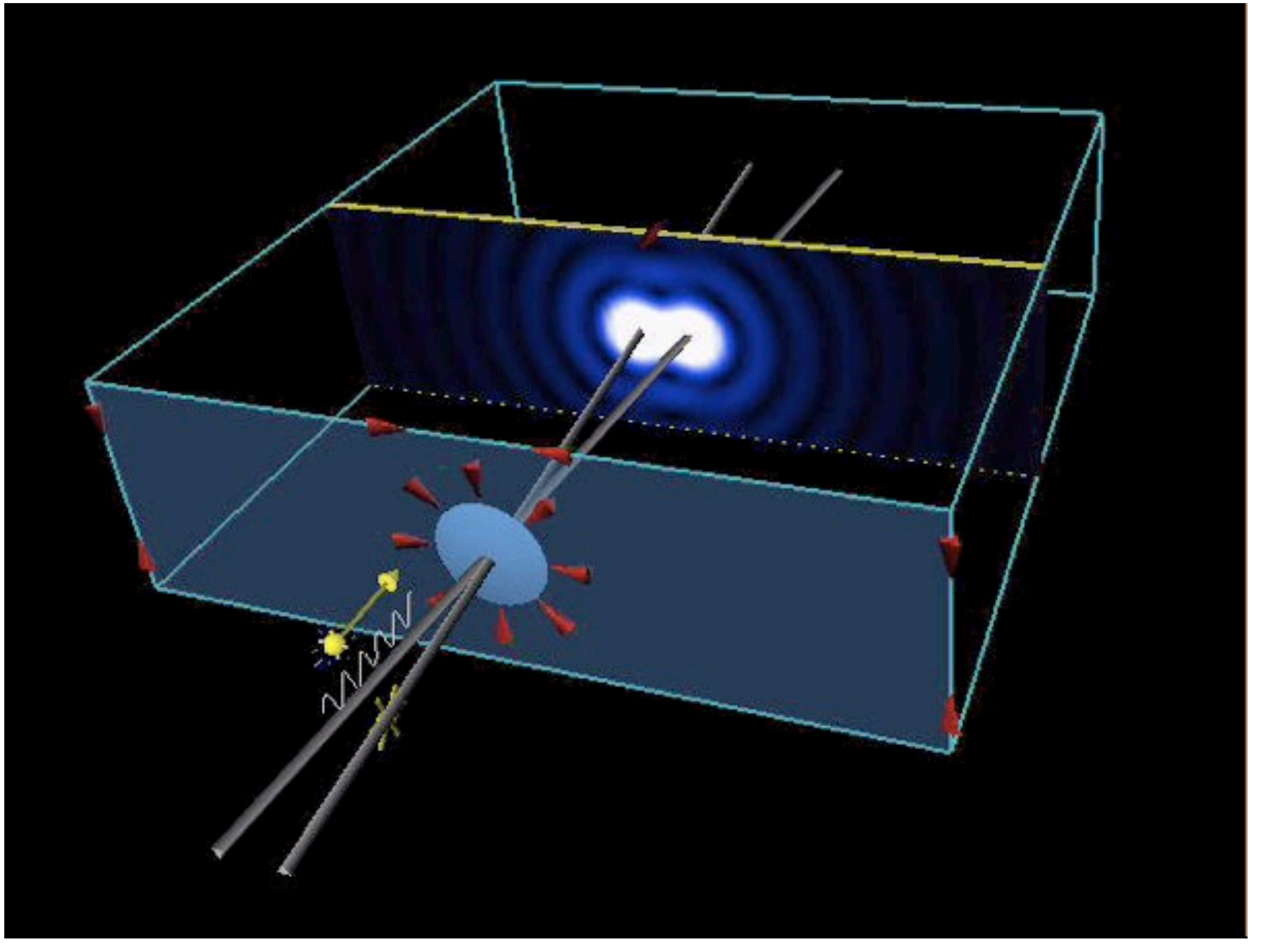

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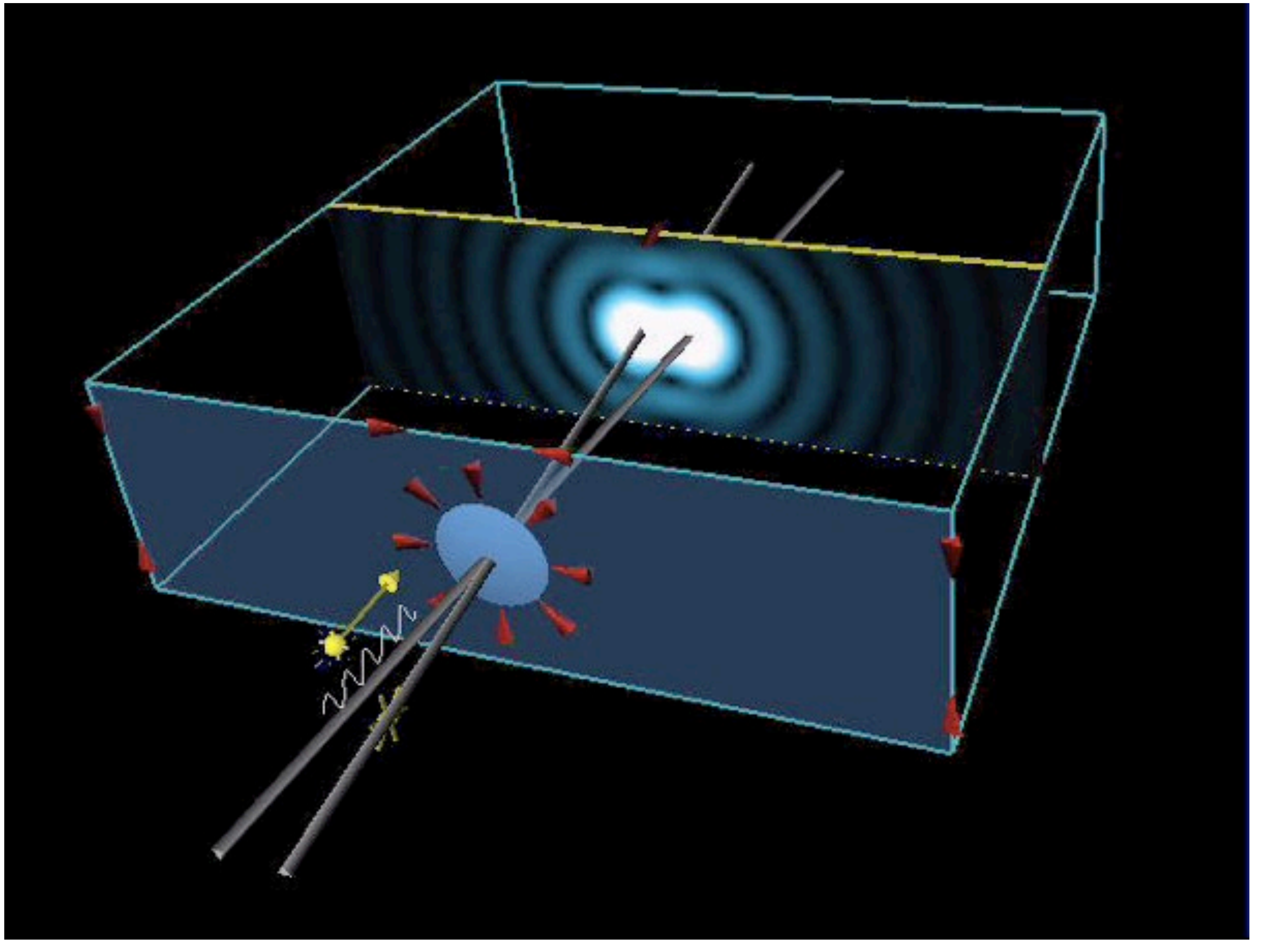

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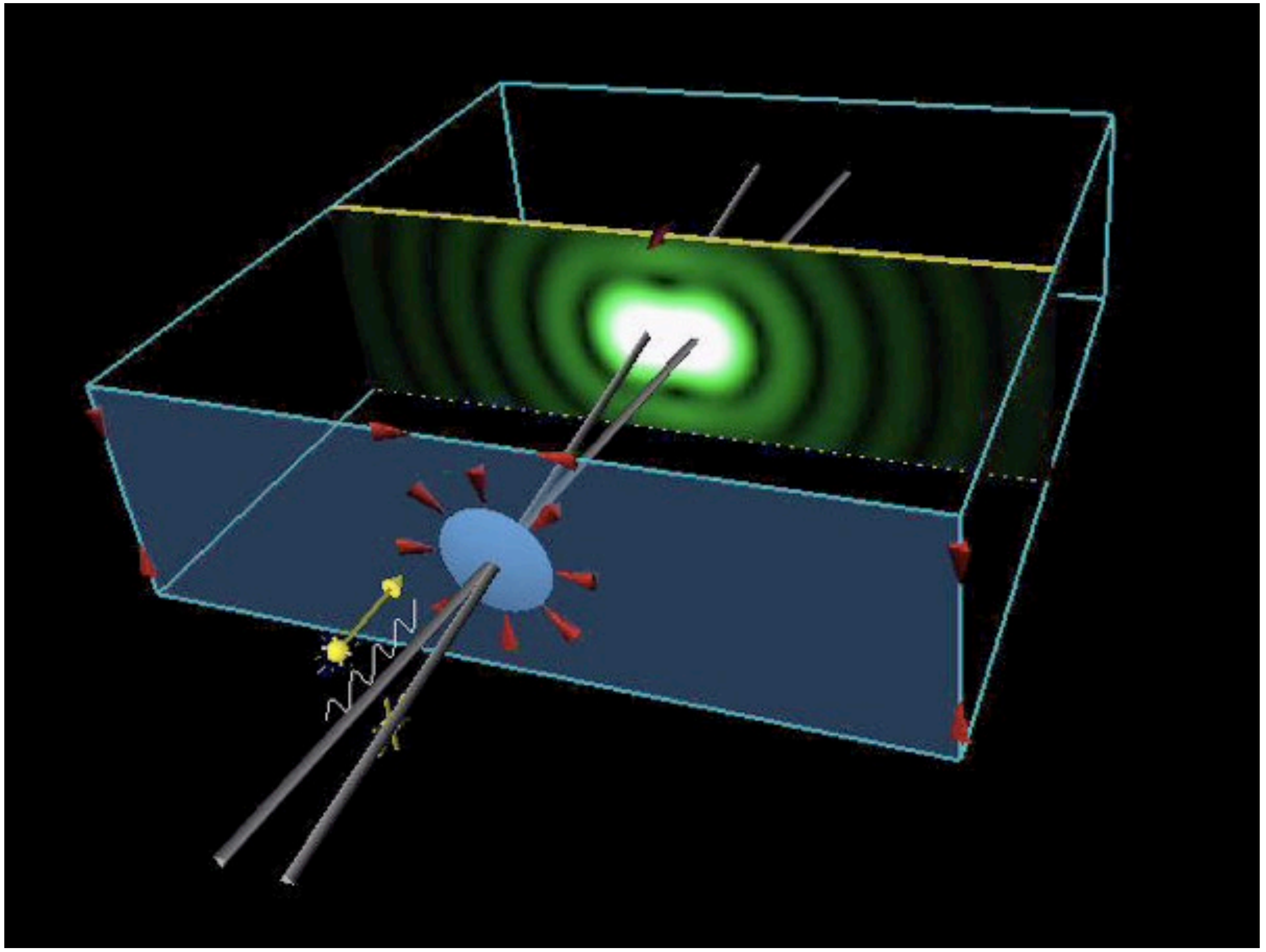

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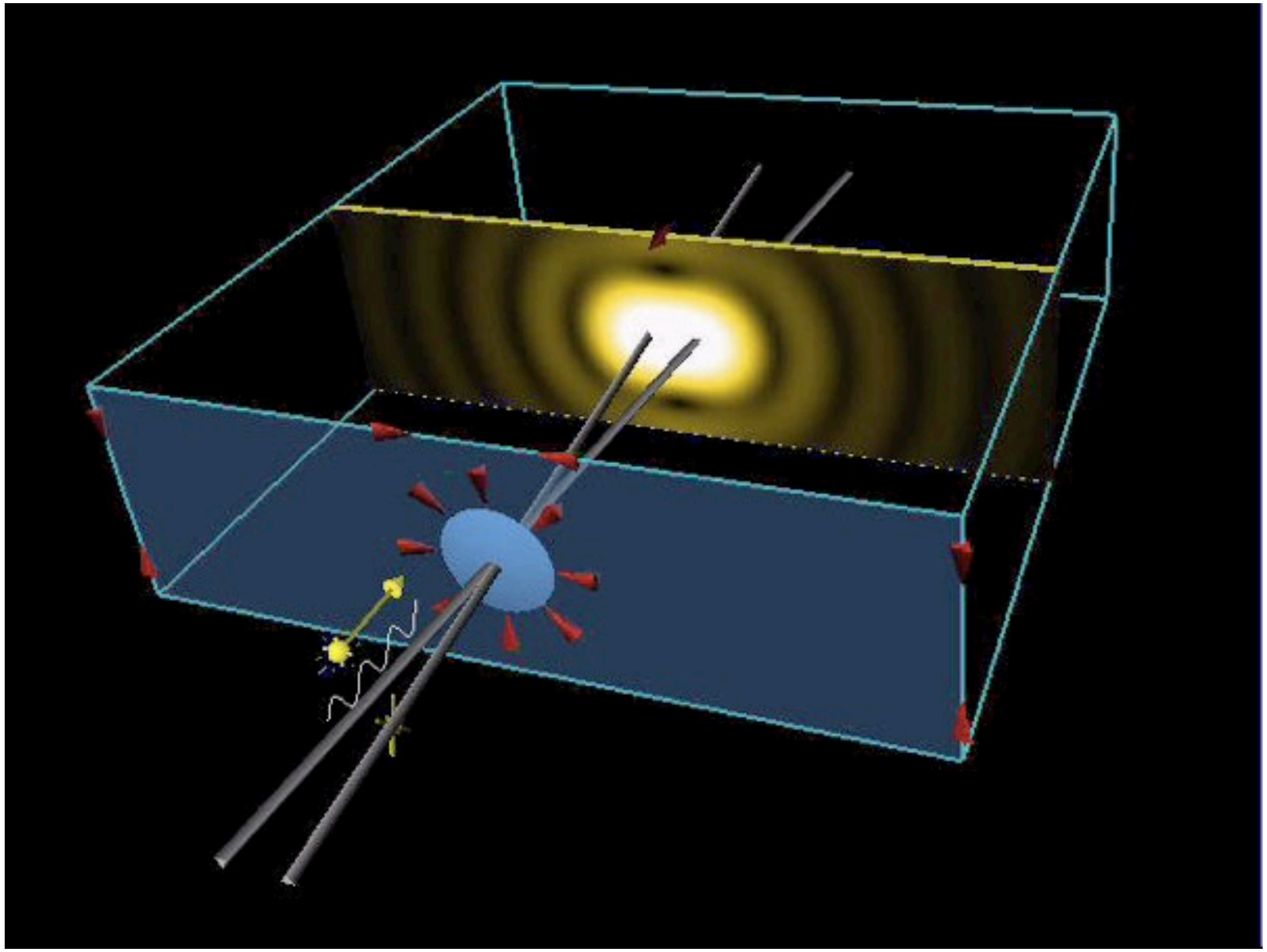

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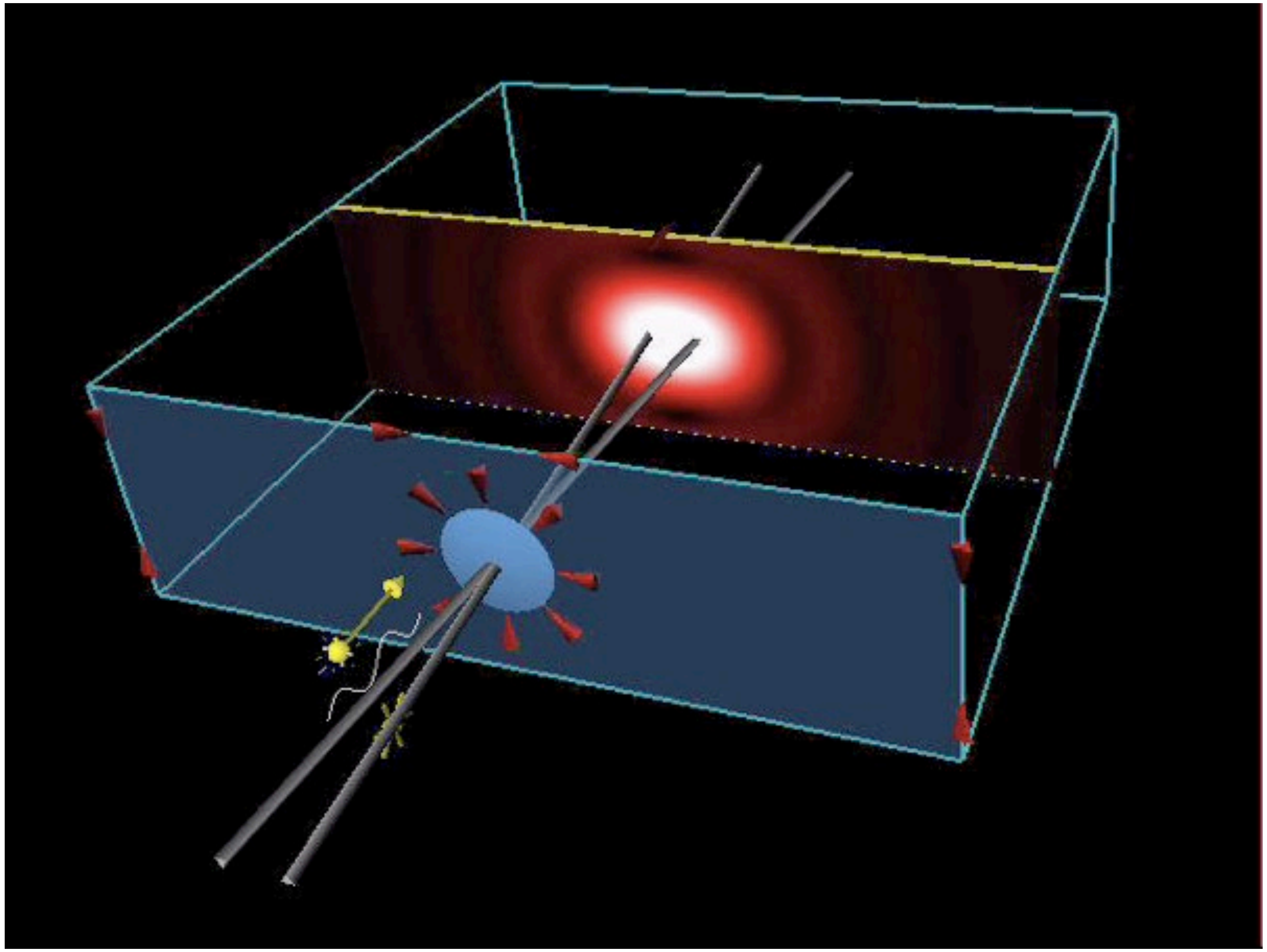
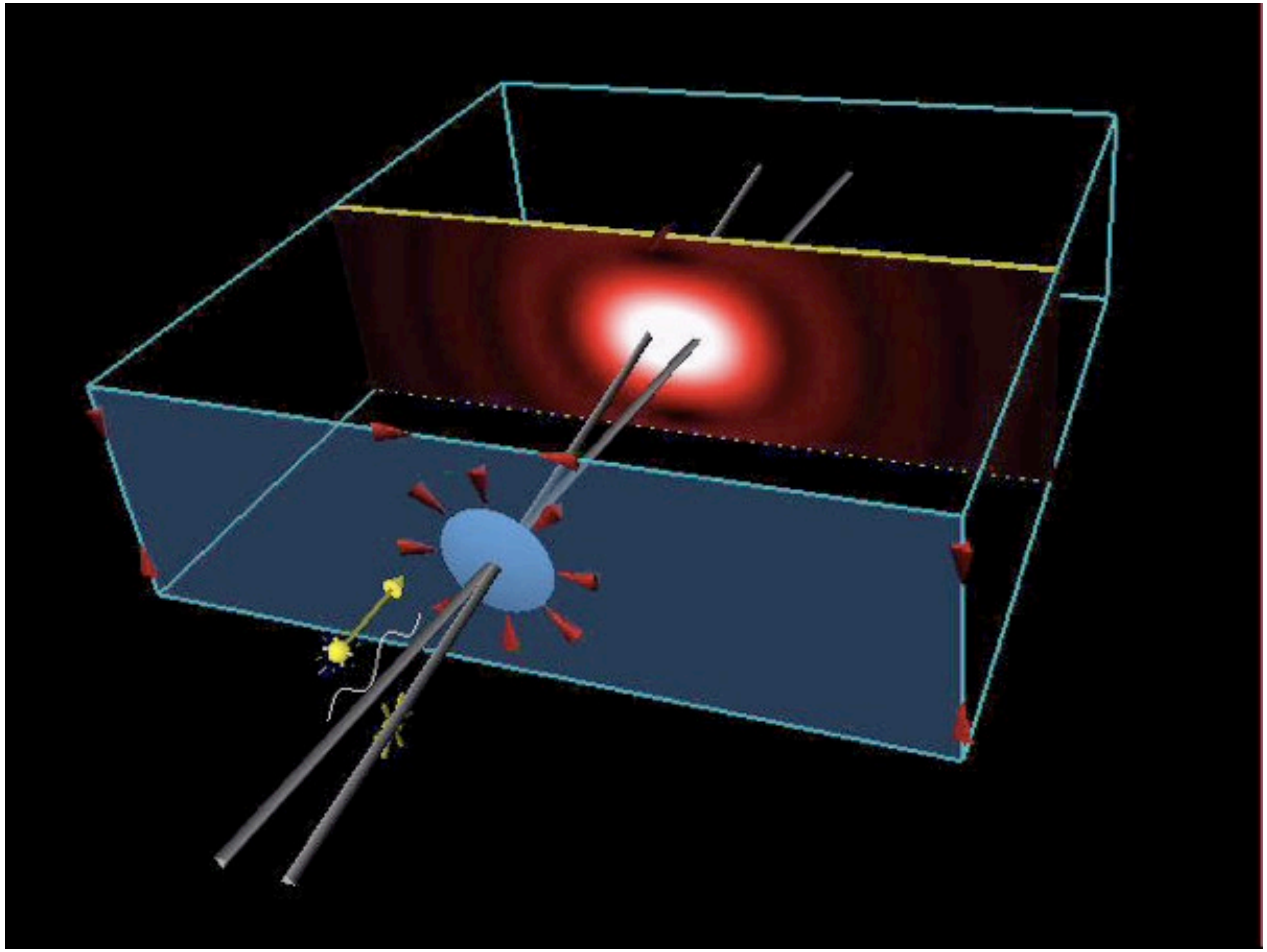


size of resolvable structures depends on the wavelength

Question: Which resolution should we choose?

# Wavelength and resolution



size of resolvable structures depends on the wavelength

Question: Which resolution should we choose?

Depends on the system and phenomena we are interested in!

# Strategy: Use a lower-resolution version

low-pass filter →

- long-wavelength information is preserved
- much less information necessary

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- long-wavelength information is preserved
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... however, it's not that easy in nuclear physics.

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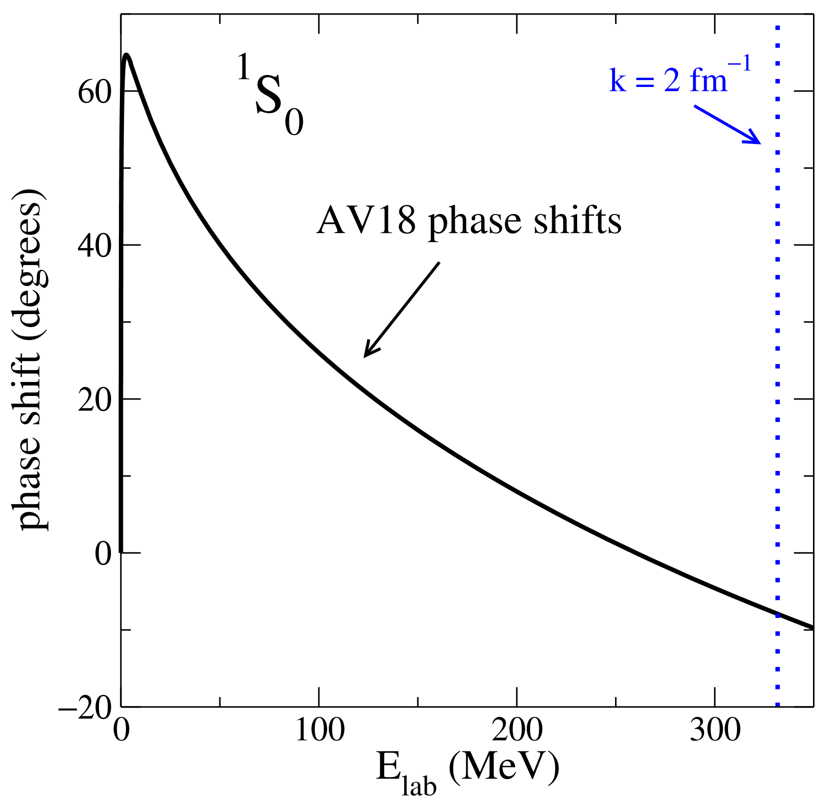


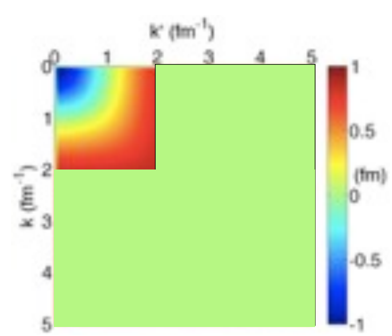


low-pass filter

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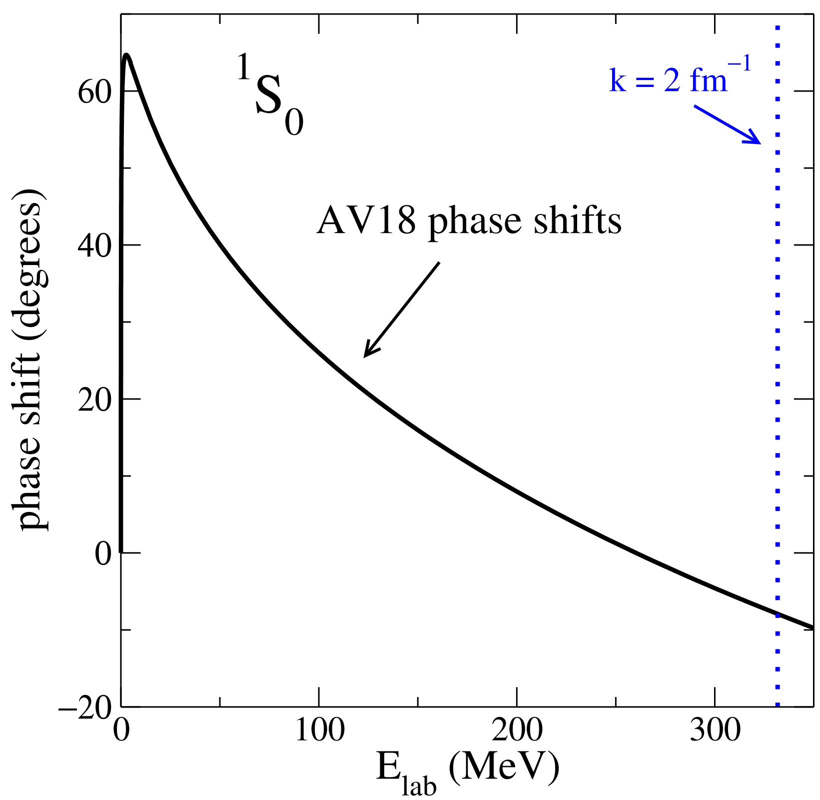
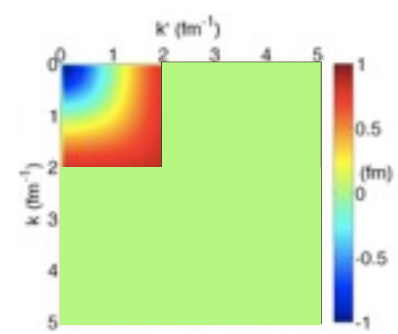
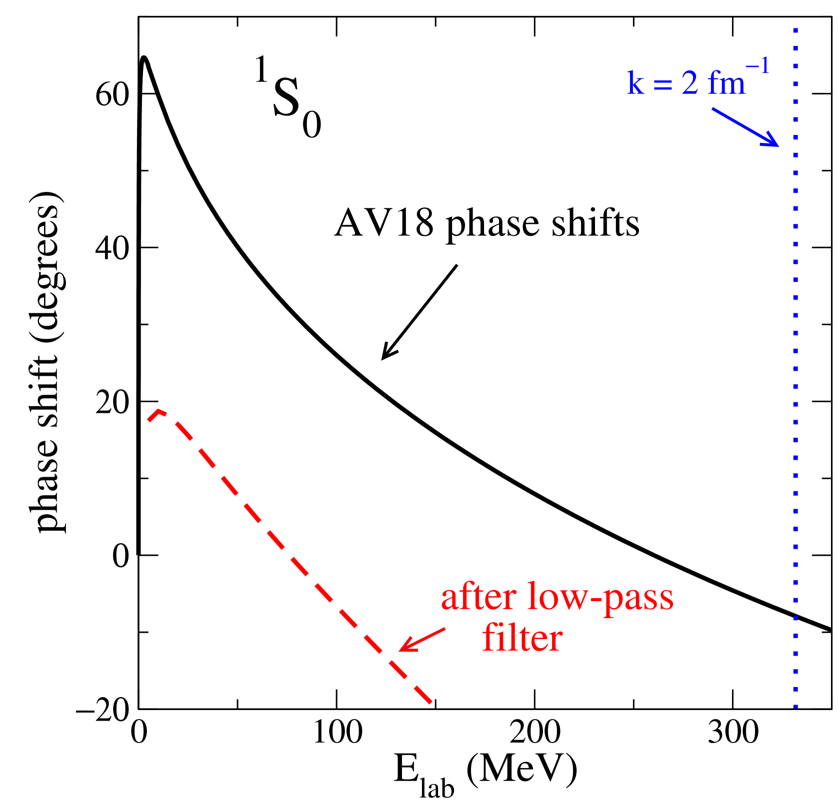



low-pass filter




- truncated interaction fails completely to reproduce original phase shifts
- problem: low- and high momentum states are coupled by interaction!

# SRG evolution in momentum space

- evolve the antisymmetrized 3N interaction

*special thanks to*
*J. Golak, R. Skibinski, K. Topolnicki*

$$\overline{V}_{123} = {}_i\langle pq\alpha| \, (1 + P_{123} + P_{132}) V_{123}^{(i)} (1 + P_{123} + P_{132}) \, |p'q'\alpha'\rangle_i$$

- embed NN interaction in 3N basis:

$$V_{13} = P_{123} V_{12} P_{132}, \quad V_{23} = P_{132} V_{12} P_{123}$$

with $\ {}_3\langle pq\alpha|V_{12}|p'q'\alpha'\rangle_3 = \langle p\tilde{\alpha}|V_{\rm NN}|p'\tilde{\alpha}'\rangle \, \delta(q-q')/q^2$

- use $P_{123}\overline{V}_{123} = P_{132}\overline{V}_{123} = \overline{V}_{123}$

$$\Rightarrow \quad \begin{aligned} d\overline{V}_{123}/ds = {} & C_1(s, T, V_{\rm NN}, P) \\ & + C_2(s, T, V_{\rm NN}, \overline{V}_{123}, P) \\ & + C_3(s, T, \overline{V}_{123}) \end{aligned}$$