Challenges of ν-transport toward the full 3D supernova simulations: solving Boltzmann equations and ν-processes

Numazu near Mt. Fuji

Wikipedia (trimmed)

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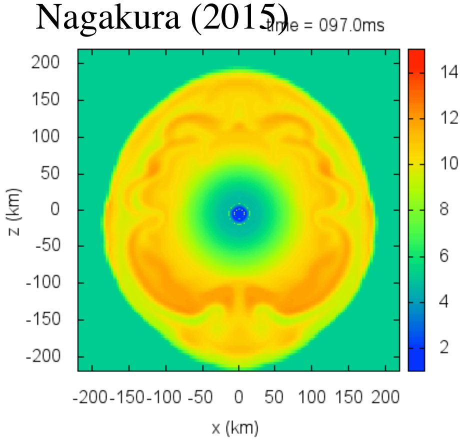
Nagakura (2015)



6D Boltzmann solver is working for:

ν-transfer in 3D supernova cores
2D core-collapse simulations on K-computer

10P Flops, rank 4th

Kobe, Japan

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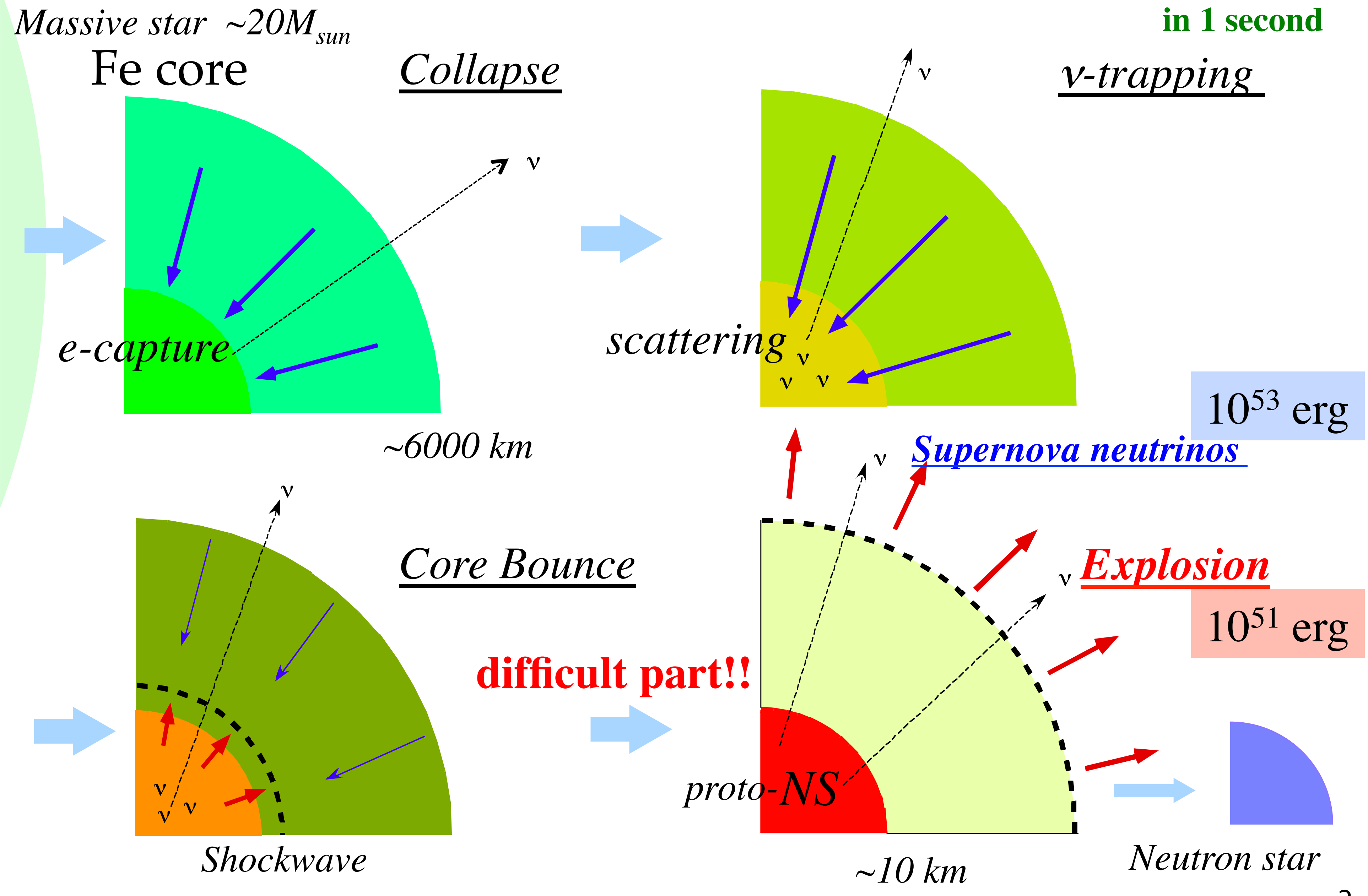
Core-collapse SNe: collapse, bounce and explosion

in 1 second



Massive star ~20M_{sun}

Fe core

Collapse

e-capture

~6000 km

ν-trapping

scattering

10^{53} erg

Supernova neutrinos

Core Bounce

Shockwave

difficult part!!

Explosion

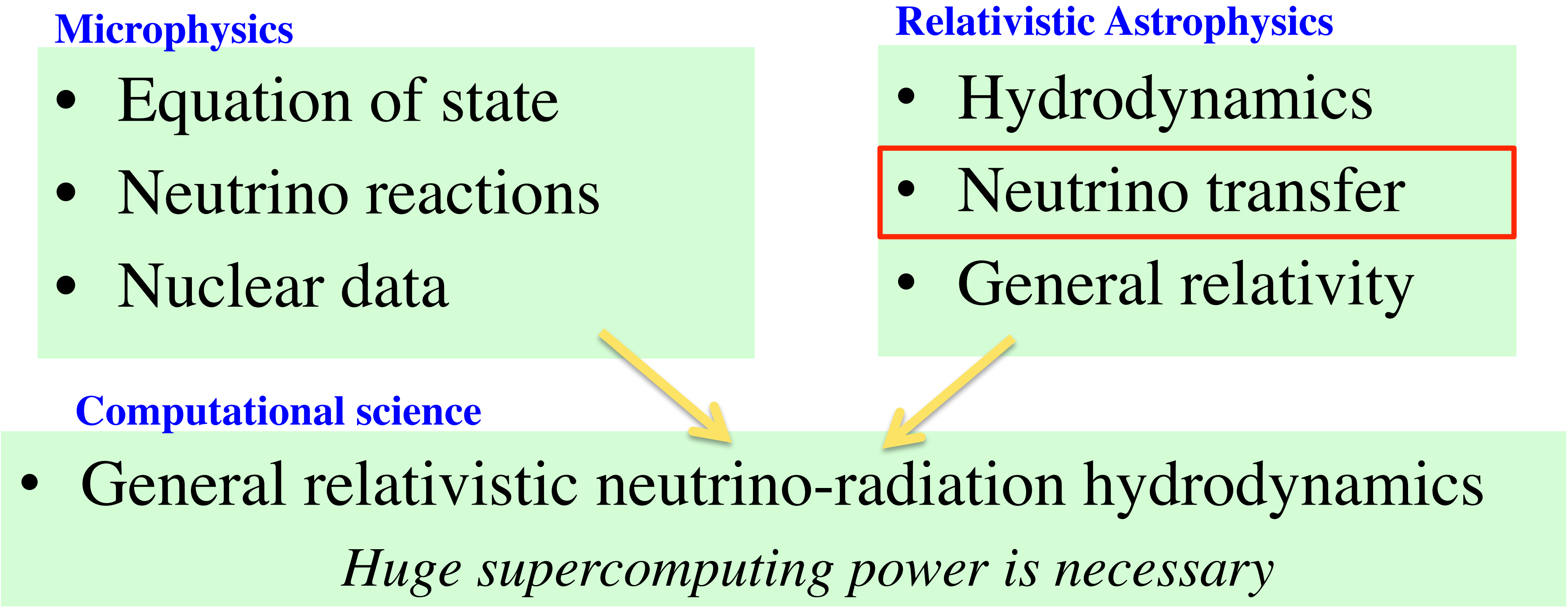
10^{51} erg

proto-NS

~10 km

Neutron star

Multi-physics in core-collapse supernovae



Microphysics

- Equation of state
- Neutrino reactions
- Nuclear data

Relativistic Astrophysics

- Hydrodynamics
- Neutrino transfer
- General relativity

Computational science

- General relativistic neutrino-radiation hydrodynamics

Huge supercomputing power is necessary

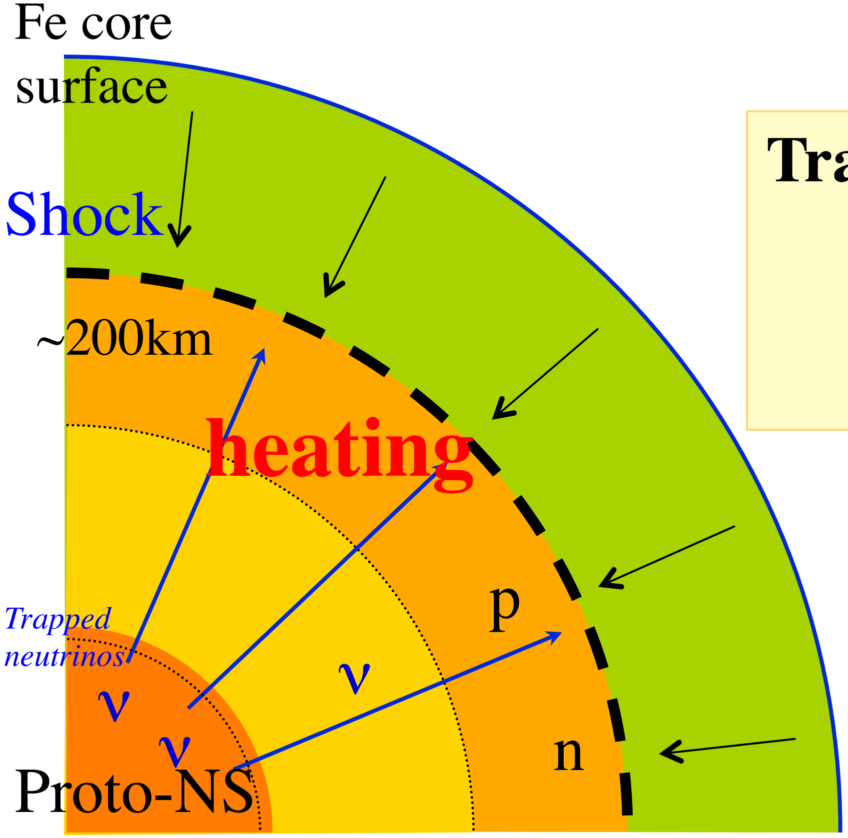
Focus on neutrino transfer

- Importance of ν-transfer: lessons from 1D spherical
- 6D Boltzmann solver: characteristic in 3D supernova core
    - Evaluate approximate methods & provide data for new schemes
- 2D core-collapse simulations: issues toward the full 3D

Neutrino heating mechanism for revival of shock

Heating by neutrino absorption

$$\nu_e + n \rightarrow e^- + p,$$
$$\bar{\nu}_e + p \rightarrow e^+ + n$$

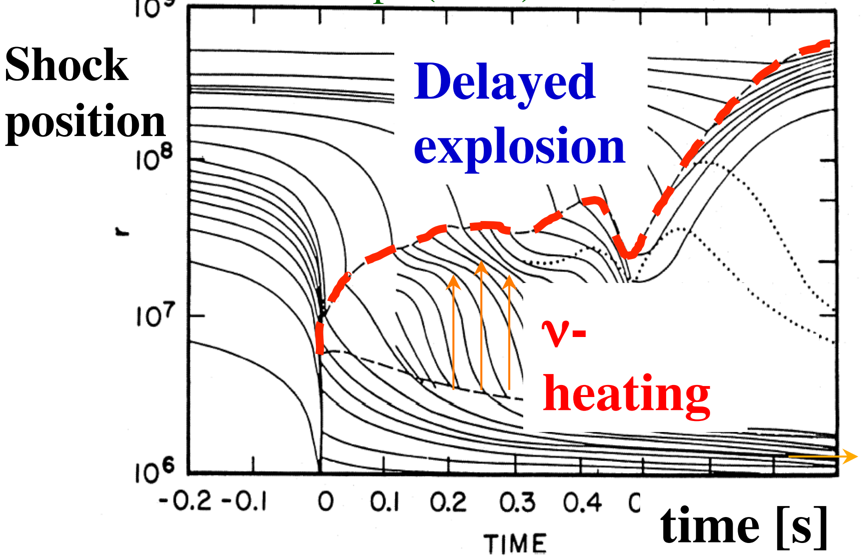


100ms after bounce

Transfer of energy from ν Janka A&A (1996)

$$E_{\nu-heat} \sim 2\times10^{51}\left(\frac{\Delta M}{0.1M_{solar}}\right)\left(\frac{\Delta t}{0.1s}\right)erg$$

Bethe & Wilson ApJ (1985) *"Legendary simulation"*



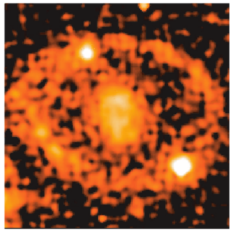
Neutrino energy/flux from trapped neutrinos → **neutrino transfer**

Neutrino heating and hydro instabilities

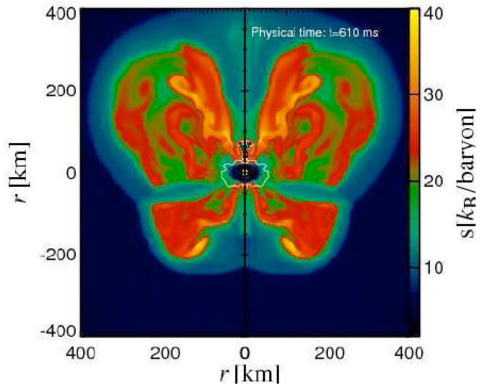
- Convection, SASI, rotation, magnetic etc   - Observations

→ neutrino-transfer in multi-dimensions

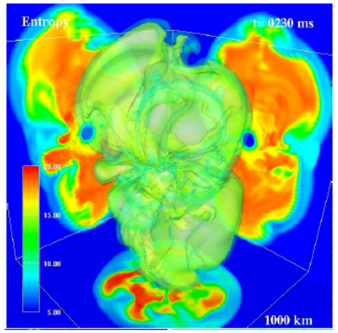
SN1987A



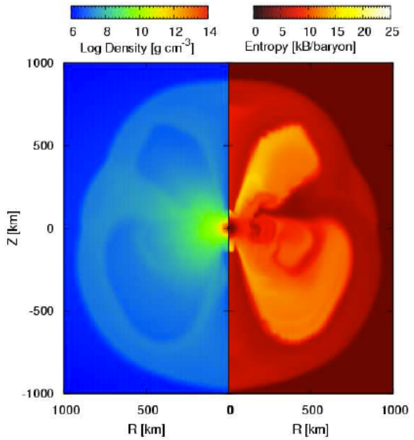
Wang (2002)



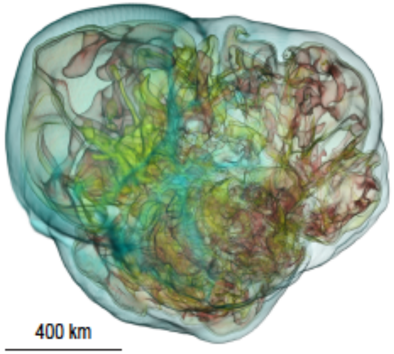
Marek et al, ApJ (2009)



Takiwaki et al. (2012)



Suwa et al. (2010) PASJ

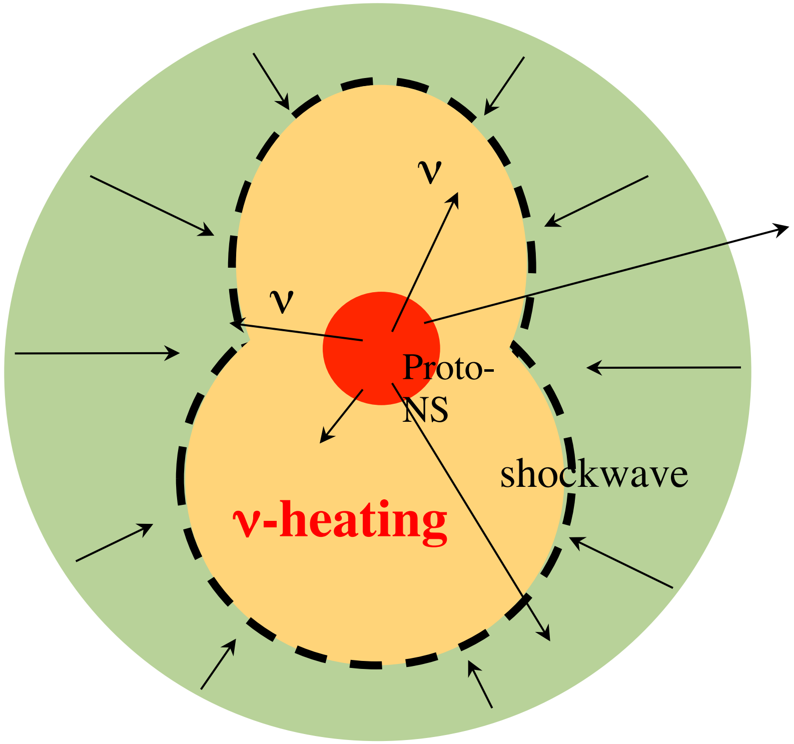


Lentz et al. (2015)

Deformation of shock
Convection & SASI

➡

Enough time
for ν-heating



Need precise evaluation

ν-heating occurs in the intermediate region

- Need neutrino-transfer for energy, angle distribution

$f(E_\nu, \theta_\nu)$

ex. Diffusion approx. is not enough

- Even ~10 % change of ν-heating may affect the outcome: explosion

Competing with other effects

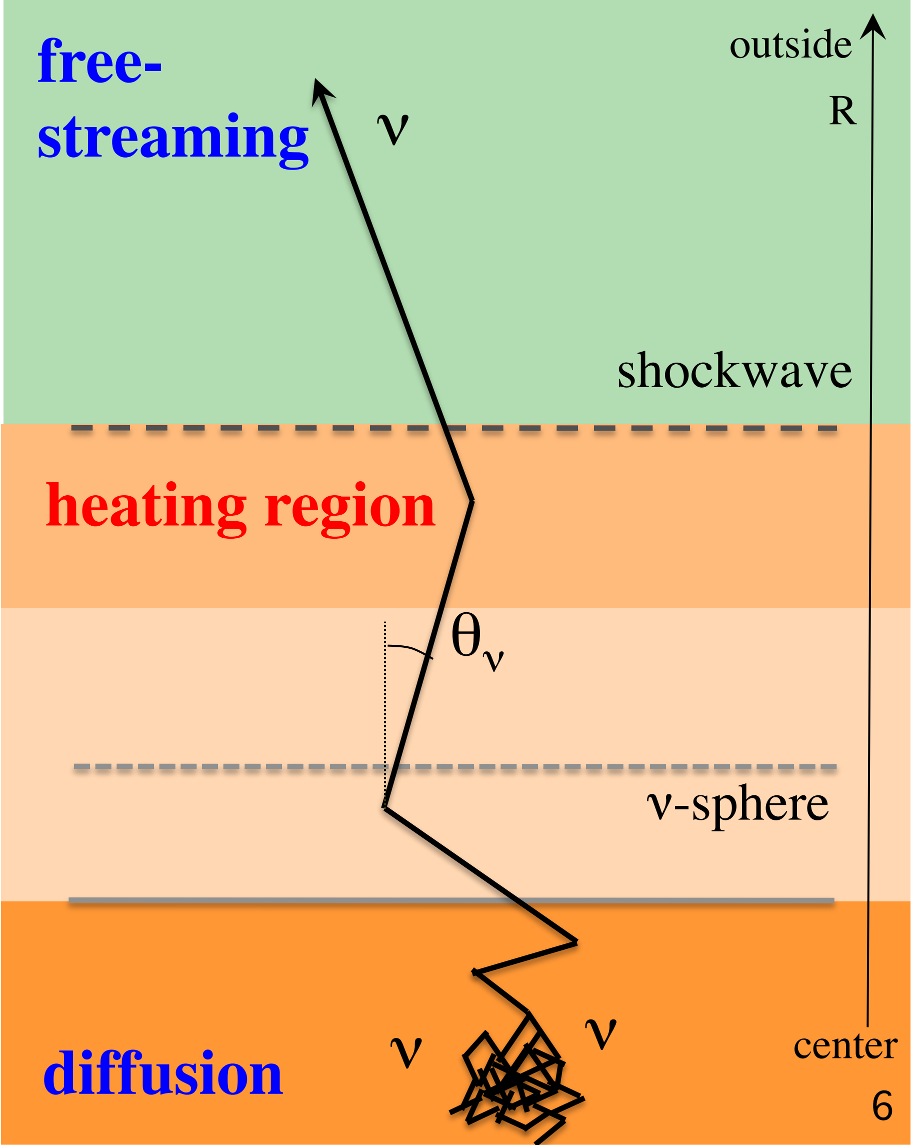
ν-heating rate Janka A&A (1996)

$$Q_\nu^i \approx 110 \frac{MeV}{s \cdot N} \left(\frac{L_\nu E_\nu^2}{R_7^2 < \mu >} X_i \right)$$

average energy, flux: E_ν, L_ν

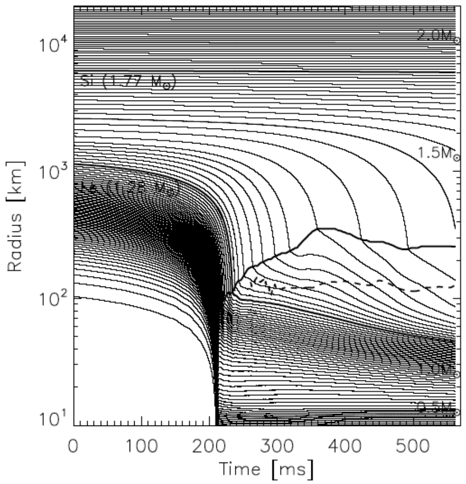
flux factor: $< \mu > = < \cos\theta_\nu > = 0 \sim 1$

between diffusion & free-streaming



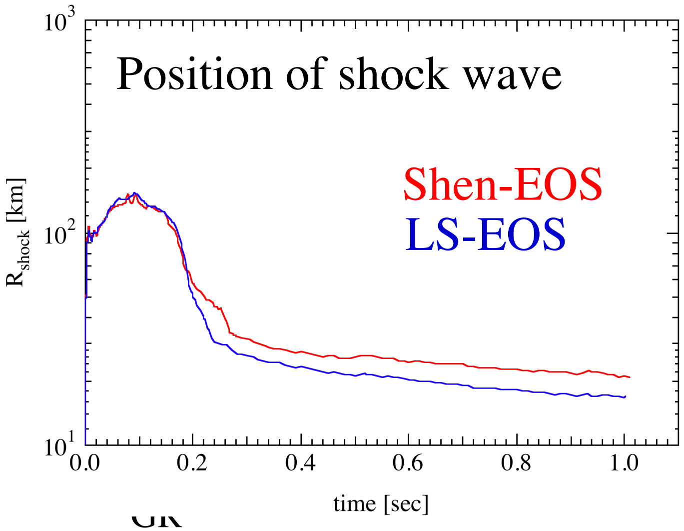
Achievement of ν-transfer in 1D (spherical)

- Shift from approximate to exact methods
  - Light bulb, Leakage → Diffusion approximations
  - Exact 1D transfer (Moment formalism, Boltzmann solver)
- First principle calculation in spherical symmetry 2000~
  - General relativistic ν-radiation hydrodynamics
- No explosion in spherical symmetry



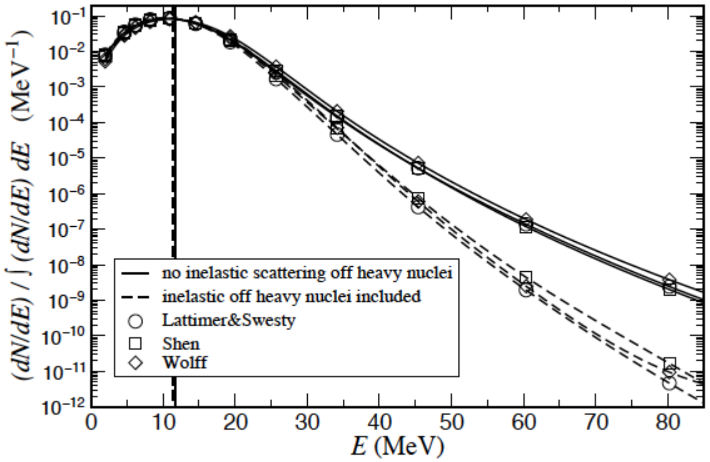
$15M_{solar}$ Rampp-Janka (2000)

Effects of EOS



$15M_{solar}$ Sumiyoshi et al. (2005)

Effect on ν-signal

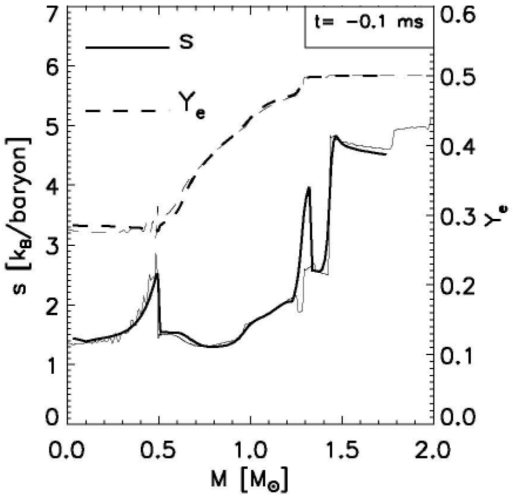


Langanke et al. (2008)

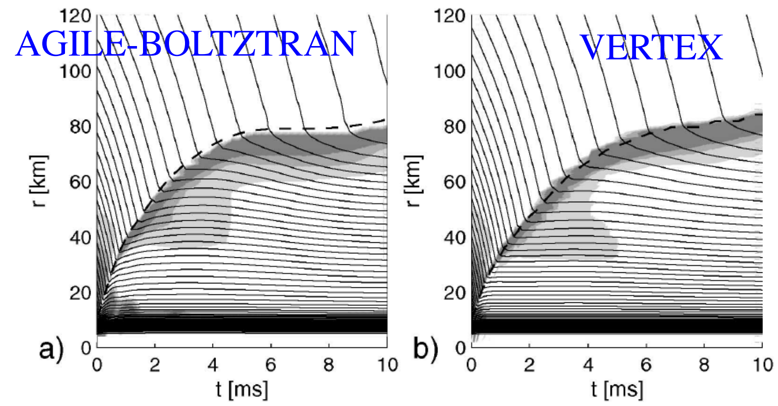
Validation of ν-transfer in 1D (spherical)

- Comparison of methods: check advantage & defects
  - Moment formalism vs Boltzmann solver
  - Boltzmann solver vs Diffusion, Monte Carlo

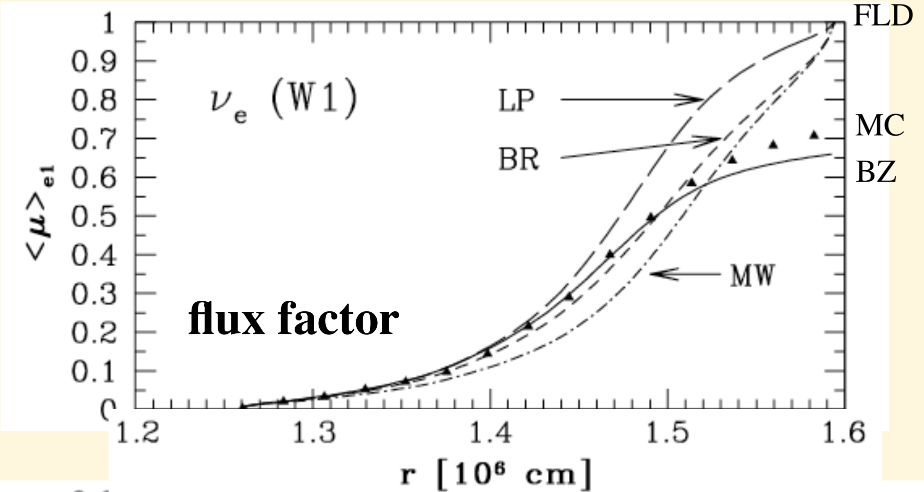
S, Ye-profiles at bounce

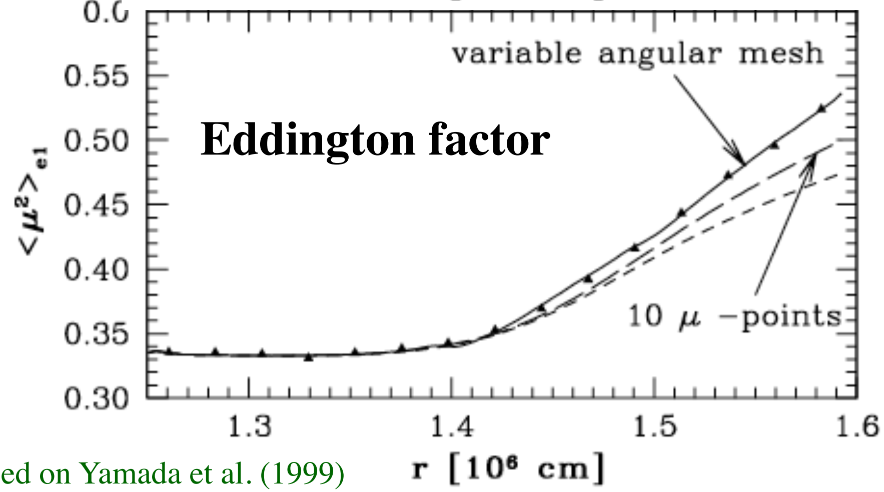


Liebendoerfer et al. (2005)


AGILE-BOLTZTRAN

VERTEX


flux factor


Eddington factor

Based on Yamada et al. (1999)

Ongoing progress of ν-transfer in 2D/3D

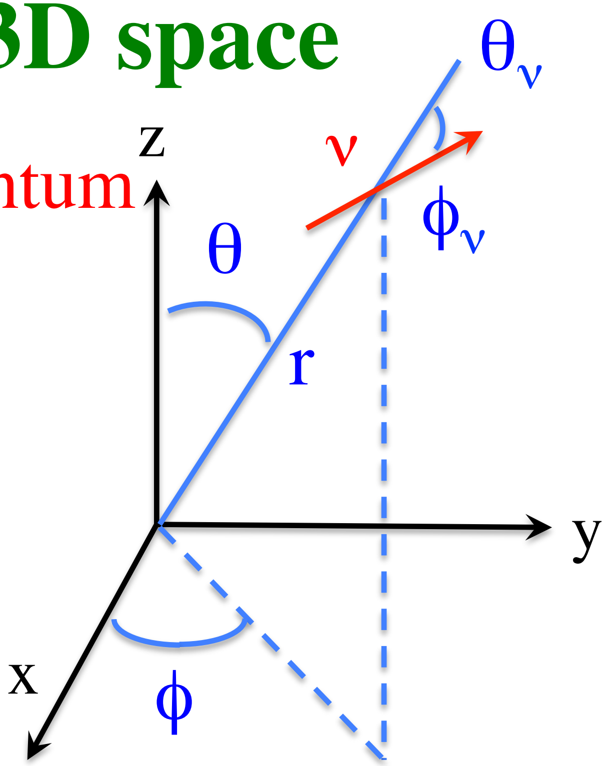
See also Messer, Pan, O'Connor, Richers

- Approximate methods currently state-of-the-art
  - Diffusion/IDSA, Moment method with closure relation
  - Ray-by-ray (along radial transport, moment/diffusion)
- Full evaluations of ν-transfer challenging
  - Monte Carlo, Moment method *Cardall, Shibata, Richers, Kuroda, Just*
  - Boltzmann in 5D, Post bounce 2D dynamics *Ott et al. (2008)*
  - Boltzmann in 6D, Fixed 3D profiles *Sumiyoshi (2012)*
- Examine approximations, core-collapse
  - Check and improve approximate methods
    - Diffusion in 2D *(Ott, Brandt)* Ray-by-ray in 3D *(Sumiyoshi 2015)*
  - Sp. Rel. Boltzmann in 5D + Hydrodynamics 2D
    - with Lorentz boosts in collision term *Nagakura (2014)*

Solving neutrino transfer in 3D space

How to handle 6D Boltzmann eq.

Solve neutrino transfer in 3D space



- Work in 6D: 3D space + 3D ν-momentum

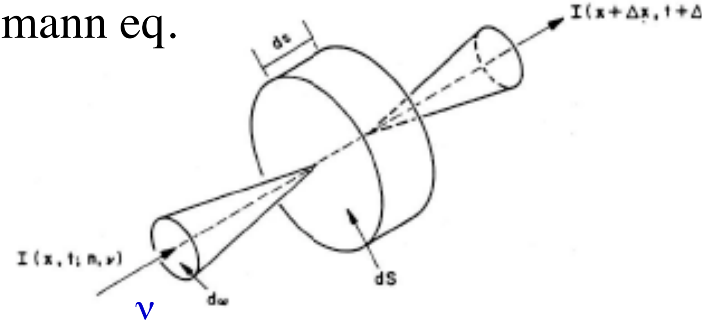
$$f_\nu(r,\theta,\phi;\ \varepsilon_\nu,\theta_\nu,\phi_\nu;\ t)$$

  - Neutrino energy (ε_ν), angle (θ_ν, ϕ_ν)

- Time evolution of 6D-distribution

$$\frac{1}{c}\frac{\partial f_\nu}{\partial t} + \vec{n}\cdot\vec{\nabla} f_\nu = \frac{1}{c}\left(\frac{\delta f_\nu}{\delta t}\right)_{collision}$$

Boltzmann eq.



  - Left: Neutrino number change
  - Right: Change by neutrino reactions

- Energy, angle-dependent reactions
  - Compositions in dense matter (EOS table)

Levels of ν-transfer: angle moments

- Boltzmann eq. $$\frac{1}{c}\frac{\partial f_\nu}{\partial t}+\vec{n}\cdot\vec{\nabla}f_\nu=\frac{1}{c}\left(\frac{\delta f_\nu}{\delta t}\right)_{collision}$$

  → Direct solve in 6D

Integration by angle to reduce computational cost

- 0th moment $$\frac{1}{c}\frac{\partial E_\nu}{\partial t}+\frac{1}{c}\nabla\cdot\vec{F}_\nu=Q_\nu^0$$

  → Diffusion approximation $\vec{F}_\nu=-D\nabla E_\nu$

  Flux limiter D

- 1st moment $$\frac{1}{c}\frac{\partial \vec{F}_\nu}{\partial t}+c\nabla\cdot\vec{P}_\nu=\vec{Q}_\nu^1$$

  → Closure relation $P_\nu^{ij}=T^{ij}E_\nu$

  Eddington factor T^{ij}

Boltzmann eq. in spherical coordinate

Pomraning, Mihalas^2, Castor

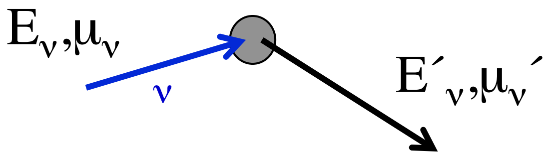
$$\frac{1}{c}\frac{\partial f_\nu}{\partial t}+\frac{\mu_\nu}{r^2}\frac{\partial}{\partial r}(r^2 f_\nu)+\frac{\sqrt{1-\mu_\nu^2}\cos\phi_\nu}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f_\nu)+\frac{\sqrt{1-\mu_\nu^2}\sin\phi_\nu}{r\sin\theta}\frac{\partial f_\nu}{\partial\phi}$$

$$+\frac{1}{r}\frac{\partial}{\partial\mu_\nu}[(1-\mu_\nu^2)f_\nu]+\frac{\sqrt{1-\mu_\nu^2}\cos\theta}{r\sin\theta}\frac{\partial}{\partial\phi_\nu}(\sin\phi_\nu f_\nu)=\frac{1}{c}\left(\frac{\delta f_\nu}{\delta t}\right)_{collision}$$

- Discrete in conservative form (S_n method)

Sumiyoshi & Yamada, ApJS (2012)

- Implicit method in time
  - Stability, equilibrium, time step
  - Stiff eq.: different time scales
- Collision term for ν-reactions



Multi-energy, angle

$$\frac{1}{c}\left(\frac{\delta f_\nu}{\delta t}\right)_{collision}=j_{emission}(1-f_\nu)-\frac{1}{\lambda_{absoption}}f_\nu+C_{inelastic}[\int f_\nu(E'_\nu,\mu'_\nu)dE'_\nu]+\ldots$$

$\mu_\nu=\cos\theta_\nu$

- absorption, emission, scattering and pair process…

Microphysics in supernova core

- Neutrino reactions via weak interactions

- **Emission & absorption:**

$e^- + p \leftrightarrow \nu_e + n$ $\quad$ $e^- + A \leftrightarrow \nu_e + A'$

$e^+ + n \leftrightarrow \bar{\nu}_e + p$

- **Scattering:**

$\nu_i + N \leftrightarrow \nu_i + N$ $\quad$ $\nu_i + A \leftrightarrow \nu_i + A$

$\nu_i + e \leftrightarrow \nu_i + e$

- **Pair-process:**

$e^- + e^+ \leftrightarrow \nu_i + \bar{\nu}_i$

$N + N \leftrightarrow N + N + \nu_i + \bar{\nu}_i$

Multi-species:

$\nu_e, \bar{\nu}_e, \nu_{\mu/\tau}, \bar{\nu}_{\mu/\tau}$

Bruenn (1985), Burrows (2006)

Energy-dependent

$\sigma_n(E_\nu) \sim \sigma_0 E_\nu^2$

at $E_\nu \sim 10\text{-}100$ MeV

At wide ρ, T, Y_i

$\rho \sim 10^5\text{-}10^{15}$ g/cm^3

$T \sim 0\text{-}10^{11}$ K

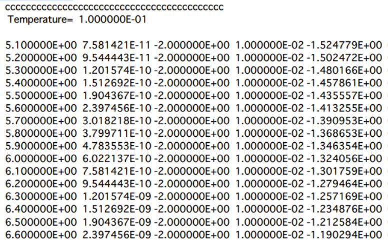
$Y_i \sim 0\text{-}0.5$

- Equation of state of dense matter: nuclear physics

- Composition: proton, neutron, nuclei, e^-, e^+
- Thermodynamics: chemical potentials pressure, entropy,…

Lattimer-Swesty (1993), Shen (1998, 2011)

EOS data tables:



```
ccccccccccccccccccccccccccccccccccccccccccc
 Temperature=  1.000000E-01

5.100000E+00  7.581421E-11 -2.000000E+00  1.000000E-02 -1.524779E+00
5.200000E+00  9.544443E-11 -2.000000E+00  1.000000E-02 -1.502472E+00
5.300000E+00  1.201574E-10 -2.000000E+00  1.000000E-02 -1.480166E+00
5.400000E+00  1.512692E-10 -2.000000E+00  1.000000E-02 -1.457861E+00
5.500000E+00  1.904367E-10 -2.000000E+00  1.000000E-02 -1.435557E+00
5.600000E+00  2.397456E-10 -2.000000E+00  1.000000E-02 -1.413255E+00
5.700000E+00  3.018218E-10 -2.000000E+00  1.000000E-02 -1.390953E+00
5.800000E+00  3.799711E-10 -2.000000E+00  1.000000E-02 -1.368653E+00
5.900000E+00  4.783553E-10 -2.000000E+00  1.000000E-02 -1.346354E+00
6.000000E+00  6.022137E-10 -2.000000E+00  1.000000E-02 -1.324056E+00
6.100000E+00  7.581421E-10 -2.000000E+00  1.000000E-02 -1.301759E+00
6.200000E+00  9.544443E-10 -2.000000E+00  1.000000E-02 -1.279464E+00
6.300000E+00  1.201574E-09 -2.000000E+00  1.000000E-02 -1.257169E+00
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6.600000E+00  2.397456E-09 -2.000000E+00  1.000000E-02 -1.190294E+00
```

http://user.numazu-ct.ac.jp/~sumi/eos

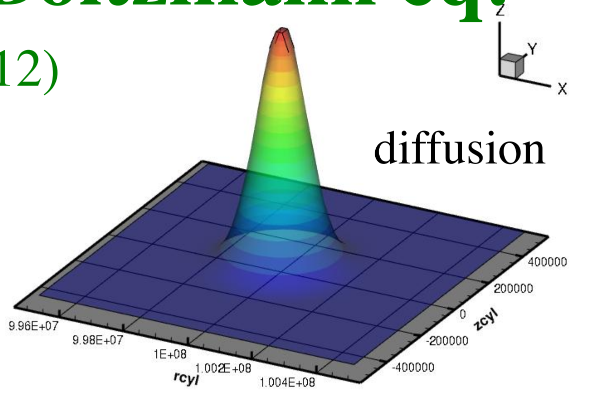
Validation of code to solve 6D Boltzmann eq.

Sumiyoshi & Yamada ApJS (2012)

- Analytic approach
  - Gaussian packet, free-streaming
  - Formal solution by integral
  - Approach to equilibrium solution
- Comparison with 1D-code
  - Fixed background
  - Density, flux, moments
  - Reaction rates, mean free path

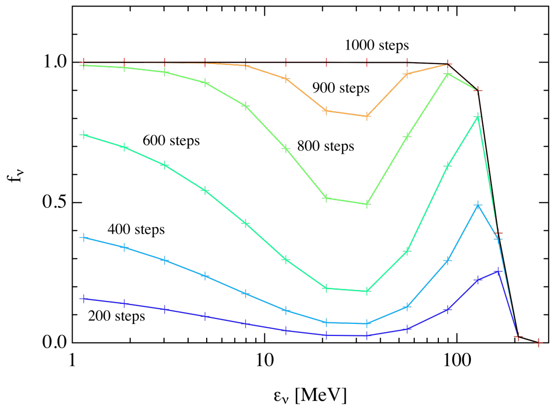
Utilize 1D code of GR ν-radiation hydrodynamics

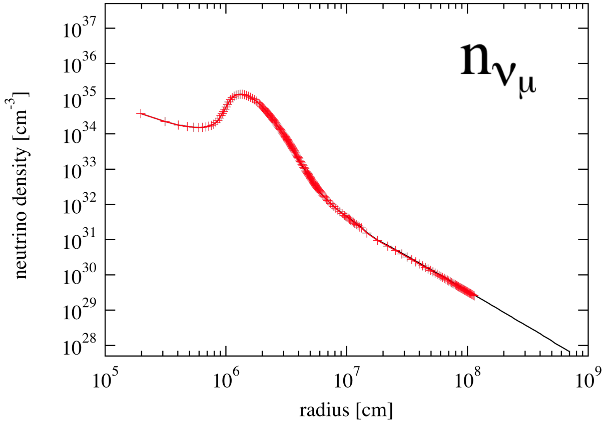
Sumiyoshi et al. ApJ (2005)



diffusion

equlibrium





6D Boltzmann solver works indeed

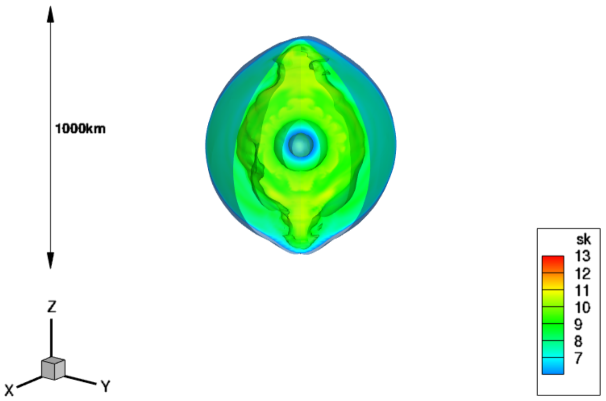
Applications to 3D supernova cores

Examine Ray-by-ray approximation
Closure relation for moment methods

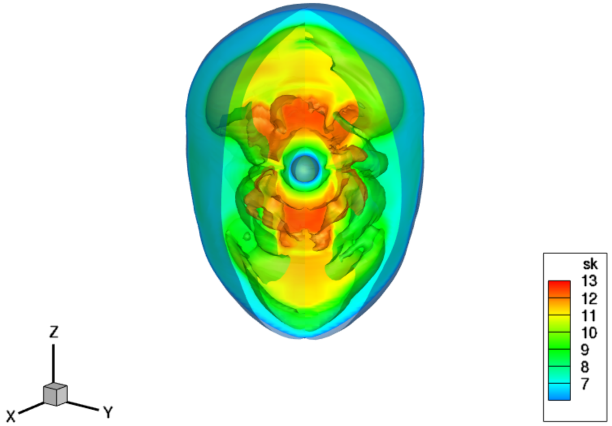
Systematic study using 3D SN core

Sumiyoshi et al. ApJS (2015)

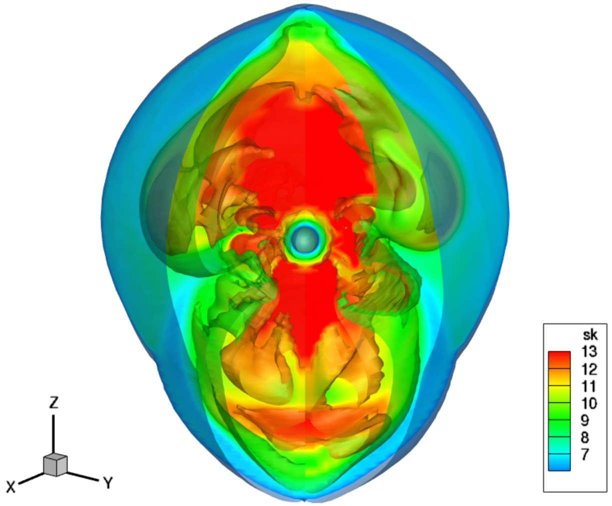
11.2M_{sun} Explosion



100 ms

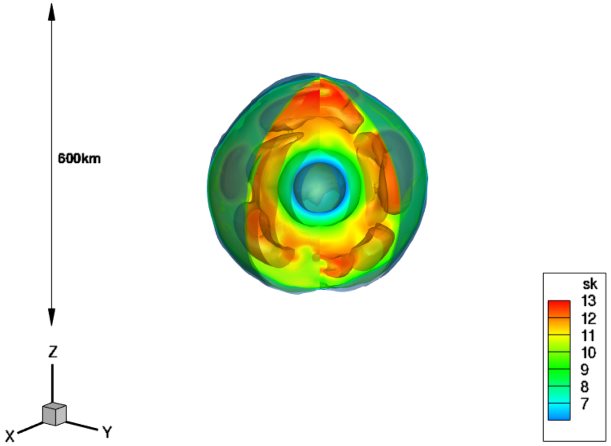


150 ms

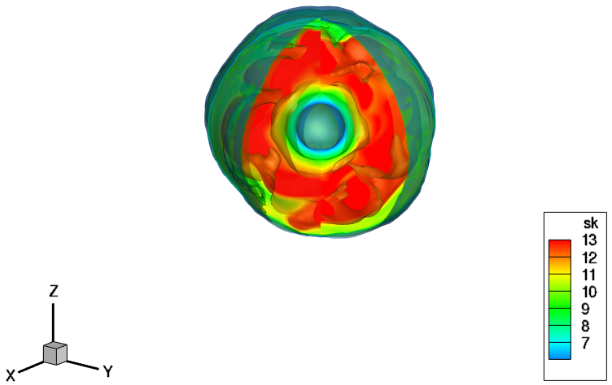


200 ms

27.0M_{sun} Non explosion



150 ms



200 ms

Takiwaki et al. (2012, 2014)

6D Boltzmann with fixed background

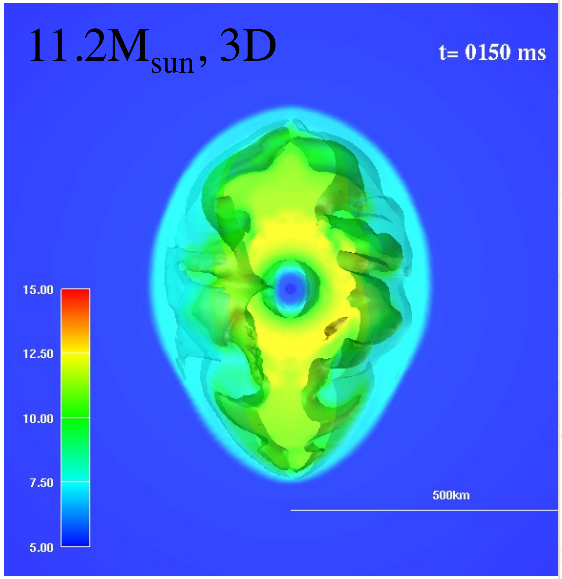
Sumiyoshi et al. ApJS (2015)

Fix hydro. variables, solve time evolution by 6D Boltzmann eq.

- Evaluate stationary state of the neutrino distributions in 6D
- Study neutrino transfer in 3D, heating rates, angle moments

3D supernova core at 150ms

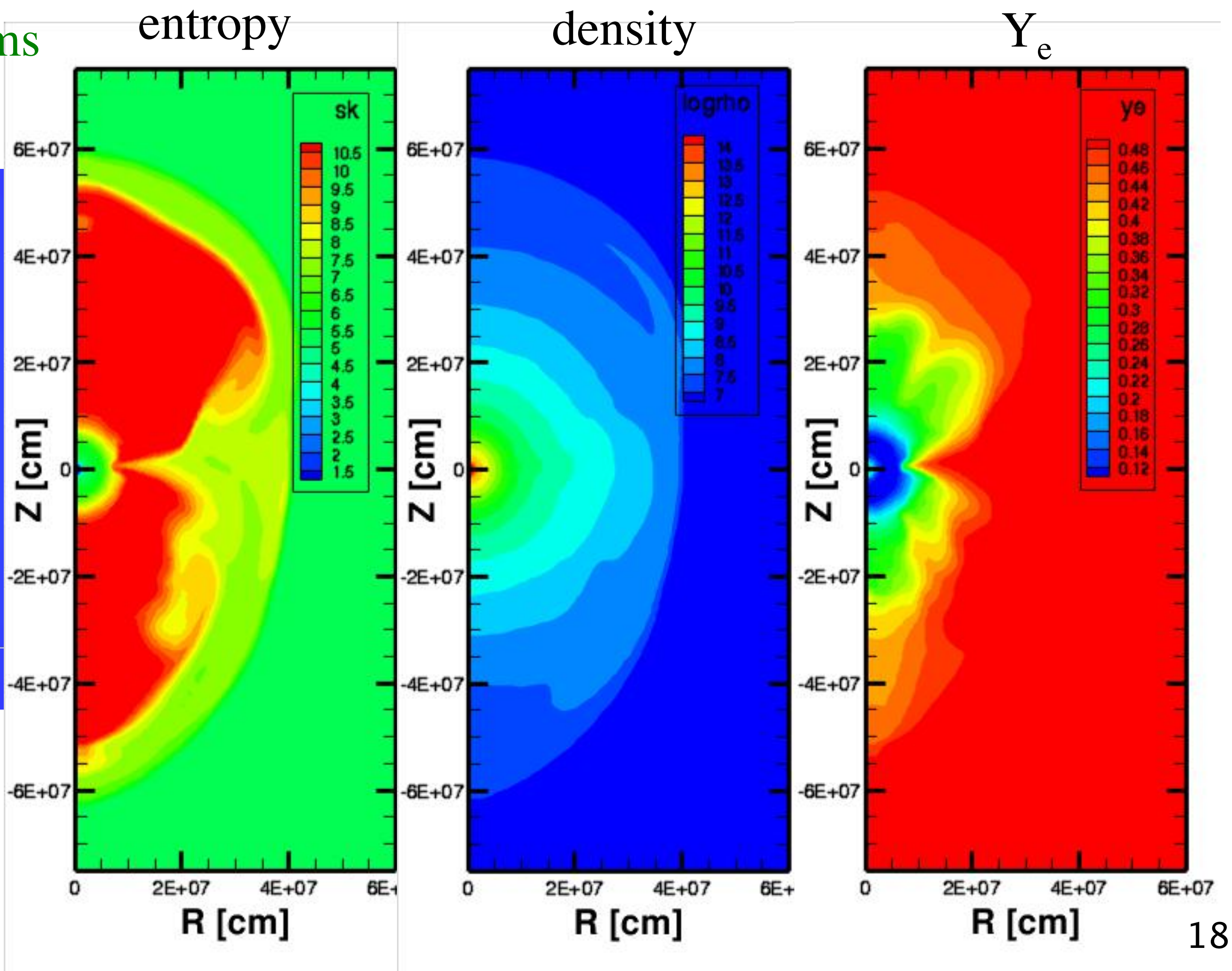
entropy



150 msec after bounce

R_{shock}~250-400km

From Takiwaki et al. ApJ (2012)



Comparison with approximate method

- Full-Boltzmann 6D:

$$\frac{1}{c}\frac{\partial f_\nu}{\partial t} + \frac{\mu_\nu}{r^2}\frac{\partial}{\partial r}(r^2 f_\nu) + \frac{\sqrt{1-\mu_\nu^2}\cos\phi_\nu}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f_\nu) + \frac{\sqrt{1-\mu_\nu^2}\sin\phi_\nu}{r\sin\theta}\frac{\partial f_\nu}{\partial\phi}$$

$$+\frac{1}{r}\frac{\partial}{\partial\mu_\nu}[(1-\mu_\nu^2)f_\nu] + \frac{\sqrt{1-\mu_\nu^2}\cos\theta}{r\sin\theta}\frac{\partial}{\partial\phi_\nu}(\sin\phi_\nu f_\nu) = \frac{1}{c}\left(\frac{\delta f_\nu}{\delta t}\right)_{collision}$$

- Ray-by-ray approximation: Switch OFF non-radial advection

$$\frac{1}{c}\frac{\partial f_\nu}{\partial t} + \frac{\mu_\nu}{r^2}\frac{\partial}{\partial r}(r^2 f_\nu) + \cancel{\frac{\sqrt{1-\mu_\nu^2}\cos\phi_\nu}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta f_\nu)} + \cancel{\frac{\sqrt{1-\mu_\nu^2}\sin\phi_\nu}{r\sin\theta}\frac{\partial f_\nu}{\partial\phi}}$$

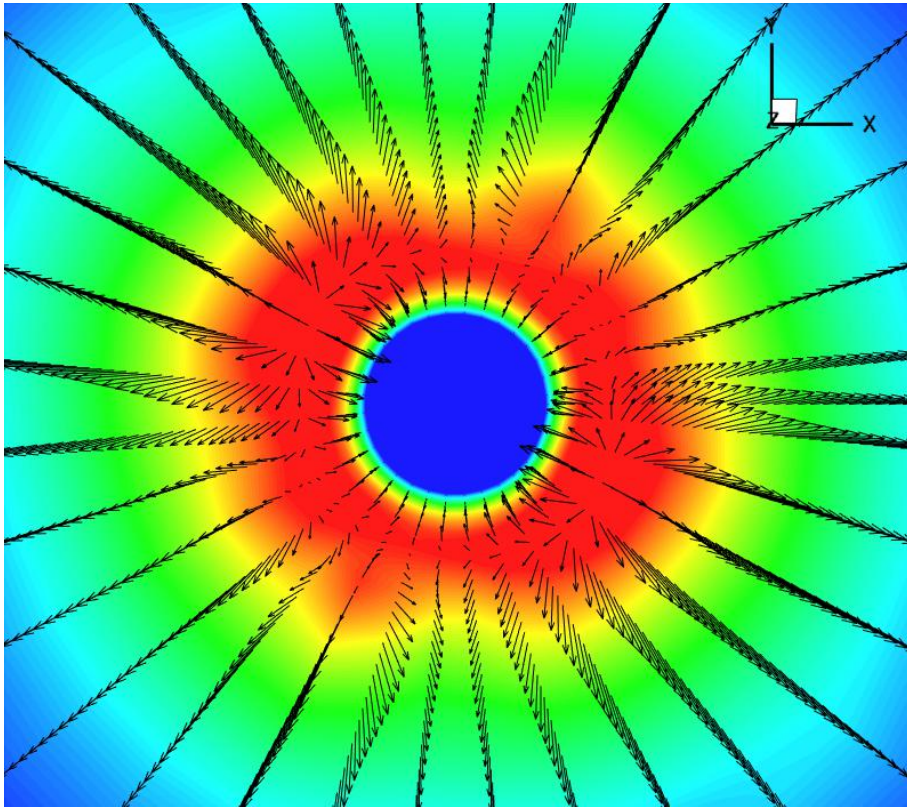
$$+\frac{1}{r}\frac{\partial}{\partial\mu_\nu}[(1-\mu_\nu^2)f_\nu] + \cancel{\frac{\sqrt{1-\mu_\nu^2}\cos\theta}{r\sin\theta}\frac{\partial}{\partial\phi_\nu}}(\sin\phi_\nu f_\nu) = \frac{1}{c}\left(\frac{\delta f_\nu}{\delta t}\right)_{collision}$$

Sumiyoshi et al. (2014)

6D Bolzmann in 3D SN core

Describes non-radial transport

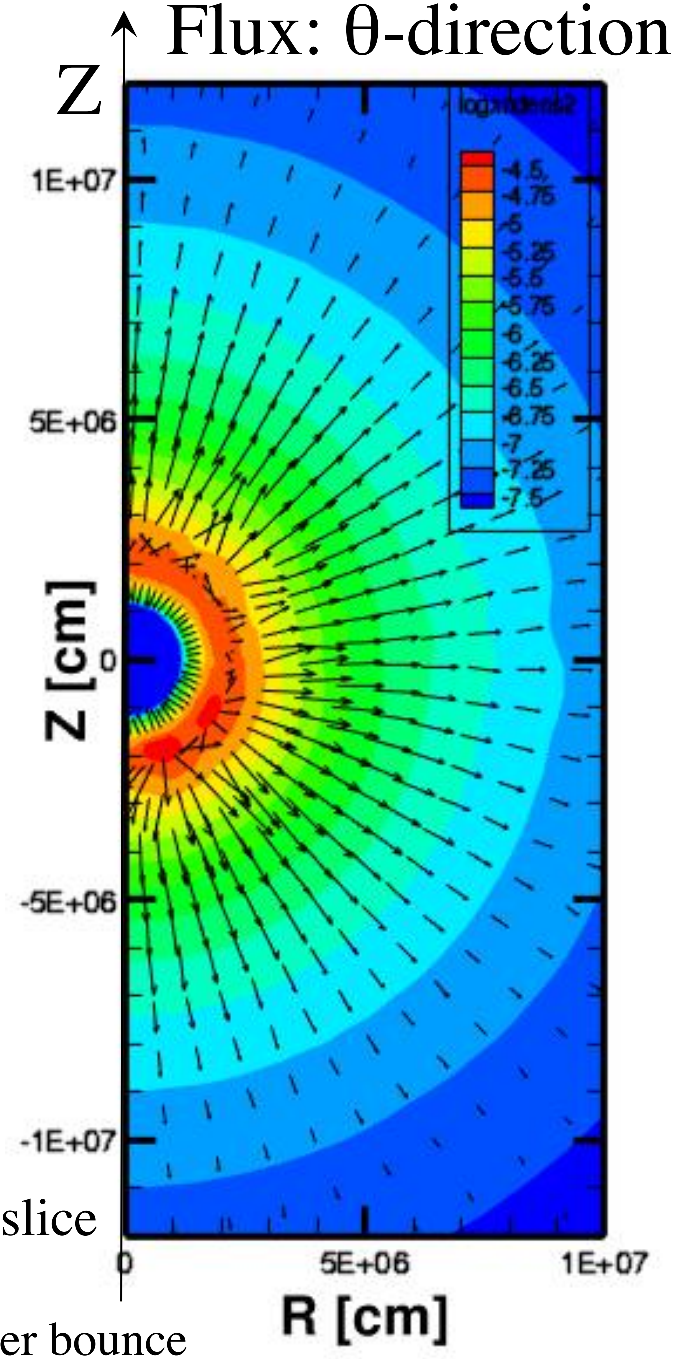
Flux: ϕ-direction



$\bar{\nu}_e$ density: color (flux: arrow)

View from north-pole

Flux: θ-direction



View from side: ϕ-slice

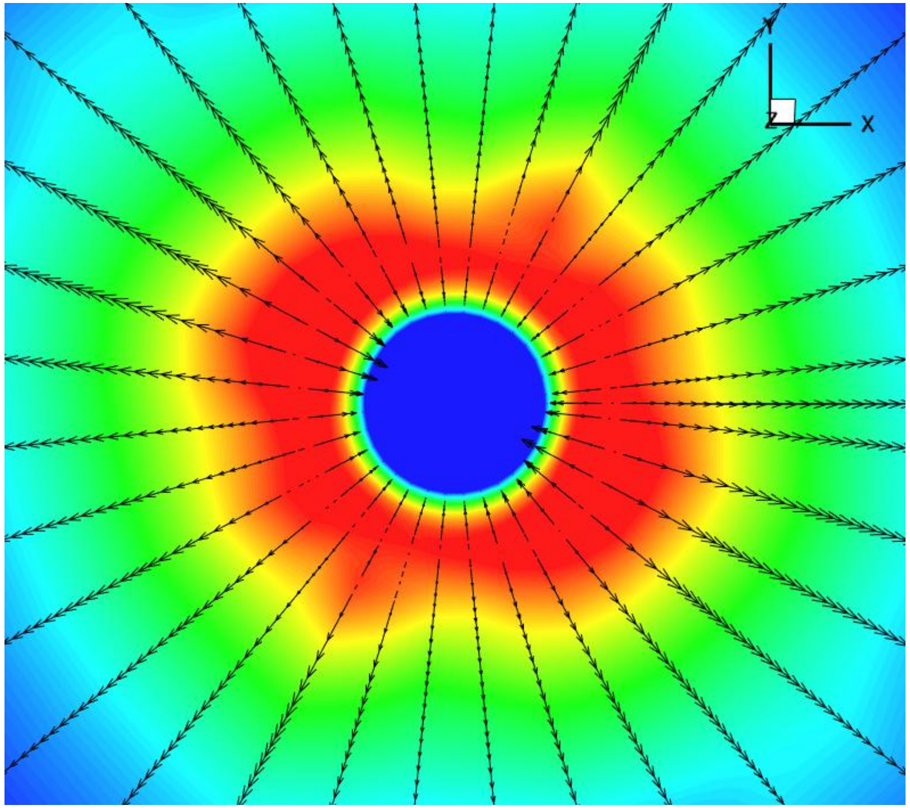
Sumiyoshi et al. (2015)

3D Supernova core 150msec after bounce

Ray-by-ray (RbR) in 3D SN core

Only radial transport

Flux: no ϕ-direction

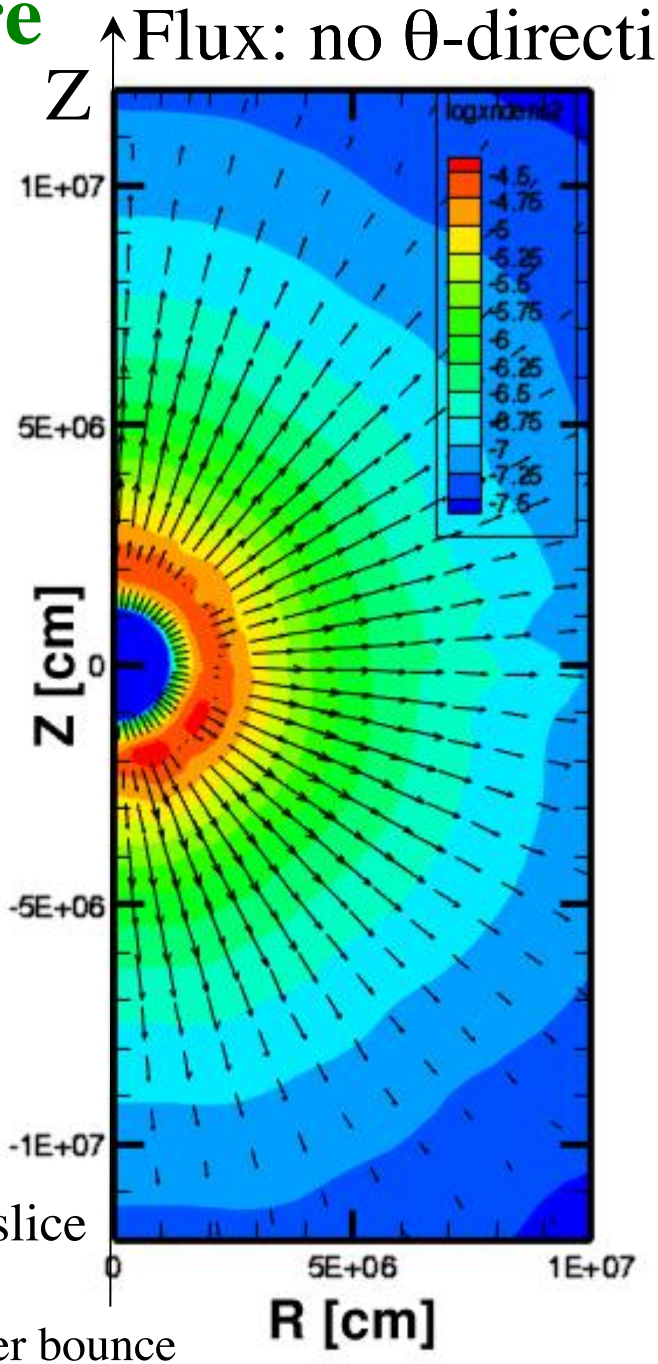


$\bar{\nu}_e$ density: color (flux: arrow)

View from north-pole

View from side: ϕ-slice

Flux: no θ-direction



Sumiyoshi et al. (2015)

3D Supernova core 150msec after bounce

Comparison with approximation

- **Ray-by-ray**

- Only radial transfer

- Anisotropy enhanced

- **6D Boltzmann**

- Non-radial transfer

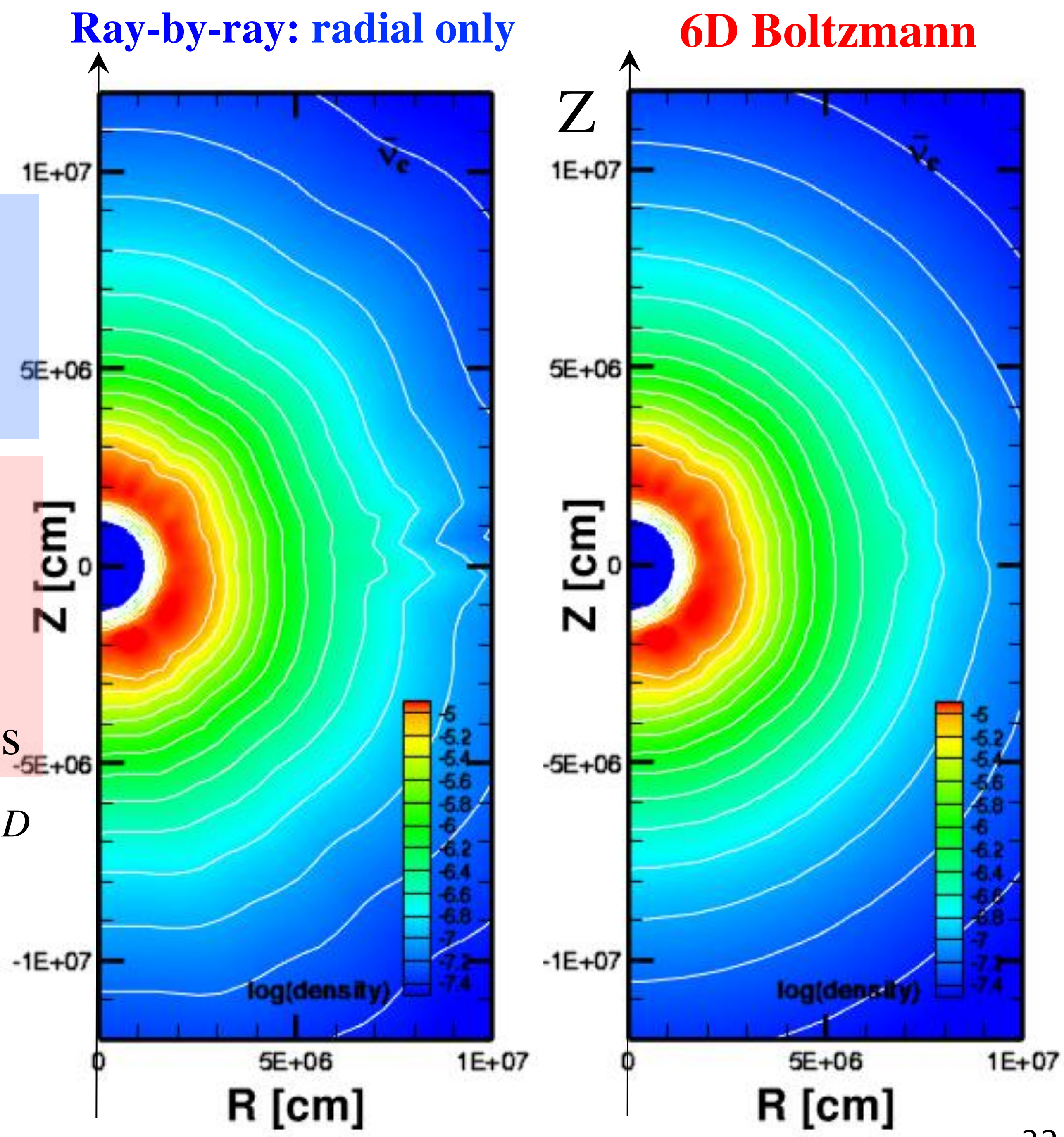
- Integrated values from various directions

Consistent with Ott-Brandt in 2D

$\bar{\nu}_e$ density: color

View from side: ϕ-slice

Sumiyoshi et al. (2015)

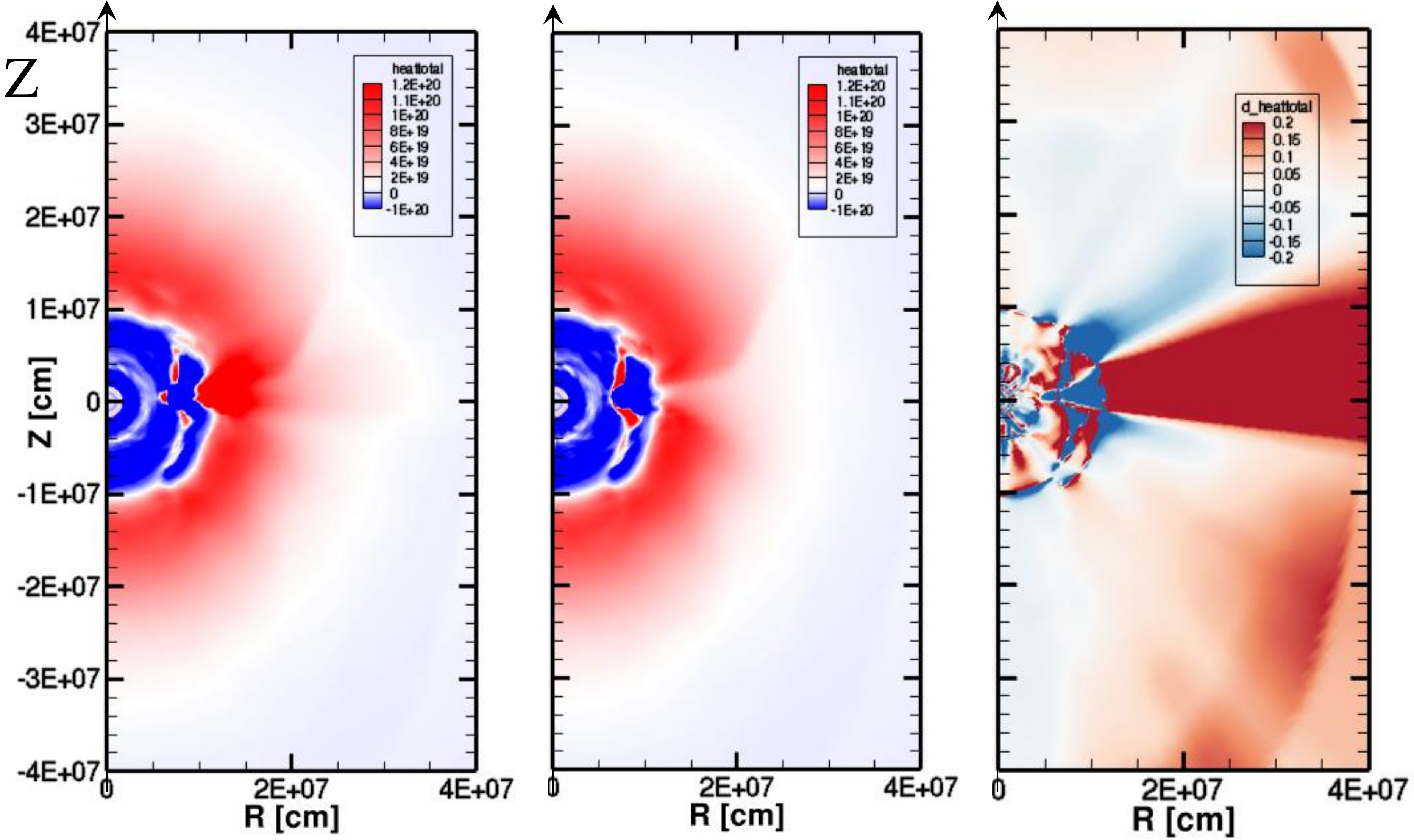


150msec

Comparison: ν-heating rate

$$\delta = \frac{Q_{RbR} - Q_{6D}}{Q_{6D}}$$

Ray-by-ray: radial only **6D Boltzmann** **Deviation of RbR**



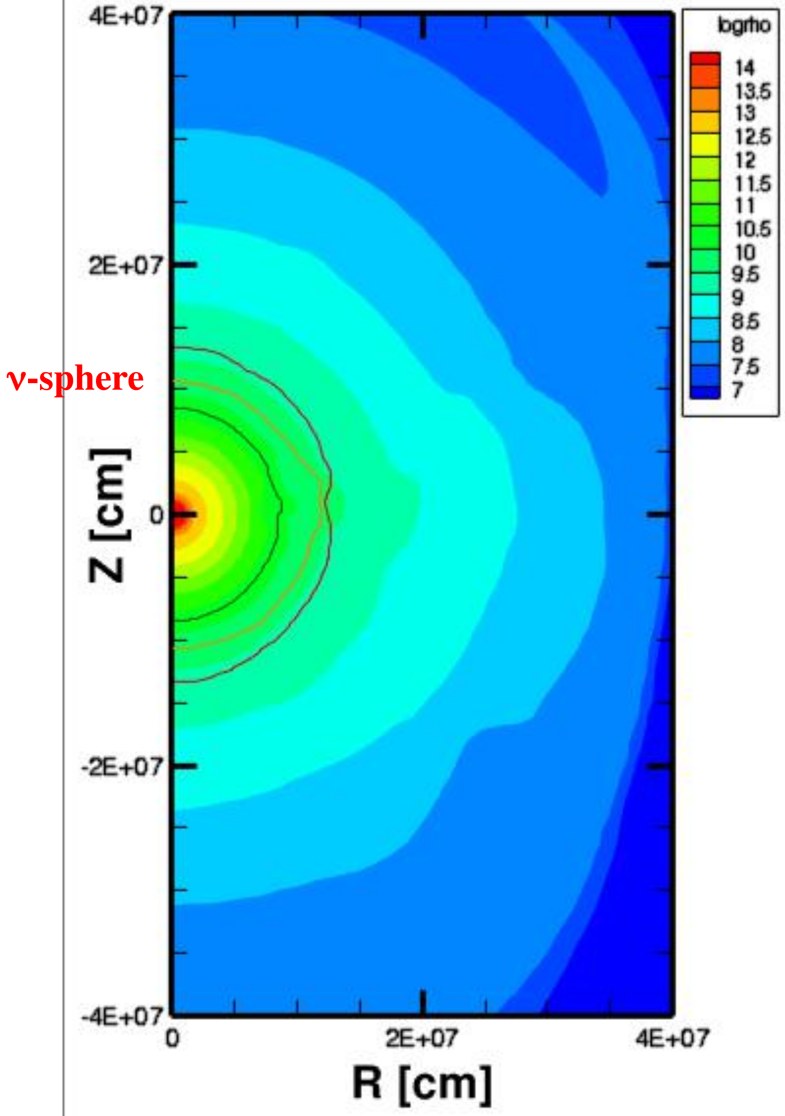
Sumiyoshi et al. (2013,2014)

Red: heating, Blue: cooling

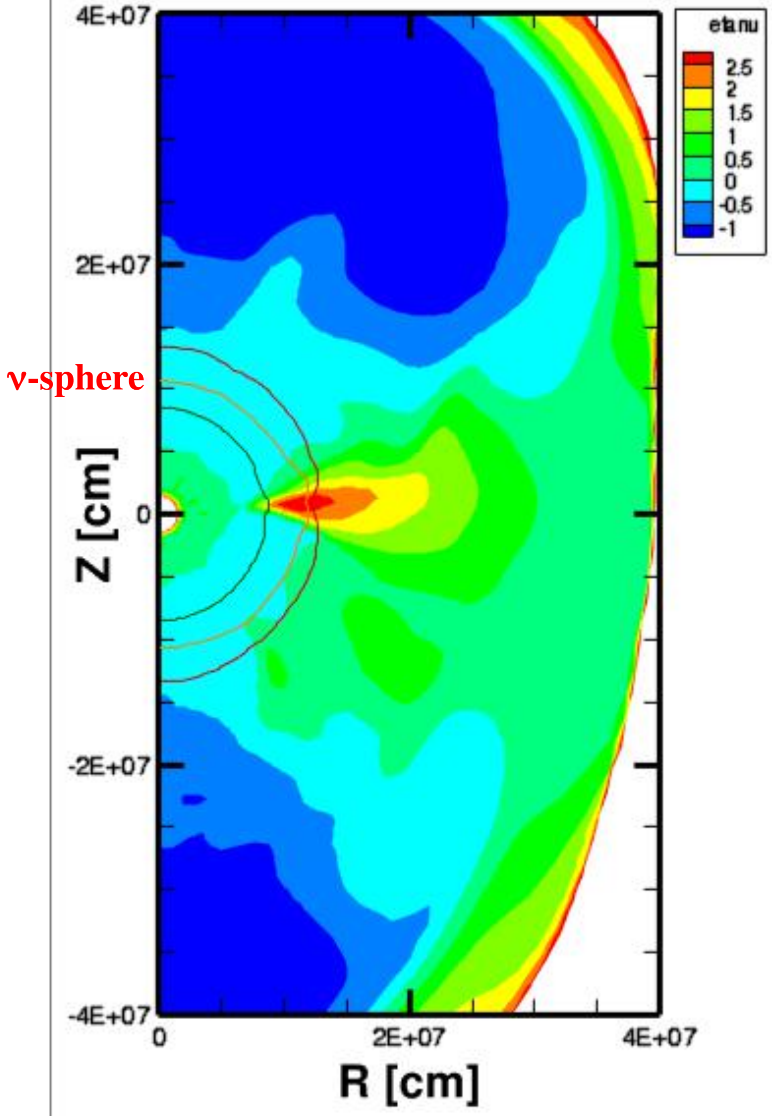
$11M_{sun}$, 150msec

Local fluctuations of neutrino degeneracy: hotspot

density



degeneracy for neutrinos



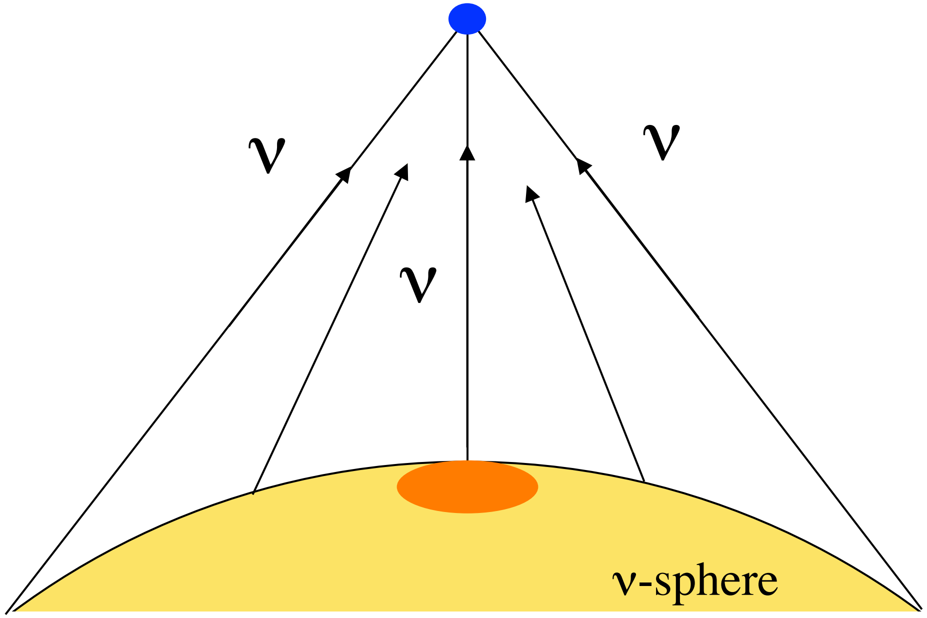
Sumiyoshi et al. (2013,2014)

$11M_{sun}$, 150msec

Evaluation of neutrino fluxes

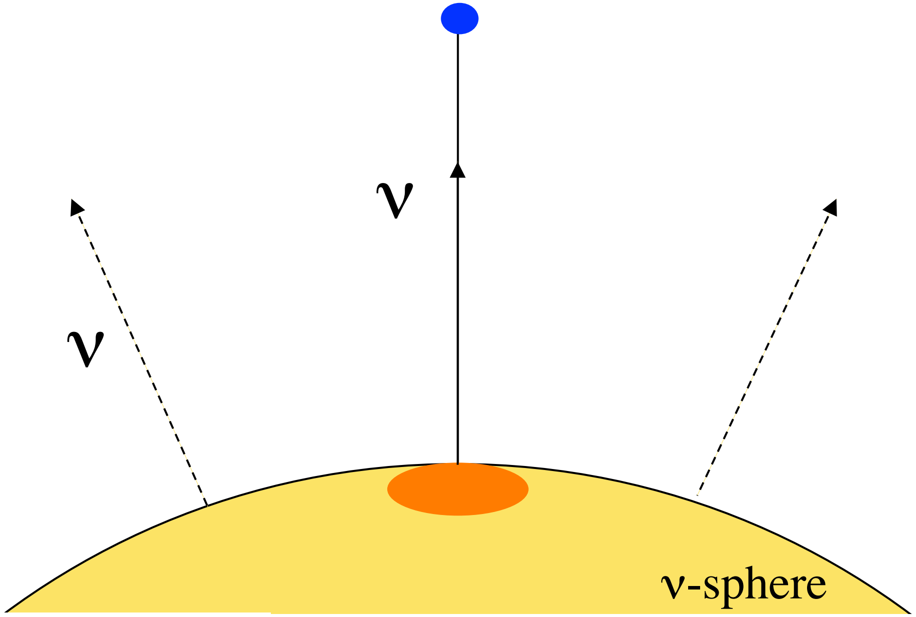
- 6D Boltzmann

Integration from many directions



- Ray-by-ray (RbR)

Contribution from 1 radial direction

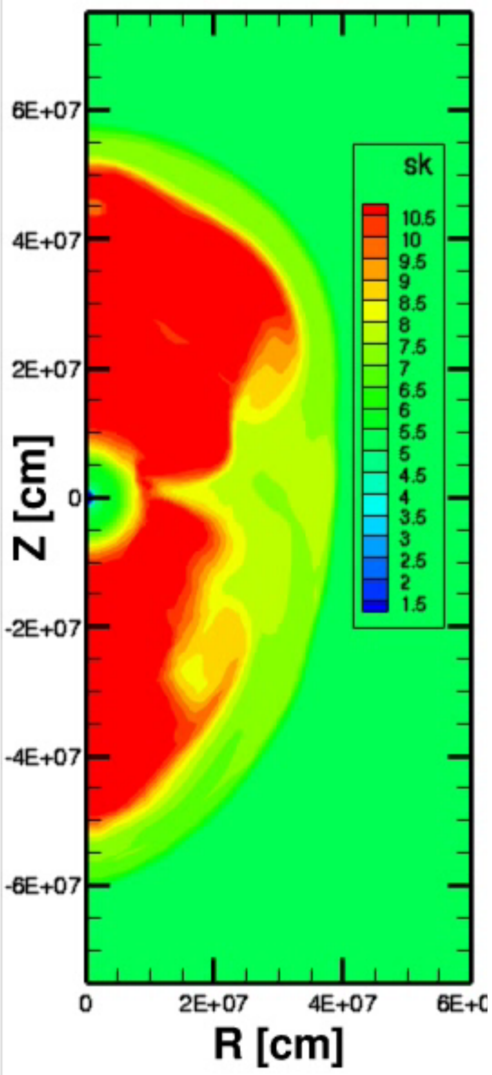


Further study on the approximations is necessary

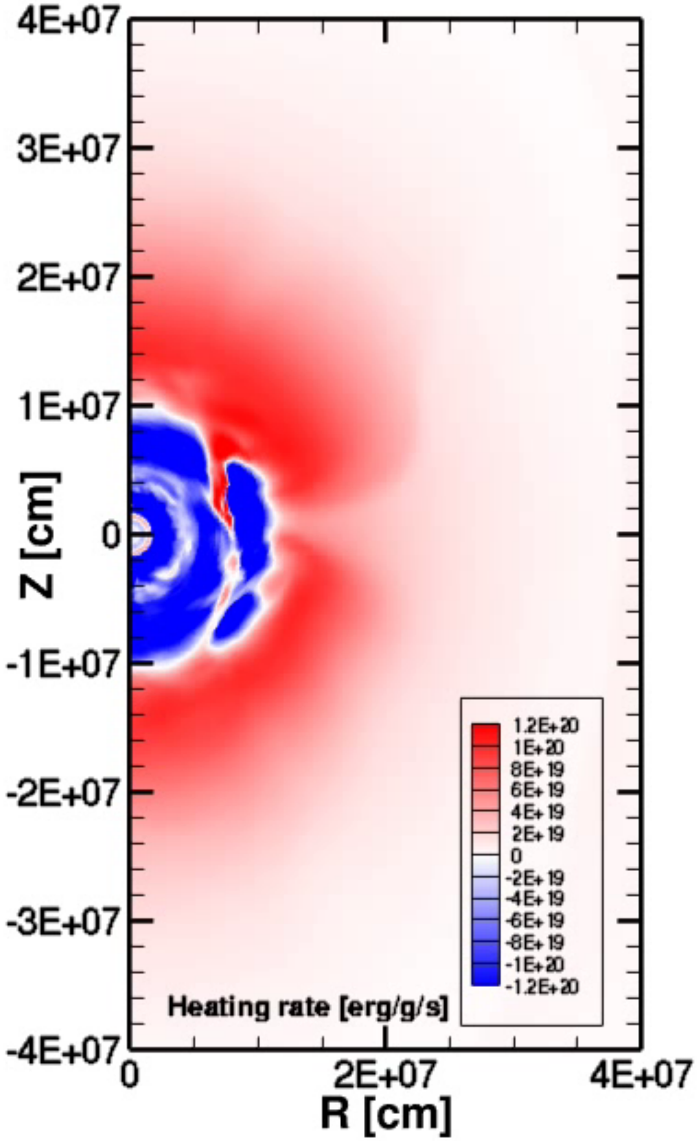
Time evolution: deviation lasts for ~10ms

145ms - 155ms

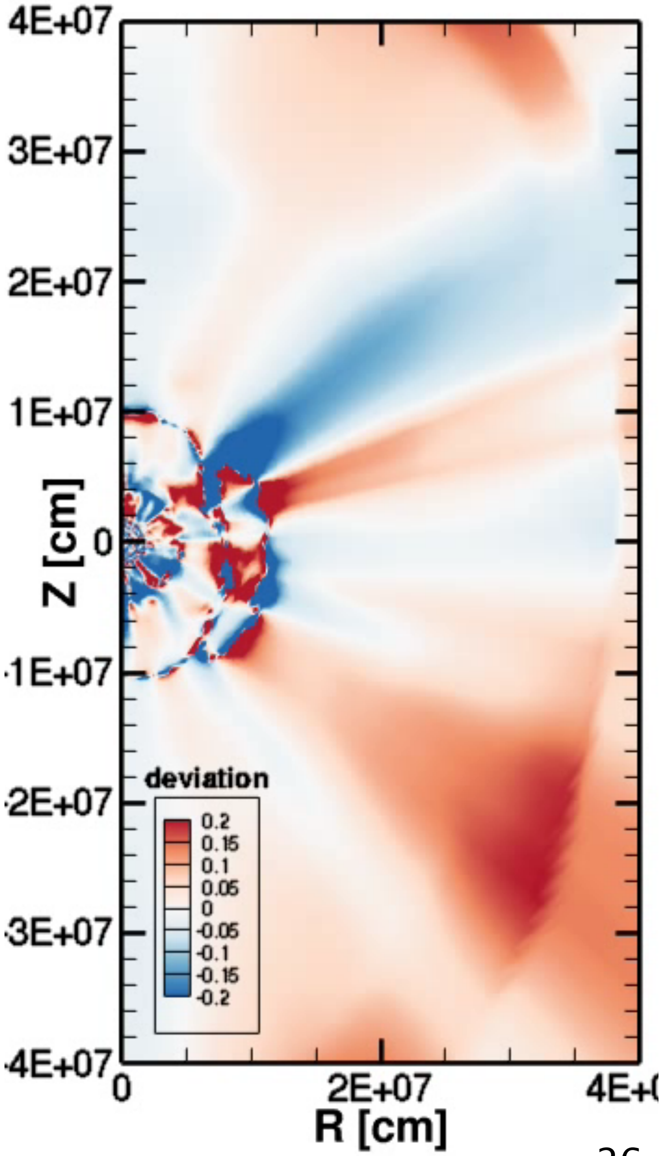
Sumiyoshi et al. (2014)



6D Boltzmann



Deviation of RbR



Providing data for approximate methods

- Comparison with closure for moment formalism
- 6D Boltzmann directly gives pressure tensor

$$P^{ij}(\varepsilon_\nu) = \int d\Omega\, \varepsilon n_i n_j f(\varepsilon,\Omega) \longrightarrow T^{ij}(\varepsilon_\nu) = P^{ij}(\varepsilon_\nu)/E(\varepsilon_\nu)$$

- Closure relation by function of flux vectors

$$\vec{\mathbf{T}} = \vec{\mathbf{I}}(1-\chi)/2 + \mathbf{nn}(3\chi-1)/2 \longrightarrow \vec{\mathbf{P}} = E\vec{\mathbf{T}}$$

$$\mathbf{f} = f\mathbf{n}$$

ex. $\chi = (1/3) + 2f^2/(2+\sqrt{4-3f^2}) = (3+4f^2)/(5+2\sqrt{4-3f^2}).$

Levermore JQSRT (1984)

6D Boltzmann

Eddington Tensor

$$T^{ij}(\varepsilon_\nu) = \frac{P^{ij}(\varepsilon_\nu)}{E(\varepsilon_\nu)}$$

Diagonal radial (Left)

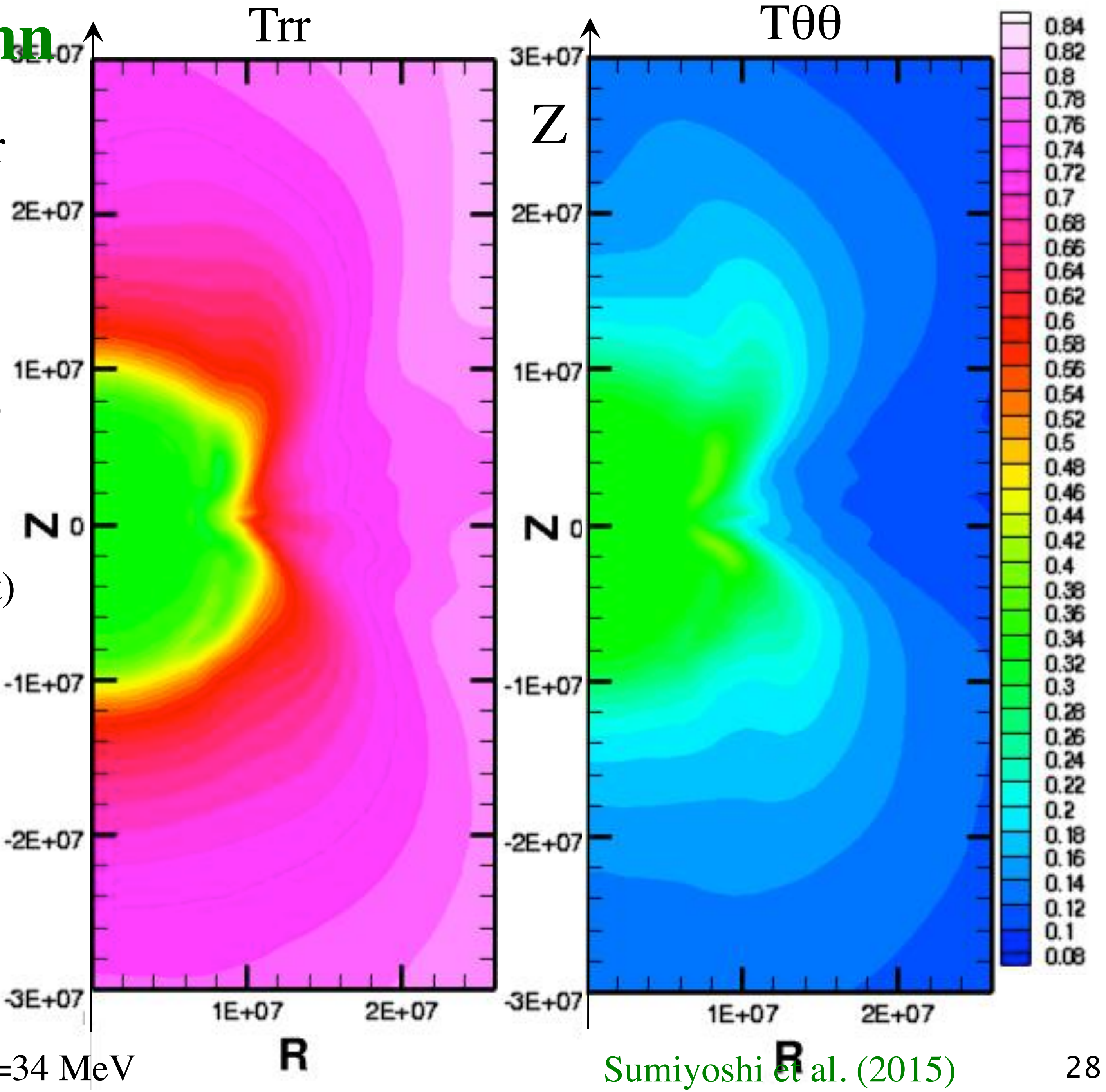
$$T^{rr}(\varepsilon_\nu) : \frac{1}{3} \to 1$$

Diagonal theta (Right)

$$T^{\theta\theta}(\varepsilon_\nu) : \frac{1}{3} \to 0$$

We have also

non-diagonal



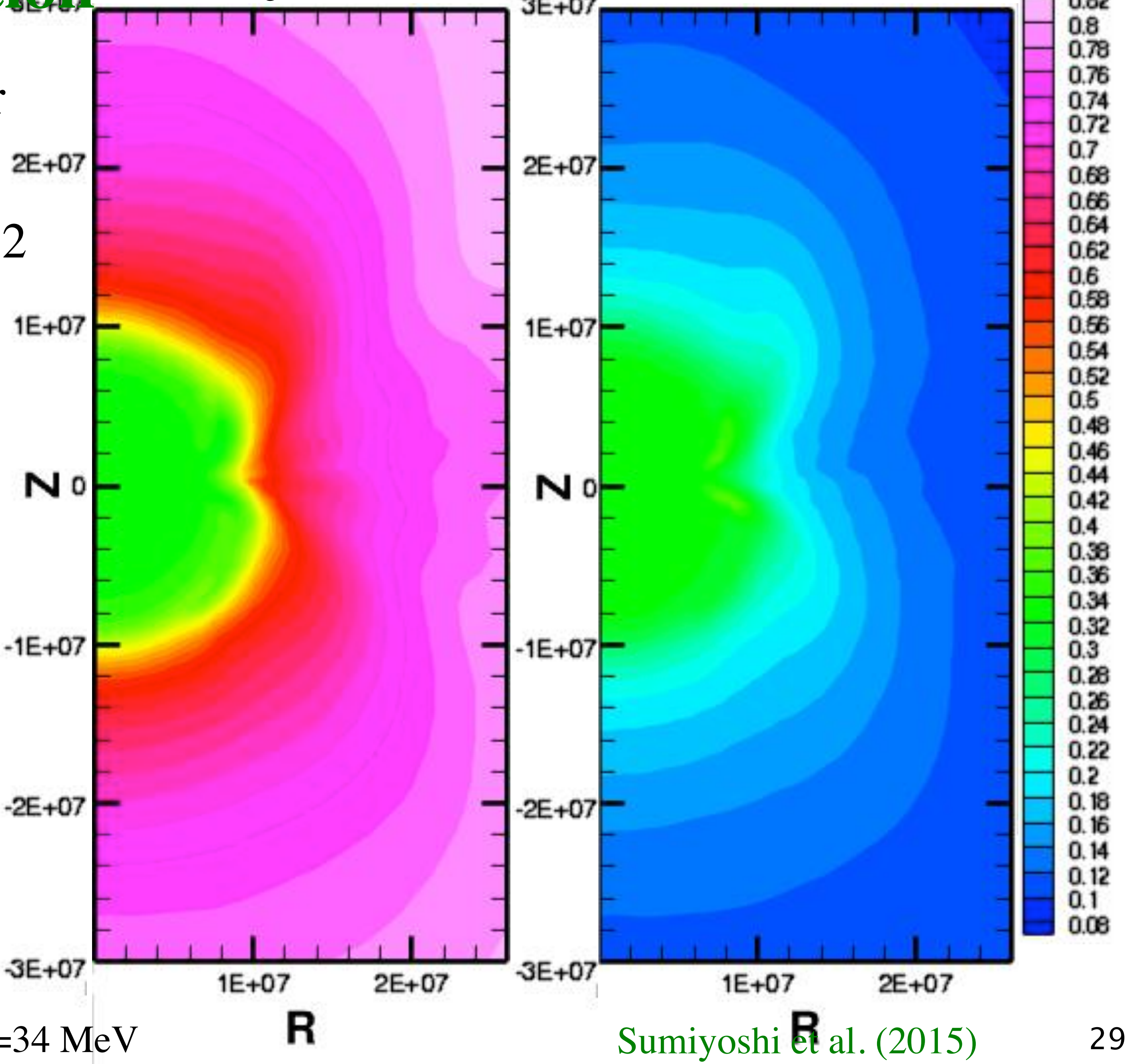
Takiwaki
3D 11.2M, 150msec, E_ν=34 MeV

Sumiyoshi et al. (2015)

Closure relation

Eddington Tensor
(Levermore)

$$T^{ij}(\varepsilon_\nu) = I^{ij}(1-\chi)/2$$
$$+n^i n^j (3\chi - 1)/2$$


ij=rr

ij=θθ

Slight differences
up to 20%

Takiwaki
3D 11.2M, 150msec, E_ν=34 MeV

Sumiyoshi et al. (2015)

Deviation

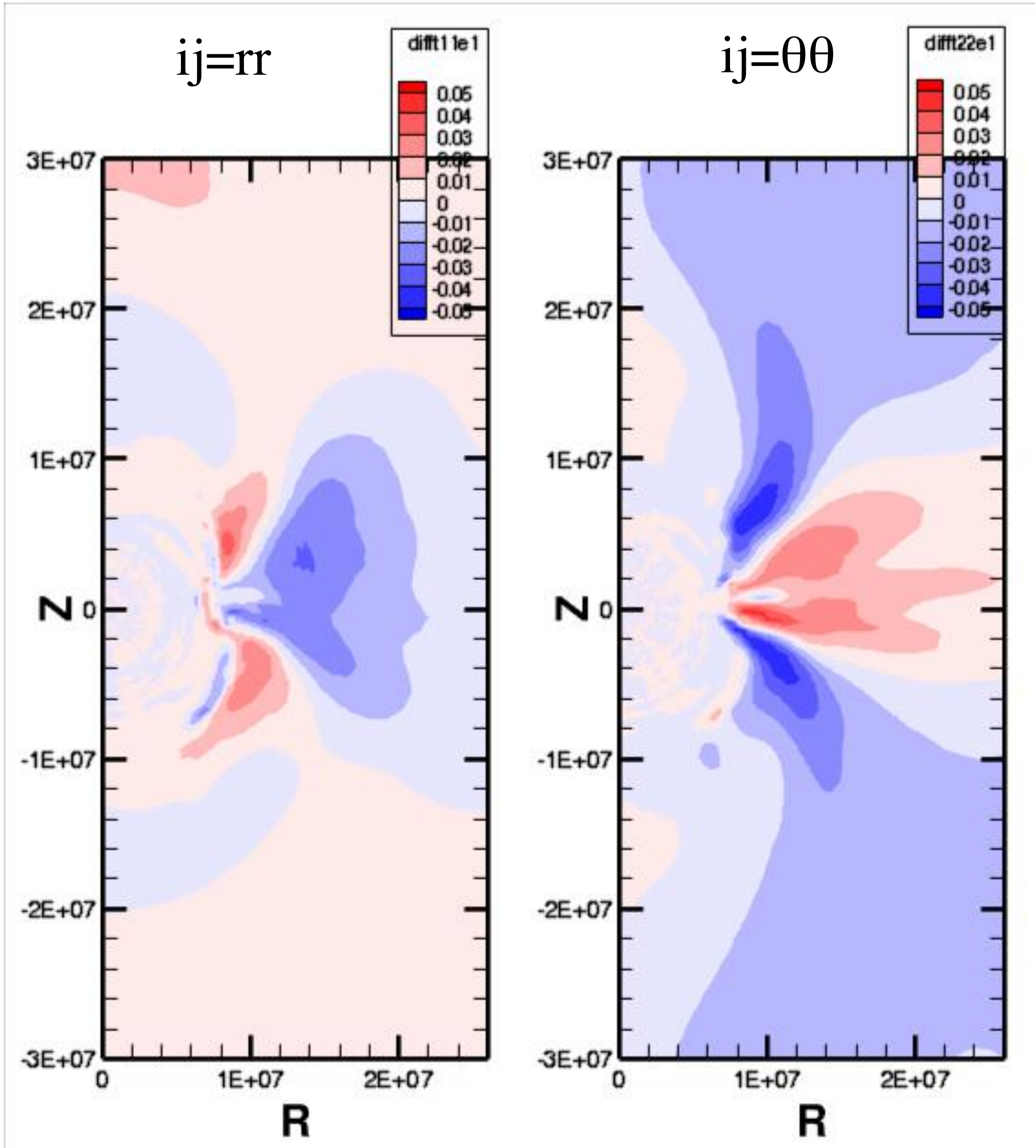
$$\Delta T^{ij} = T^{ij}_{Cl} - T^{ij}_{6D}$$

Closure

$$T^{ij}(\varepsilon_\nu) = I^{ij}(1-\chi)/2$$
$$+n^i n^j (3\chi - 1)/2$$

6D Boltzmann

$$T^{ij}(\varepsilon_\nu) = \frac{P^{ij}(\varepsilon_\nu)}{E(\varepsilon_\nu)}$$



ij=rr

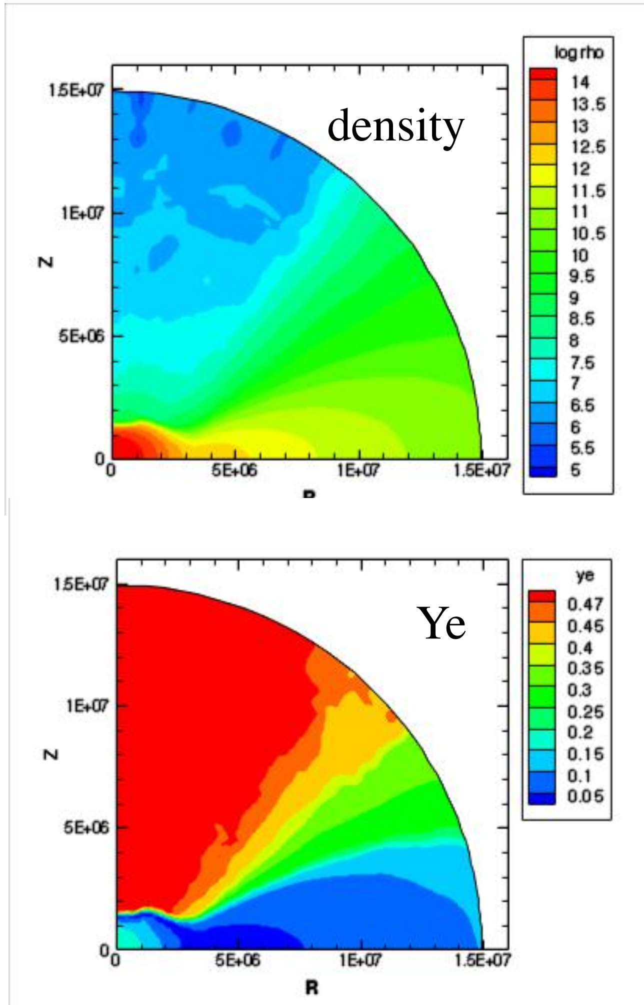
ij=θθ

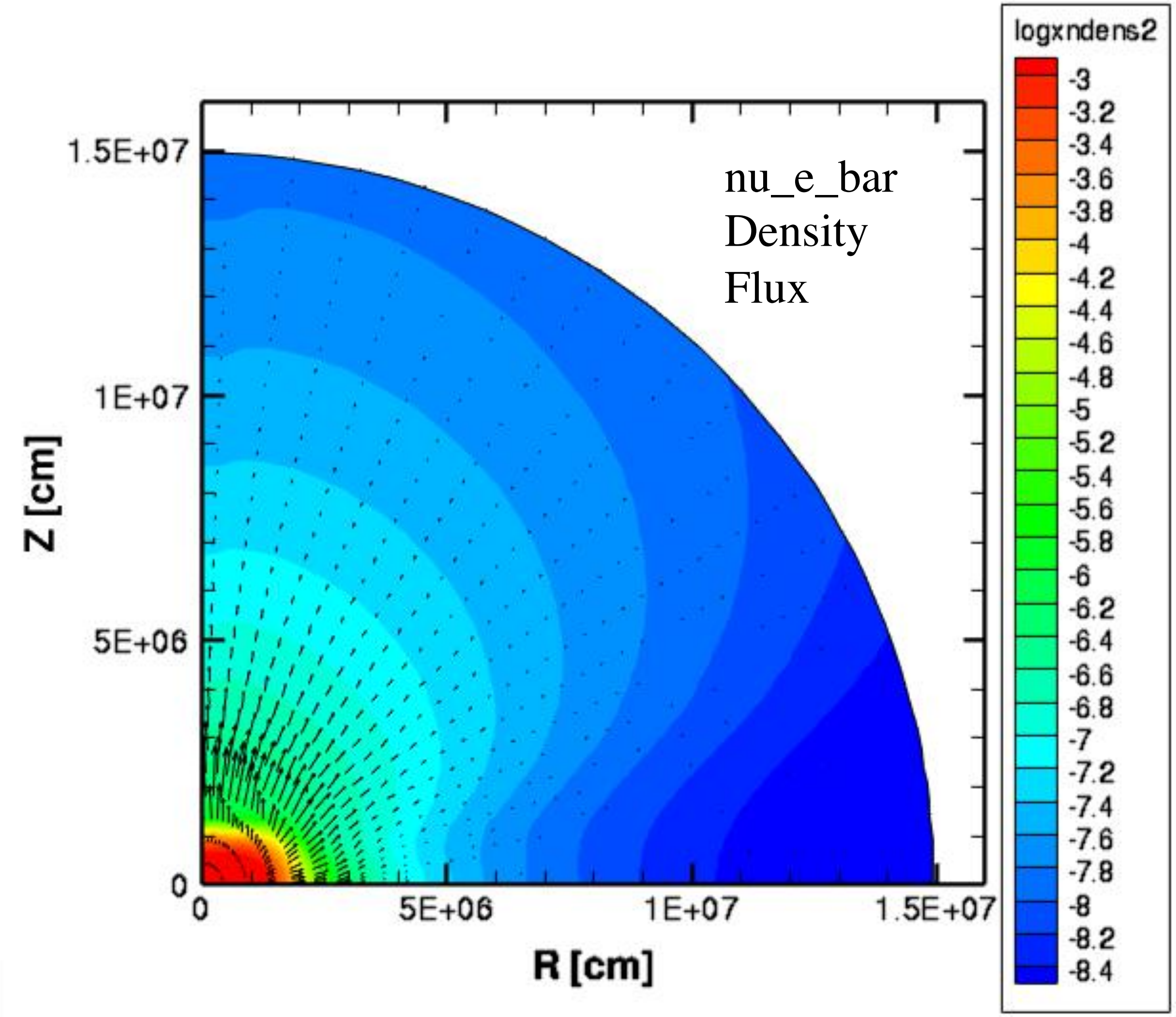
Takiwaki
3D 11.2M, 150msec, E_ν=34 MeV

Sumiyoshi et al. (2015)

6D Boltzmann to examine moments

- Sekiguchi GR simulation: 2D rotating Collapsar



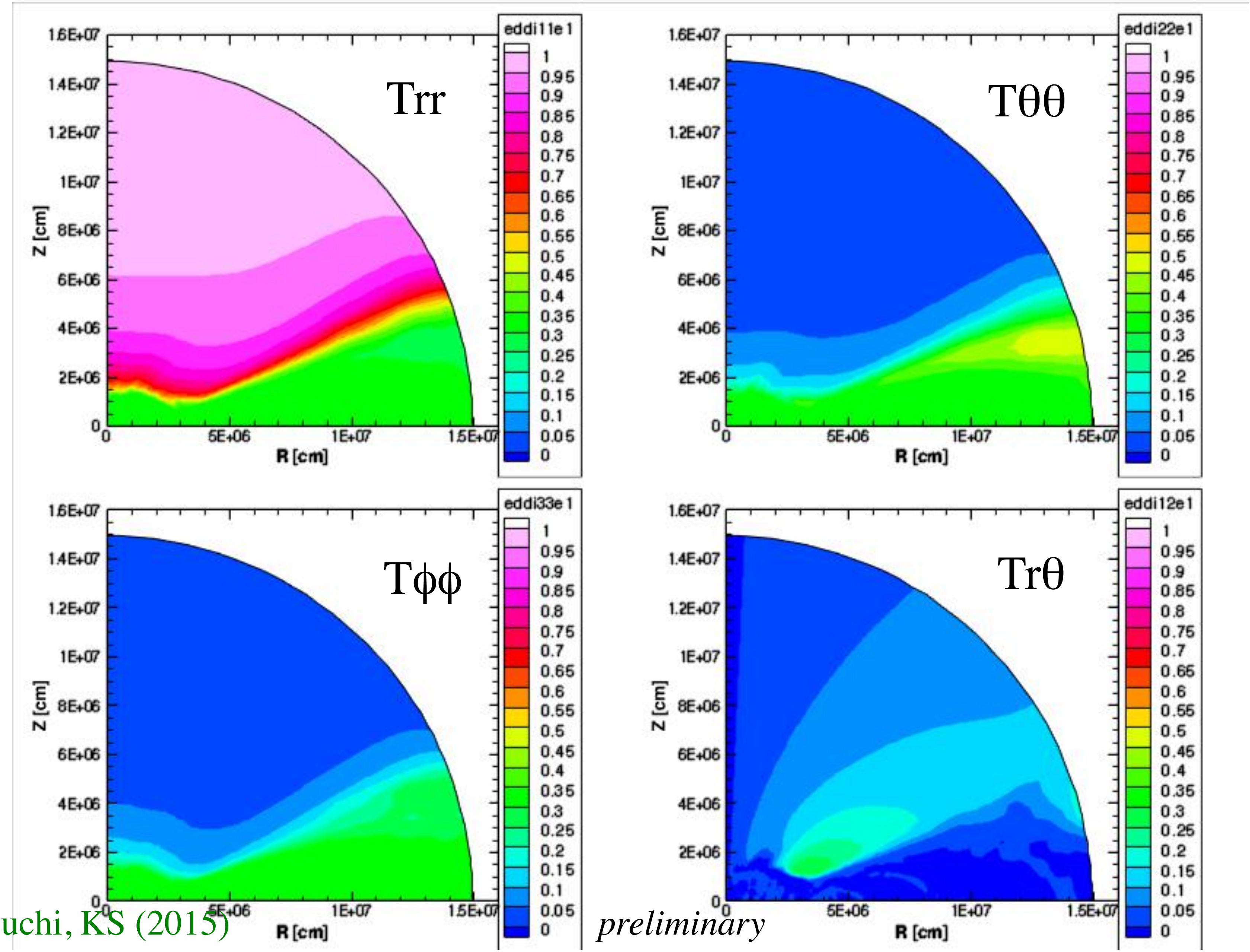


Core of UN08, 100M$_{sun}$, Z$_{solar}$ /200

Sekiguchi, KS (2015)

preliminary

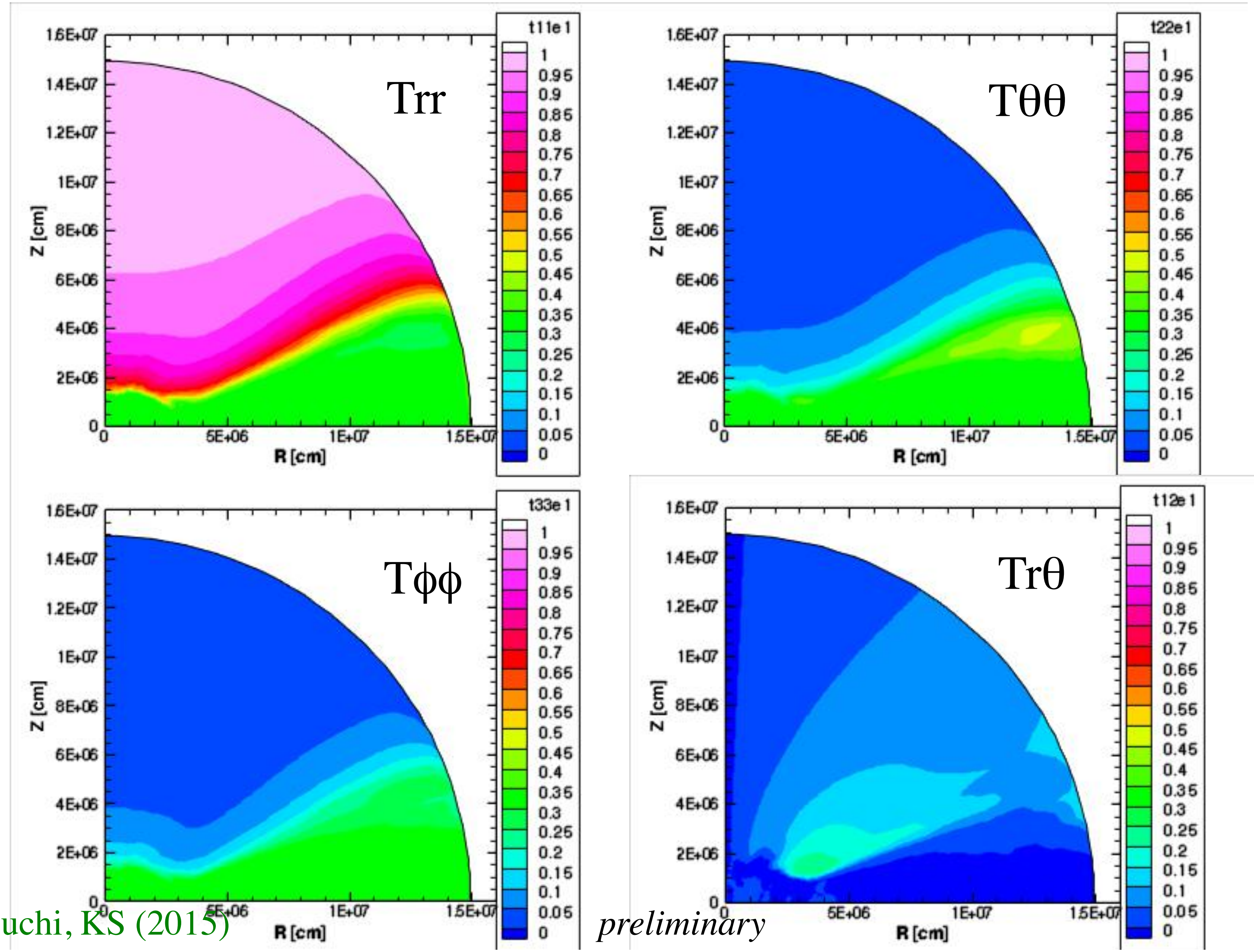
Eddington tensor by 6D Boltzmann



Sekiguchi, KS (2015)

preliminary

Eddington tensor by Closure relation

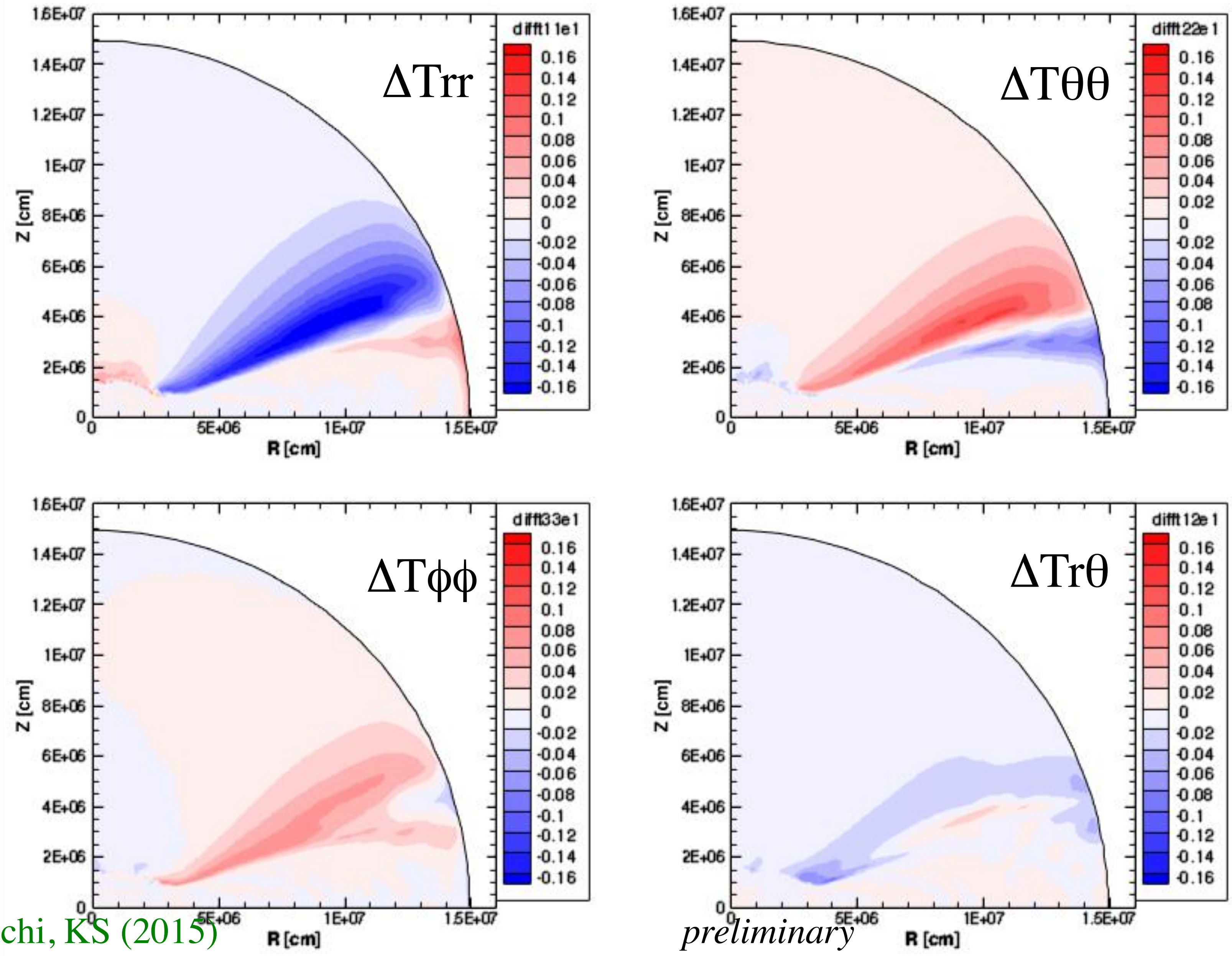


Sekiguchi, KS (2015)

preliminary

Deviation of Eddington Tensor

$$\Delta T^{ij} = T^{ij}_{Cl} - T^{ij}_{6D}$$



Sekiguchi, KS (2015)

preliminary

6D Boltzmann solver & Hydro

Applications to 2D core-collapse dynamics

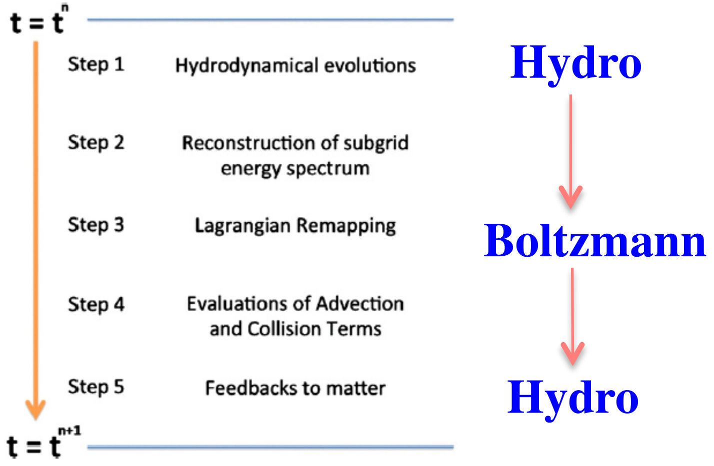
Nagakura, Iwakami (2015)

K-computer

Sp. Rel. Boltzmann & hydrodynamics in 2D

Nagakura et al. ApJS (2015) cf. Ott (2008)

- Coupling with Hydrodynamics and 2D Gravity Newtonian
  - Semi-implicit updates by multi-step
- Lorentz boosts in collision term
  - Energy shift & angle aberration
- Bruenn + GSI e-capture rates
  - e-scattering, pair processes
- Furusawa EOS
  - NSE multi-composition, TM1

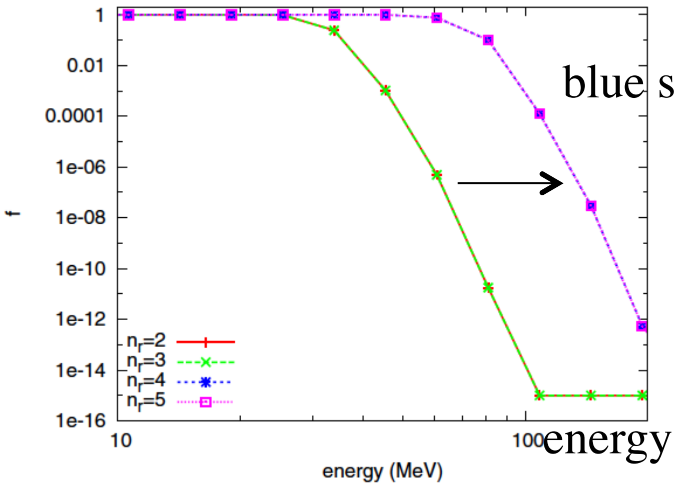


$t = t^n$

Step 1	Hydrodynamical evolutions	**Hydro**
Step 2	Reconstruction of subgrid energy spectrum	
Step 3	Lagrangian Remapping	**Boltzmann**
Step 4	Evaluations of Advection and Collision Terms	
Step 5	Feedbacks to matter	**Hydro**

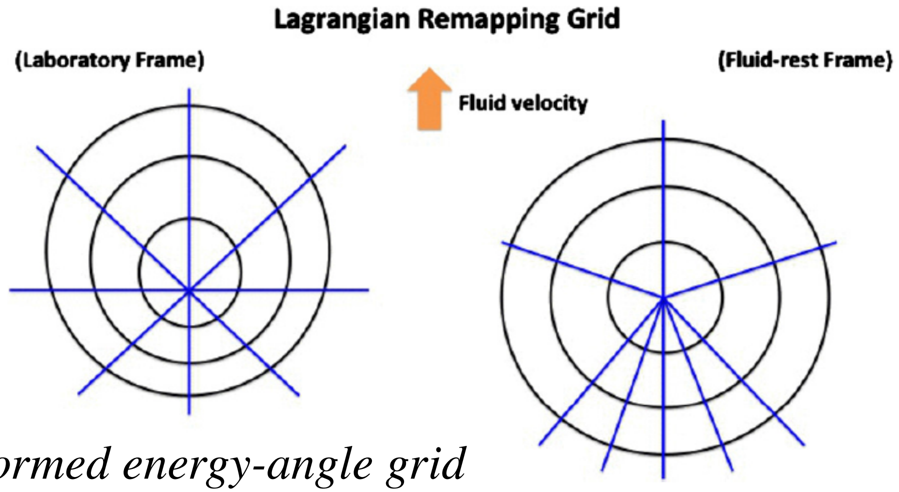
$t = t^{n+1}$

ν-distribution in the fluid rest frame



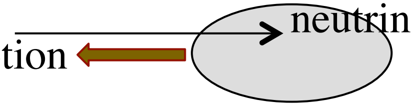
blue shifted

energy



deformed energy-angle grid

Lorentz transformation of neutrino distributions



matter motion

neutrino propagation

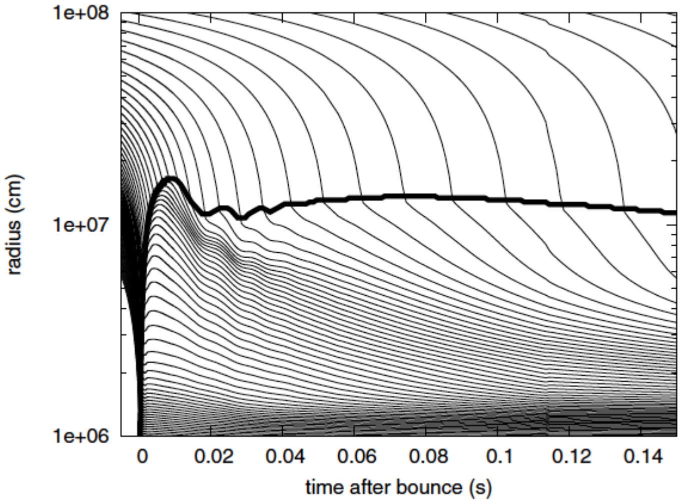
Applications of Boltzmann-Hydro Code

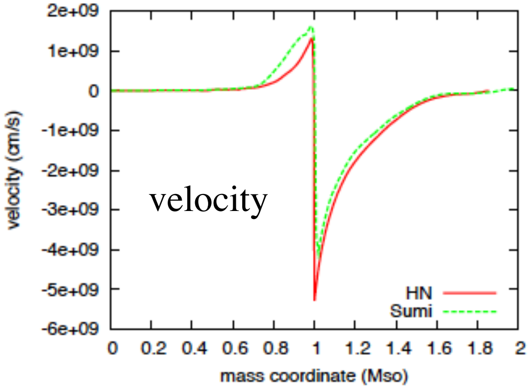
6D Boltzmann works in supernova dynamics

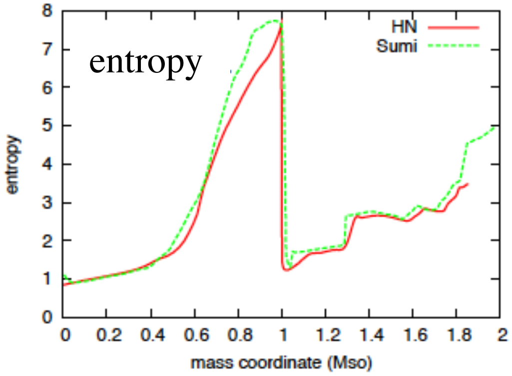

Nagakura


Iwakami

- 1D Collapse of $15M_{sun}$
  - Check with 1D GR code
  - Collapse, bounce and stall

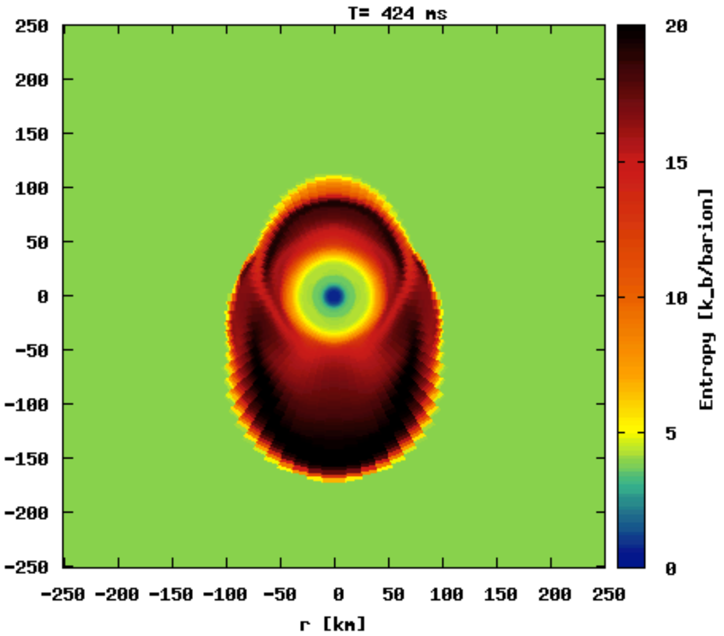






- 2D core collapse of $15M_{sun}$

entropy



  - Convection, SASI
  - Low resolution
  - ~ 400 ms

Simulation & Movie by Iwakami

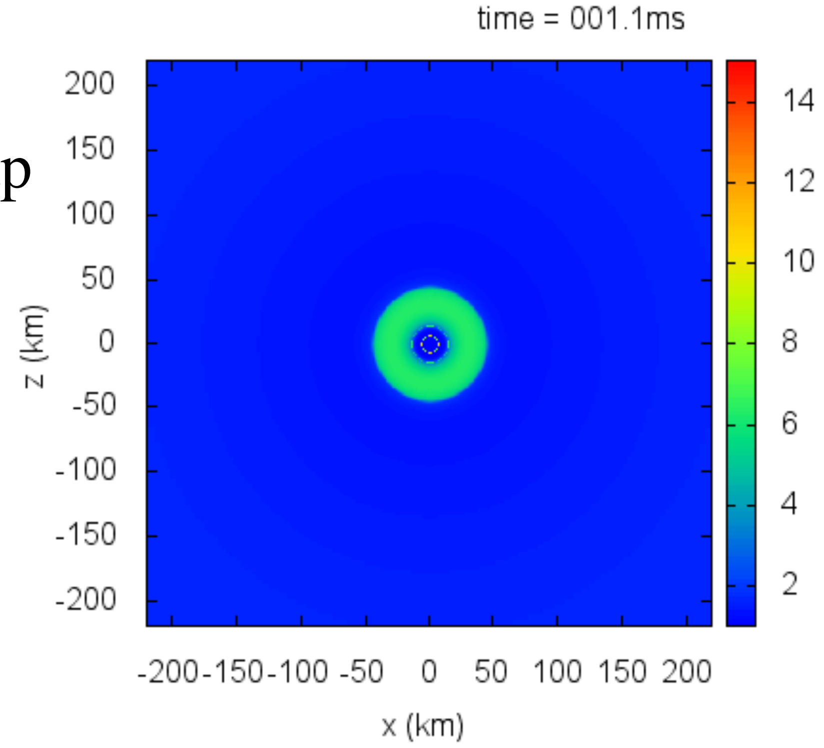
2D core-collapse by Boltzmann-Hydro Code

- Running at K-computer: Fe core, 11.2M_{sun}, 15M_{sun} stars

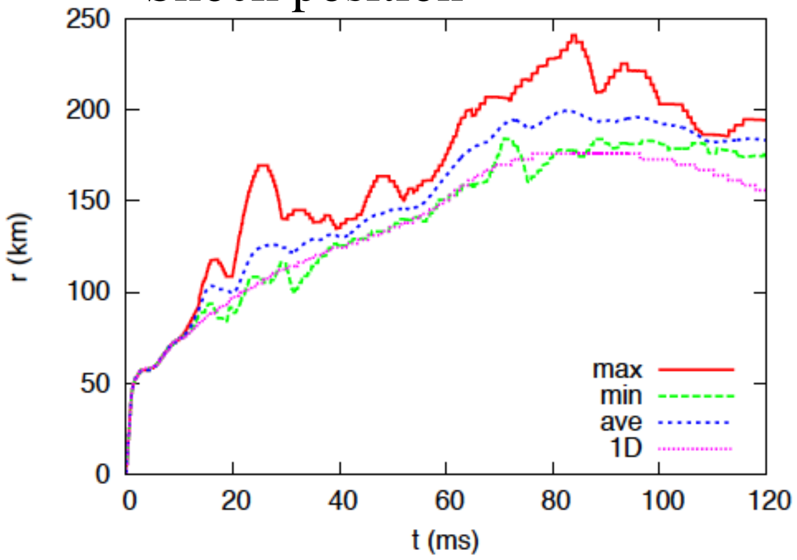
WHW02

Entropy color map

11.2M_{sun}



Shock position



Now at ~140ms after bounce

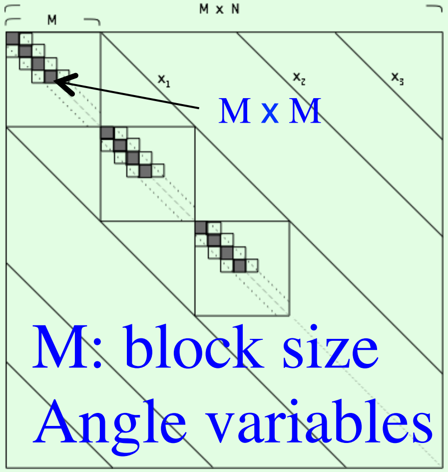
Stay tuned

- N_{space}=384x128, N_{ν}=20x10x6 @ K-computer
- 4M node-hour: 5x10^6 steps

Simulation & Movie by Nagakura

Necessary computational resources

- Matrix: memory $N_{space}\,N_e \times M^2$
  operation $N_{space}\,N_e \times M^3$
  Space: N_{space}, Neutrino: $N_\nu = N_e M$
  Parallel (MPI+OpenMP) by space r, θ, ϕ



M x M

M: block size

Angle variables

- Static: 6D Boltzmann + 3D background
  - N_{space}=256x64x32, N_ν=14x6x12 @ KEK Hitachi SR
- Dynamics: 5D Boltzmann + 2D Hydrodynamics
  - N_{space}=384x128, N_ν=20x10x6 @ K-computer
  - 5x10^6 steps to see if explodes in 0.5s

\- Need Exa-flops machines for full 6D+3D simulations

Sumiyoshi et al. ApJS (2011), Kotake et al. PTEP (2012)

Dawn of full ν-transfer in 3D supernovae

- From approximate to exact ν-transfer in 2D/3D
  - – pin down the uncertainty from ν-transfer
  - – establish the explosion mechanism as in 1D case

    Need to determine effects precisely around threshold
- New tools to solve 6D Boltzmann equation
  - – 3D non-radial transport, heating rates
    - Ray-by-ray method, Moment closure
  - – 2D supernova dynamics running: explosion?
    - Preparing 3D supernova dynamics *(full at Exa-scale)*
- Comparisons of methods is important
  - – 6D Boltzmann to examine approximations

Project in collaboration with

- Numerical simulations
  - H. Nagakura
  - W. Iwakami
  - S. Yamada
- Supernova research
  - T. Takiwaki
  - K. Kotake
  - Y. Sekiguchi
- Supercomputing
  - H. Matsufuru
  - A. Imakura
  - T. Sakurai
- EOS tables & neutrino rates
  - H. Shen, K. Oyamatsu, H. Toki
  - C. Ishizuka, A. Ohnishi
  - S. Furusawa, S. Nasu
  - S. X. Nakamura, T. Sato

Supported by PI M. Shibata

- *HPCI Strategic Program Field 5*

Supernovae is one of the target simulations of K-computer and Exa-scale machine

- *HPC resources at KEK, YITP, UT, RCNP*

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